Ministry of Higher Education and Scientific Research
Ahmed Zabana University of Relizane
Faculty of Science and Technology
Département of Civil Engineering and Public Works

Handout of:

Reinfoced Concrete Course

Level : - 3rd Year bachelor's in Public Works

- 3rd Year bachelor's in Civil Engineering

- 2rd Year Civil Engineering student

Auther : Dr. ZINE Abdallah

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Foreword

This document contains Reinforced Concrete courses for the third year of the Bachelor's degree in Public Works (3TP), third year of the Bachelor's degree in Civil Engineering (3GC), and second year of the Civil Engineering degree (2 ING).

It allows students to acquire knowledge about the physical and mechanical characteristics of reinforced concrete and to learn the design of sections subjected to simple loads (tension, compression, and simple bending) according to the BAEL 91 rules revised in 99 and CBA93. The goal of this booklet is to present the calculation methods for the basic elements of reinforced concrete constructions.

It should be noted that mastering the content of these courses requires prior knowledge of the concepts of strength of materials and construction materials.

This document is composed of six (06) chapters structured as follows:

- **Chapter 1:** Formulation and mechanical properties of reinforced concrete
- **Chapter 2:** Regulatory requirements
- **Chapter 3:** Adhesion and anchorage
- **Chapter 4:** Simple compression
- **Chapter 5:** Simple tension
- **Chapter 6**: Calculation of reinforced concrete sections subjected to simple bending

Chapter I

Formulation and Mechanical Properties of Reinforced Concrete

1 Concrete :

1.1 Definition :

Concrete is a mixture of a binder (cement) + water + aggregates (sand and gravel), proportioned so as to obtain a suitable consistency during placement and the required properties after hardening.

The properties that guide the study of concrete composition are:

a. Mechanical strength/resistance, mainly compressive strength and, for certain special uses, tensile strength.
b. Resistance to aggressive agents.
c. Instantaneous deformation.
d. Time-dependent (delayed) deformation.
e. Workability.

1.1.1 Aggregates :

Aggregates are a set of mineral grains known as fines, sand, fine gravel, or stones, depending on their size, which ranges between 0 and 80 mm (see Table 1.1).

For aggregates used in reinforced concrete, two main types are distinguished:

- Alluvial aggregates, known as rounded aggregates (shape acquired by erosion);
- Quarry aggregates with angular shapes (obtained by blasting and crushing).

Table I.1 Aggregate categories according to grain size

Designation	Fines	Sand	Fine Gravel	Pebbles and Crushed Stones
Categories following grain size in mm	< 0,080	Fine: 0,080 to 0,315 Medium: 0,315 to 1,25 Coarse:1,25 to 5	Small: 5 to 8 Medium: 8 to 12,5 Coarse:12,5 to 20	Small: 20 to 31,5 Medium: 31,5 to 50 Coarse: 50 to 80

1.1.2 Cement

Cement is a hydraulic binder, meaning that it is capable of setting and hardening in the presence of water. It appears as a very fine powder which, when mixed with water, forms a paste that sets and progressively hardens over time. Different types of cement and different strength classes are distinguished. (see tables 1.2 et1.3).

The choice of cement depends on the following criteria:

- High performance in the short-term, e.g. CEM I 52.5R or CEM III/A 52.5R;
- Concreting temperature:
  - Cold weather: CEM I 52.5 or CEM I 42.5;
  - Hot weather: CEM III/C 32.5;
- Presence of sulfates, e.g. CPJ CEM II/B-S 42.5N-ES.

Table I.2 Different types of common cement

Types of Cement	Designations
Portland cement	CPA-CEM I
Portland composite cement	CPJ-CEM II/A
	CPJ-CEM II/B
Blast furnace cement	CHF-CEM III/A
	CHF-CEM III/B
	CLK-CEM III/C
Pozzolanic cement	CPZ-CEM IV/A
	CPZ-CEM IV/B
Slag and ash cement	CLC-CEM V/A
	CLC-CEM V/B

Tableau I.3 Different classes of common cements

	Compressive Strength (MPa) (EN 196-1 standard/norm)			
	At a young age		At 28 days	
	2 days	7 days	Minimum	Maximum
32,5		(17,5)	≥ 32,5	≤ 52,5
32,5 R	≥ 10		≥ 32,5	≤ 52,5
42,5	≥ 10		≥ 42,5	≤ 62,5
42,5 R	≥ 20		≥ 42,5	≤ 62,5
52,5	≥ 20		≥ 52,5	
52,5 R	≥ 30		≥ 52,5	
R : early onset of rapid hardening				

1.2 Mecanical Caracteristics of Concrete :

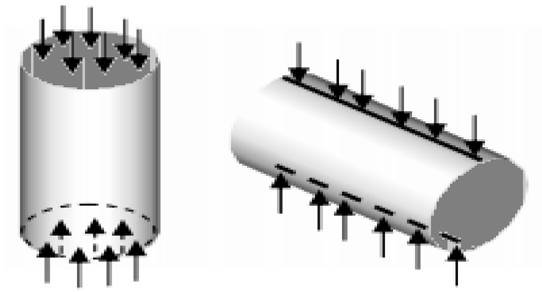
In most structures, some parts are subjected to compressive stresses and others to tensile stresses. Concrete is a material that resists compression very well but tension very poorly, whereas steel resists tension very well. This is why steel reinforcement bars are placed in areas where tensile forces occur, oriented in the direction of these forces. Microcracks may appear in the concrete in these areas under the effect of tensile stresses, but the steel will prevent the cracks from widening and will absorb the tensile forces on their own.

Concrete is therefore characterized by:

- High compressive strength, denoted f_{cj}.
- Low tensile strength, denoted f_{tj}.

1.2.1 laboratory tests:

Experimentally, compressive strength is generally measured on cylindrical specimens with a diameter of 16 cm and a height of 32 cm, or on cubic specimens. Tensile strength is obtained either by the splitting tensile test (Brazilian test) or by a flexural test on prismatic specimens.



a) Compression test b) Tensile test

Fig.I.1 Compression and tensile tests on specimens 16x32 cm²

1.2.2 Characteristic Compressive Strength:

For design purposes in common cases, concrete is defined by its characteristic compressive strength at 28 days. This strength is denoted fc28, where:

- f: strength
- c: compression
- 28: age in days

The characteristic compressive strength fcj is determined after a series of crushing tests on standardized cylindrical or cubic specimens. It is expressed in MPa. (1 MPa = 1 N/mm²)

This strength varies with the age of the concrete, and the regulation provide laws describing the evolution of fcj (compressive strength at j days) as a function of the concrete age j (in days).

$J \leq 28$ days	$fc28 \leq 40$ MPa	$fcj = j.\ fc28/(4{,}76+0{,}83j)$
	$fc28 > 40$ MPa	$fcj = j.fc28/(1{,}40+0{,}95j)$
$J > 28$ days	$fcj = fc28$	for strength calculations.
	$fcj = 1{,}1.fc28$	for deformation calculations.

1.2.3 Characteristic Tensile Strength:

The tensile strength of concrete at age *j* days, denoted f_{tj} and expressed in MPa, is conventionally defined by:

$$f_{tj} = 0{,}6 + 0{,}06.\ f_{cj} \quad \text{in MPa}$$

Example : for fcj = 25 MPa ; f_{tj} = 0,6 +0,06x25 = 2.1 MPa

***1.2.4* Deformations and Longitudinal Moduli of deformation of Concrete :**

Young's modulus takes two values depending on whether instantaneous deformations (Eij) or long-term deformations (time-dependent or delayed deformations) (Evj) are considered.

a- Instantaneous deformation:

The longitudinal deformation modulus is obtained from Hooke's law. Under normal stresse applied for a duration of less than 24 hours (≤ 24h), an instantaneous longitudinal deformation modulus of concrete Eij is considered.

$$E_{ij} = 11000\sqrt[3]{f_{cj}}$$ in MPa According to BAEL 91 rules

b- Time-dependent deformation

A concrete element subjected to compression undergoes an instantaneous deformation immediately upon application of the load. If the load remains applied, this deformation will continue to increase due to creep (deformation over time under constant load) and will be up to three times greater than the instantaneous deformation.

The delayed deformation modulus due to the delayed deformations of concrete, associated with long-duration loads (more than 24 hours), is:

$$E_{vj} = 3700\sqrt[3]{f_{cj}}$$

According to BAEL 91 [1] rules

1.2.5 Poisson's Ratio:

It is accepted that Poisson's ratio, related to the elastic deformations of uncracked concrete under the serviceability limit state (SLS), is equal to $\nu=0.2$ for deformation calculations.
For cracked concrete under the ultimate limit state (ULS), Poisson's ratio is taken as $\nu=0$ for stress calculations.

ν = transverse strain/ longitudinal strain = $\varepsilon' / \varepsilon$

1.2.6 Density:

- Normal-weight concrete: 2200 – 2400 kg/m³
- Lightweight aggregate concrete: 700 – 1500 kg/m³
- Heavyweight aggregate concrete: 3500 – 4000 kg/m³
- Reinforced concrete: approximately 2500 kg/m³

2. Steel for Reinforced Concrete:

2.1 Manufacturing Processes:

Steel is an alloy of iron and carbon with a low carbon content. The steels used in reinforced concrete are mild, medium hard, and hard (carbon steel) steels.

The types of steel used in the construction of reinforced concrete elements are:

- Smooth bars or plain round bars (symbol Ø)
- Deformed or ribbed bars (High-bond/adhesion steel) (HA)
- Welded wire mesh (WWM)



(a) (b) (c)

Fig. I.2 Steels used in reinforced concrete: a. Smooth round steel, b. High-adhesion steel, c. Welded wire mesh

- For reinforced concrete, steel is available in three form: bars, wires, or welded meshes (Fig. 1.3).
- Standard bar length is generally 12 m for straight bars.



(a) (b) (c)

Fig. I.3 Fig. I.3 Forms of steel for reinforced concrete on the market: a. bars, b. wire in coils, c. welded wire mesh

2.2 Guaranteed Yield Strength of Reinforcing steels:

Reinforcing steels used in reinforced concrete consist of different grades, distinguished by their guaranteed yield strength and by their surface condition (plain or deformed).

Table 1.4 Guaranteed yield strength of steel for reinforced concrete.

Plain bars	**Ø**	
Grade FeE	2	235
Yield strength (MPa)	2	235
Deformed bars	**HA**	
Grade FeE	4	500
Yield strength (MPa)	4	500
Welded wire mesh	**WWM**	
Grade FeE	500	
Yield strength (MPa)	500	

- The standard diameters of common reinforcing bars are : 6, 8, 10, 12, 14, 16, 20, 25, 32, and 40 mm.
- A nominal diameter corresponds to a nominal cross-sectional area and a nominal perimeter (cross-sectional area and perimeter of a smooth rod with a diameter equal to the nominal diameter).

Table 1.5 gives the nominal cross-sectional area and linear mass corresponding to the different nominal diameters.

Table I.5 Table of steel sections

		Total steel cross-sections in cm²									
Diameters	Masse kg/m	1	2	3	4	5	6	7	8	9	10
6	0,222	0,28	0,57	0,85	1,13	1,41	1,70	1,98	2,26	2,54	2,83
8	0,395	0,50	1,01	1,51	2,01	2,51	3,02	3,52	4,02	4,52	5,03
10	0,617	0,79	1,57	2,36	3,14	3,93	4,71	5,50	6,28	7,07	7,85
12	0,888	1,13	2,26	3,39	4,52	5,65	6,79	7,92	9,05	10,18	11,31
14	1,210	1,54	3,08	4,62	6,16	7,70	9,24	10,78	12,31	13,85	15,39
16	1,580	2,01	4,02	6,03	8,04	10,05	12,06	14,07	16,08	18,10	20,11
20	2,466	3,14	6,28	9,42	12,57	15,71	18,85	21,99	25,13	28,27	31,42
25	3,850	4,91	9,82	14,73	19,63	24,54	29,45	34,36	39,27	44,18	49,09
32	6,313	8,04	16,08	24,13	32,17	40,21	48,25	56,30	64,34	72,38	80,42
40	9,864	12,57	25,13	37,70	50,26	62,83	75,40	87,96	100,53	113,09	125,66

Example : The total cross-section of 6 HA12 is 6.79 cm2.

2.3 Mechanical Caracteristics of Steel:

The characteristics of steel are determined after a series of direct tensile tests.

These characteristics apply to steel bars, welded wire mesh, and wire supplied in coils.

The mechanical characteristics used as the basis for calculations of reinforced concrete elements are:

2.3.1 Modulus of Elasticity of Steel :

Based on a tensile test, the modulus of elasticity of the steel is taken to be:

$E_s = 200\ 000\ MPa = 2\text{x}10^5 MPa.$

This value is constant.

2.3.2 Bond (adhesion) Characteristics:

a- Cracking coefficient: « η »

- $\eta = 1$ for plain bars
- $\eta = 1,3$ for HA with $Ø \leq 6mm$
- $\eta = 1,6$ for HA with $Ø > 6mm$

b- Anchorage coefficient: « ψ »

- $\eta = 1$ for plain bars
- $\eta = 1,5$ for HA

Chapter II
Regulatory Provisions

1. Definition of Limit States :

A limit state is a state in which a required condition of a construction (or one of its elements) is strictly satisfied and would no longer be satisfied in the event of an unfavorable variation of one of the applied actions.

2.Ultimate Limit State (ULS) and Serviceability Limit State (SLS) :

The limit state design theory considers two main limit states.

2.1. Ultimate Limit State of Resistance (ULS) :

Exceeding this limit state leads to failure of the structure. Beyond the ultimate limit state, the strength (resistance) of concrete and steel is reached, safety is no longer ensured, and the structure may collapse. This state is characterized by:

- The limit state of static equilibrium.
- The limit state of strength of one of the materials.
- The limit state of shape stability (buckling).

2.2. Serviceability Limit State (SLS)

At this limit state, the conditions for proper functioning of the structure are reached and its durability may be compromised. This state is characterized by:

- Crack opening limit state: risk of crack opening (admissible stresses).
- Concrete compression limit state: the compressive stress is intentionally limited to a reasonable value.
- Deformation limit state: maximum deflection.

When the serviceability limit state is reached, the structure's fitness for service is affected (cracking, leakage, various defects). However, its safety (strength) is not compromised.

3. Characteristic Actions

These are the different actions to which the structure is subjected. They are classified into three categories according to their frequency of occurrence.
Limit state mainly distinguishes three types of characteristic actions :

- Permanent actions.
- Variable actions.
- Accidental actions.

The values assigned to these actions are characteristic values: that is, they take into account the random nature of actions, meaning that it is not possible to precisely determine the value of any given action. These values are therefore derived from a probabilistic calculation and accept risk in only 5% or 10% of cases.
The actual value of these actions may exceed (unfavorable cases) the selected characteristic value.

3.1. Permanent Actions (Gi)

Permanent actions have a constant or very slight variation in intensity over time and are denoted by the letter G. They include:

- Self-weight of the structure : calculated from on the dimensions shown on the excution drawings, with the density of reinforced concrete taken as 2.5 t/m³.
- Loads from the superstructure and fixed equipment.
- Forces due to earth pressure or liquids with a relatively constant level.
- Deformations imposed during construction (prestressing).

3.2. Variable Actions (Qi)

Variable actions have an intensity that varies frequently and significantly over time. They are designated by the letter Q. They include:

- Operating loads (ratio of users, vehicles, etc.) classified by duration of application (temporary, long-term)
- Climatic loads (snow and wind)
- Thermal effects

3.3. Accidental Actions (Fa)

These are short-duration actions such as earthquakes, shocks, and fire etc. They are considered only when required by public regulations or contractual documents.

4. Design Situations, Combinations, and Internal Actions
4.1. Design Situations :

Limit states primarily distinguish two types of situations :

a- Persistent situation:

The duration of application is of the same order as the service life of the structure.

b- Transient situation:

The duration of application is much shorter than the service life of the structure. These include:

- Transient situations.
- Accidental situations.

4.2. Design Combinations :

For a given situation, the most unfavorable combinations of actions must be considered, since ensuring safety for these combinations guarantees safety for all others.

Three types of combinations are considered:

- Fundamental combinations (ULS)
- Accidental combinations (ULS)
- Rare combinations (SLS)

4.3. Design Internal Actions

For a given load combination, design actions are calculated, namely bending moment, torsional moment, axial or normal force, and shear force, using strength of materials methods or other appropriate methods.

4.3.1 Design Internal Actions for ULS (Resistance and Stability):

a- Fundamental combination (FC)

$$1.35G_{max} + G_{\min} + \gamma_{Qc} Q_c + \Sigma 1.3 \psi_{0i} Q_i$$

Where :

- G_{max} : sum of unfavorable permanent actions.
- G_{min} : sum of favorable permanent actions.
- Q_c : basic variable action.
- Q_i : other variable actions, known as accompanying actions.

b- Accidental combination (AC):

$$G_{max} + G_{\min} + F_A + \psi_{1i} Q_c + \Sigma \psi_{2i} Q_i$$

Where :

- F_A : nominal value of the accidental action.

4.3.2 Design Internal Actions for Serviceability Limit States (SLS):

They result from the following rare combination :

$$G_{max} + G_{\min} + Q_1 + \Sigma\psi_{0i} Q_i$$

Table II.1. Coefficients Ψ_{0i} , Ψ_{1i} Ψ_{2i} according to BAEL rules.

Type of loads		Ψ_{0i}	Ψ_{11}	Ψ_{2i}
Operating loads	Archives	0.90	0.90	0.80
	Parking lots	0.90	0.75	0.65
	Meeting rooms - Seated - Standing	 0.77 0.77	 0.65 0.75	 0.4 0.25
	Exhibition halls – Various halls	0.77	0.65	0.25
	Other premises	0.77	0.75	0.65
Climatic loads	Wind (W)	0.77	0.20	0
	Snow (Sn) - altitude ≤ à 500 m - altitude > 500m	0.77 0.77	0.15 0.30	0 0.1
	Temperature variation	0.6	0.5	0

5. Ultimate Limit State (ULS):

5.1. Fundamental Design Criteria /Assumptions / Hypotheses

H1: Conservation of plane sections: (Navier hypothesis: small deformations). Cross-sections remain plane after deformation

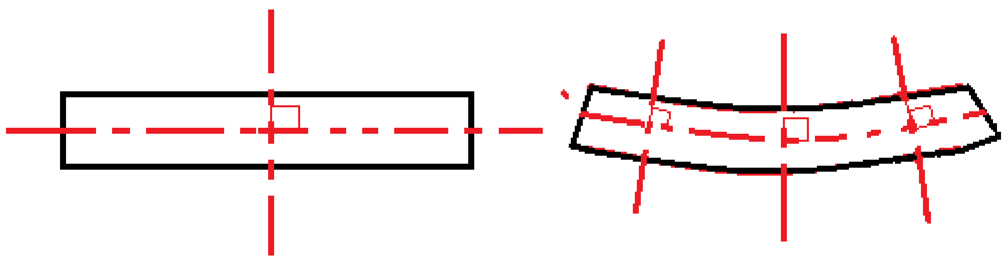


Fig II.1 Conservation of plane sections.

H2: Tensile strength of concrete is neglected due to cracking.

H3: Steel-concrete deformation compatibility : there is no relative sliding/slipping between steel reinforcement and concrete. $\varepsilon_b = \varepsilon_a$.

H4: The ultimate shortening of the concrete is limited to:

$\varepsilon_{bu}=3.5‰$ *in simple bending,* $\varepsilon_{bu}=2‰$ *in simple compression*

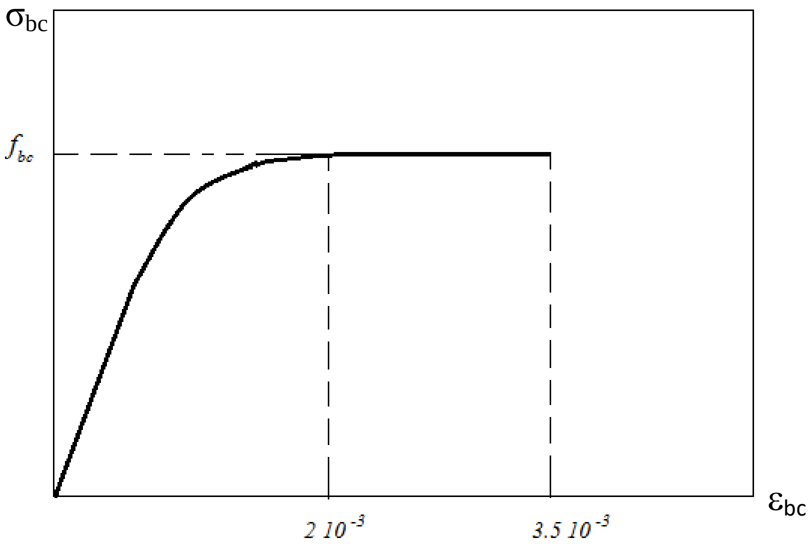


Fig II.2 Stress-strain diagram of concrete in simple compression

H5: The ultimate elongation of steel is limited to: $\varepsilon_{au}=10‰$ in tension and compression.

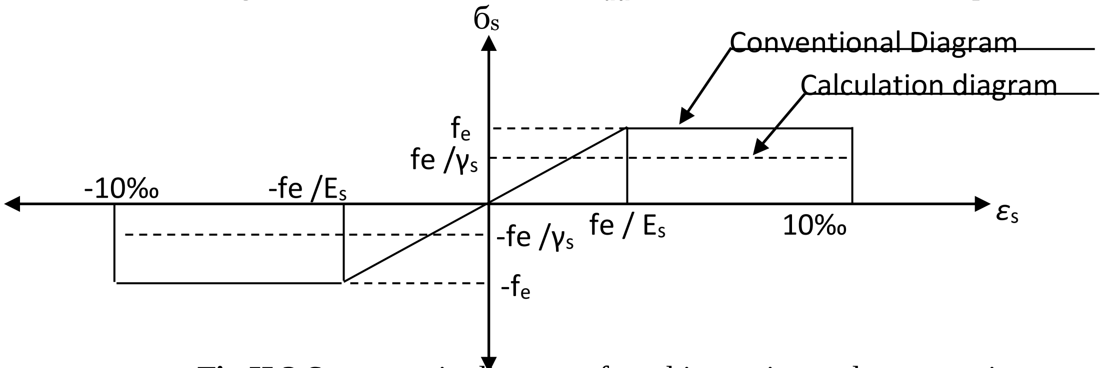


Fig II.3 Stress-strain diagram of steel in tension and compression

H6: Three-pivot diagram.

5.2. Three-Pivot Rule

The possible positions of the strain diagram of a cross-section necessarily pass through one of the three pivots A, B, or C (figure 2.4).

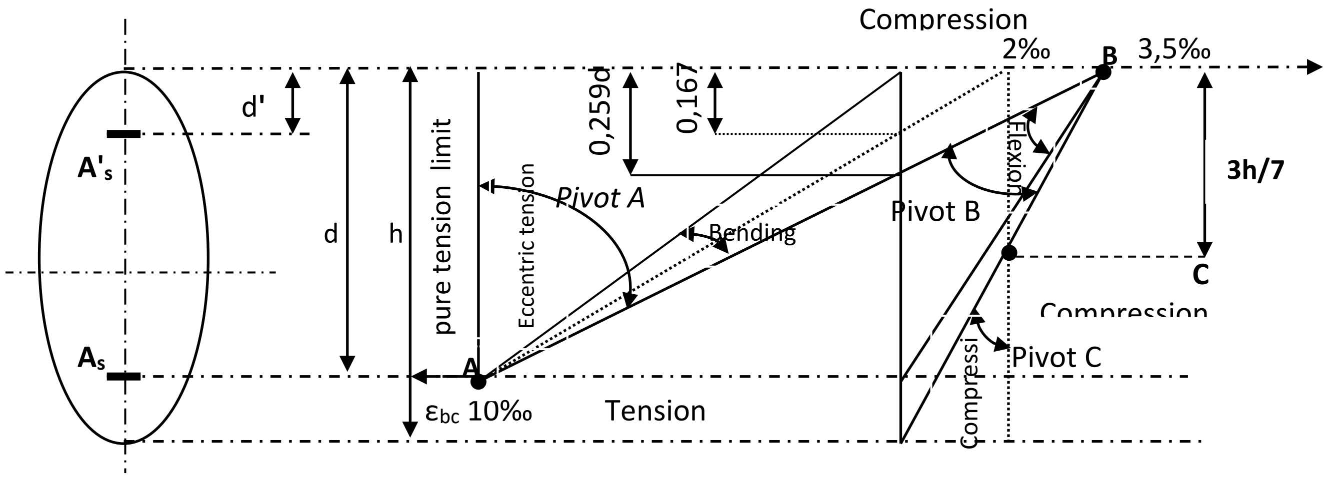


Fig II.4. Three-Pivot Diagram.

The problem consists in determining the limit positions of the strain diagram of a section such that none of the ultimate strain limits is exceeded. The section being subjected to the ultimate limit state, according to the different types of normal loading, namely: pure tension, eccentric tension, simple bending, combined bending, and pure compression.

- **Pivot A :**

Region 1 : Characterized by an ultimate elongation of the steel located on the most tensioned side, equal to 10‰.

Region 1a : Characterized by:

- Fully tensioned section
- Neutral axis outside the section
- Pure tension or tension combined with bending

Région 1b : Characterized by:

- Partially compressed section (partially tensioned)
- Neutral axis inside the section
- Simple or combined bending

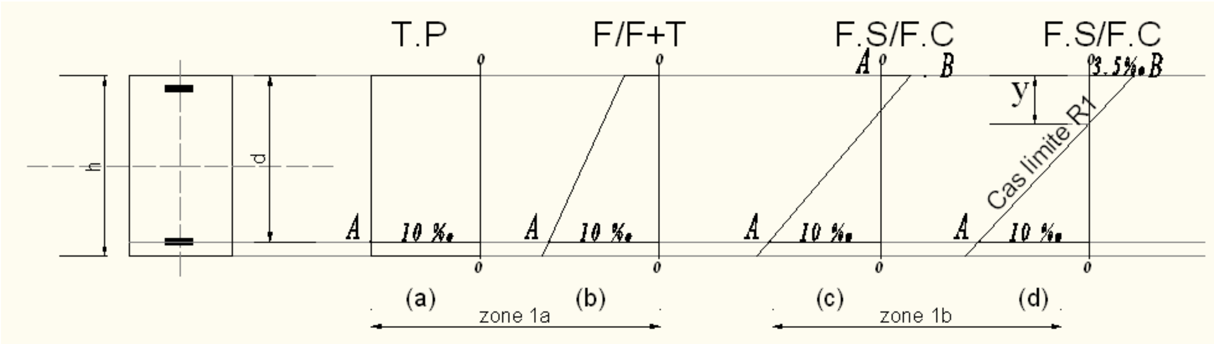


Fig II.5. Pivot A – limit strains.

Region1a :

a) Simple (pure) tension: all elongations are equal to 10‰.

b) Fully tensioned section (tensile force with small eccentricity).

Region1b :

c) Simple or combined bending. Concrete does not reach its ultimate shortening of 3.5‰.

d) Simple or combined bending. Concrete reaches its ultimate shortening of 3.5‰.
This is a limit case for Region 1 (Pivot A).

- **Definition of the limit cases for Region 1 and Region 2 (Pivot A and Pivot B) :**

From similar triangles, we obtain:

$$\frac{3,5 10^{-3}}{10 10^{-3}} = \frac{y}{d-y}$$

$$\rightarrow 13,5y - 3,5d = 0$$

$$\rightarrow y = 0,2593$$

- if $y \leq 0.2593$ d → Pivot A (Region 1)
- if $y > 0.2593$ d → Pivot B (Region 2)

➢ **Pivot B**

Region 2 : Characterized by a shortening of an ultimate compressive concrete equal to 3.5‰.

- The section is partially compressed
- The neutral axis is inside the section
- Simple or combined bending

Region 2a : The elongation of the steel is between 10 ‰ and ε_{sl} (value corresponding to the strength of the considered steel).

Region 2b : The elongation of the steel is between ε_{sl} and 0.

Region 2c : All reinforcement bars are subjected to a shortening. A small part of the concrete section remains in tension.

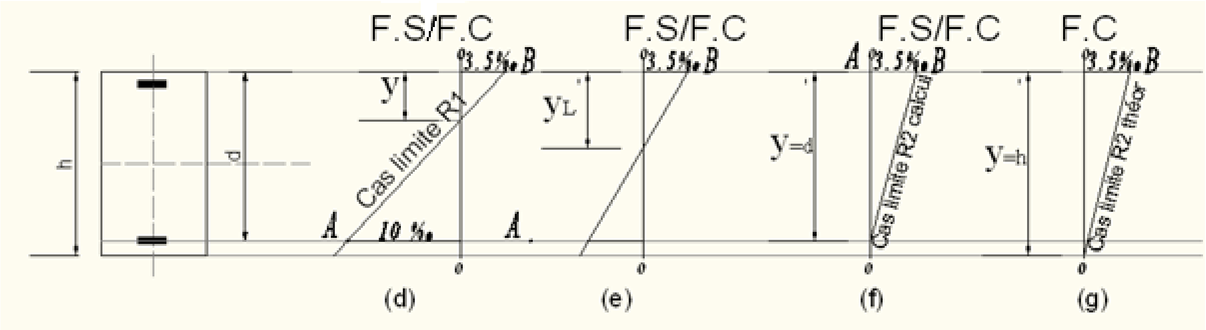


Fig II.6. Pivot B – limit strains.

d : Same as for Pivot A (Region 1).

e : Simple or combined bending when the concrete reaches its ultimate shortening and the steel elongation is less than $\varepsilon_s = \varepsilon_{sl}$

f : Simple or combined bending, with zero elongation in the steel (zero stresses in reinforcement).

g : Combined bending with compressive force when the concrete reaches its ultimate shortening of 3.5‰ and the most tensioned fiber has zero shortening.
This is a limit case for Pivot B (Region 2).

- **Definition of the limit cases for Region 2 and Region 3 (Pivot B and Pivot C)**

if $y \leq d$ → Pivot B (Region 2)
if $y > d$ → Pivot C (Region 3)

- **Pivot C**

Region 3 : Characterized by an ultimate concrete shortening equal to 2‰.

- The section is fully compressed
- The neutral axis is outside the section
- Bending with compression or pure compression

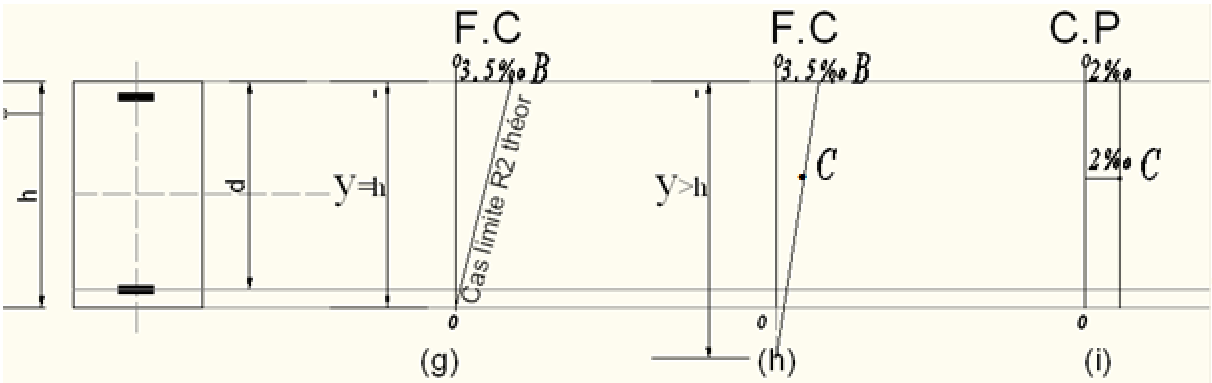


Fig II.7. Pivot C – limit strains.

g : Same as for Pivot B (Region 2).

h : Combined bending with compressive force. Neutral axis outside the section, $\varepsilon_{bc} < \varepsilon_{bcu}$=3.5‰.

i : Pure compression simple (ε_{bc}=2‰ over the entire section height h).

For cases h and i, the strain lines pass through point C (y ≥ h).

5.3. Concrete Stress–Strain Diagrams

5.3.1 Characteristic Diagram

The characteristic diagram of a concrete represents the relationship between strain and stress of the concrete when it is subjected to uniaxial stress. The diagram generally has the form shown below (Figure II.8). It makes it possible to define the secant longitudinal modulus of elasticity.

5.3.2 Idealized Diagram (for section calculation/design): Parabola-rectangle

This diagram is a mathematical model of the characteristic diagram. It consists of a parabolic arc and a straight line segment. This diagram allows for numerous definitions and simples formulations.

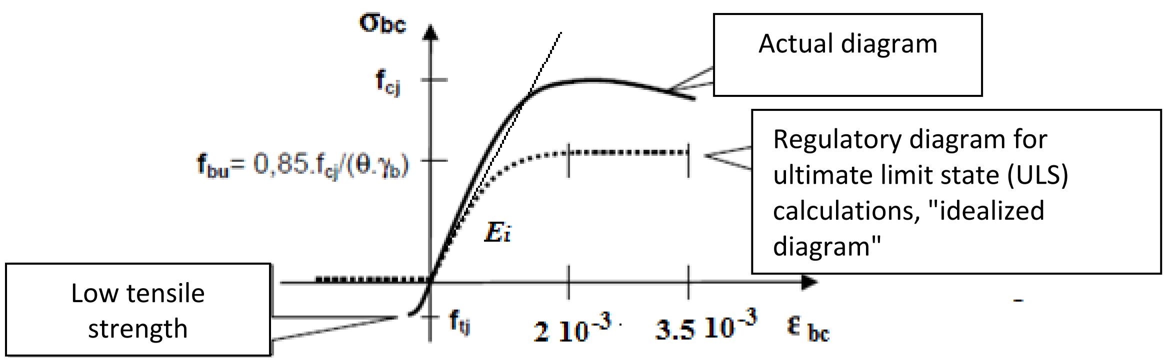


Fig II.8. Idealized concrete stress–strain diagram in compression.

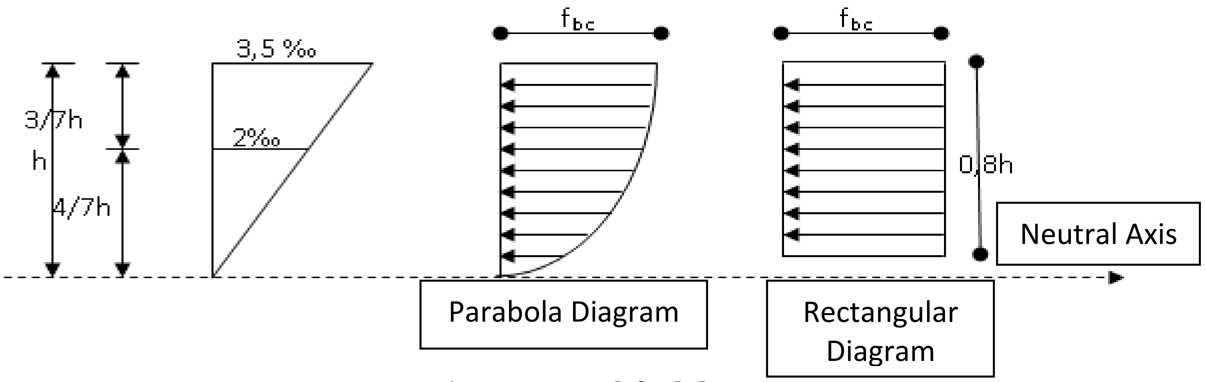


Fig.II.8. Simplified diagram

The ultimate compressive strength of concrete at the ultimate limit state σ_{bcu}:

- For sections of constant or increasing width toward the most compressed fiber:

$$\sigma_{bcu} = \frac{0,85.f_{cj}}{\theta.\gamma_b}$$

- For sections of decreasing width toward the most compressed fiber:

$$\sigma_{bcu} = \frac{0,80.f_{cj}}{\theta.\gamma_b}$$

With :

$\theta = 1$ for loads applied for more than 24 hours

$\theta = 0.9$ for loads applied between 1 hour and 24 hours

$\theta = 0.85$ for loads applied for less than 1 hour

γ_b : safety factor:

- $\gamma_b = 1.5$ for persistent or transient situations
- $\gamma_b = 1.15$ for accidental situations

NOTE :
When the section is not fully compressed (i.e., region 1 (A) or region 2 (B)), a simplified rectangular equivalent diagram can be used in the calculations, defined as follows:

- Over a distance of **0.2y** from the neutral axis, the concrete stress is zero.
- Over the remaining distance of **0.8y**, the concrete stress remains.

$$\sigma_{bcu} = \frac{0,85.f_{cj}}{\theta.\gamma_b}$$

5.4. Steel Stress–Strain Diagram

5.3.1 Characteristic Diagram

The characteristic diagram of steel represents the relationship between strain and stress of the steel when it is subjected to a uniaxial tensile (or compressive) stress. The diagram generally has the form shown below. It allows us to define the longitudinal modulus of elasticity (linear elastic portion).

5.3.2 Idealized Diagram (for section calculation/design) :

This diagram is a mathematical model of the characteristic diagram.

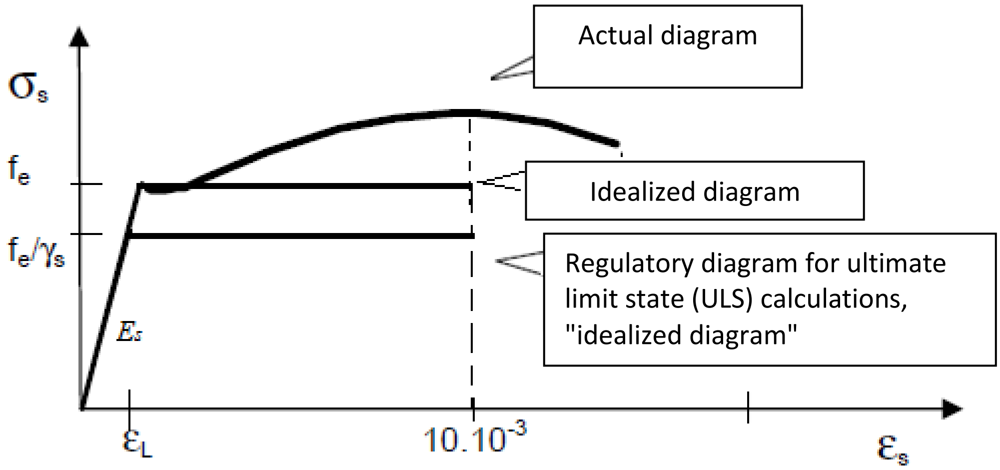


Fig II.9. Idealized steel stress–strain diagram in tension

The ultimate tensile strength of steel at the ultimate limit state σ_S is:

if : $\varepsilon_{sl} \leq \varepsilon_s \leq 10\ 10^{-3} \implies \sigma_s = \dfrac{f_e}{\gamma s}$

if $0 \leq \varepsilon_s < \varepsilon_{sl} \implies \sigma s = \varepsilon_s E_s$

With :

γ_s : safety factor:

- γ_s = 1.15 for persistent or transient situations.
- γ_s =1 for accidental situations.

6. Limit straints in the Serviceability Limit State (SLS) with regard to the durability of the structure:

The verifications to be carried out relate to the limit state of concrete compression and the limit state of crack opening.

6.1. Design (or calculation) Criteria /Assumptions / Hypotheses

Calculations are carried out under the following assumptions, with internal forces obtained from the serviceability limit state load combinations:

- H1 : Cross-sections remain plane after deformation..
- H2 : There is no relative slip between concrete and reinforcement..
- H3 : Steel and concrete are considered as linear elastic materials : shrinkage and creep of concrete are neglected.
- H4 : The tensile strength of the concrete is neglected in the calculation of reinforcement..
- H5 : By convention, the ratio between the longitudinal elasticity coefficients of steel and concrete, or equivalence coefficient, is taken to be equal to : $n=E_s/E_b=15$.
- H6 : The area of reinforcement is not deducted from the compressed concrete area.

6.2. Concrete Compression Limit State

The admissible compressive stress in the concrete must be at most equal to: $\overline{\sigma_{bc}} = 0,6 f_{cj}$

6.3 Crack Opening Limit State

- Three types of cracking (fissuring) are distinguished.

a- Slightly harmful cracking:

Cracking is considered slightly harmful if the element under study is protected (located in an enclosed and covered area).

- Example: the interior elements of a building.

In this case, verification of steel stress limit is not required.

b- Harmful cracking:

Cracking is considered harmful if the element under study is exposed to weather conditions (located in a place exposed to external climatic conditions).

- Example: the exterior elements of a building

In this case, the steel stress limit is :

$$\overline{\sigma_s} = \min\left(\frac{2}{3} f_e ; 110\sqrt{\eta\, f_{tj}}\right)$$

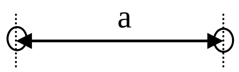
η : Cracking coefficient:

- $\eta = 1.0$ for plain bars R.L
- $\eta = 1.6$ for deformed bars H.A

- The minimum diameter of the transverse reinforcement for this cracking is 6mm

$Ø_t \geq 6$ mm

- The distance between two neighboring bars « a » : if Ø > 20mm ⟹ a ≤ 4 Ø



c- Very harmful cracking:

Cracking is considered very harmful when structural elements are exposed to an aggressive environment or must ensure watertightness.

- Example: structure at the seaside, aggressive soil, etc.

In this case, the steel stress limit is :

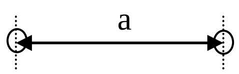
$$\overline{\sigma_s} = \min\left(\frac{1}{2} f_e ; 90\sqrt{\eta\, f_{tj}}\right)$$

η : Cracking coefficient:

- $\eta = 1.0$ for plain bars R.L
- $\eta = 1.6$ for deformed bars H.A
- The minimum diameter of the transverse reinforcement for this cracking is 8mm

$Ø_t \geq 8$ mm

- The distance between two neighboring bars « a » : if Ø > 20mm ⟹ a ≤ 3 Ø



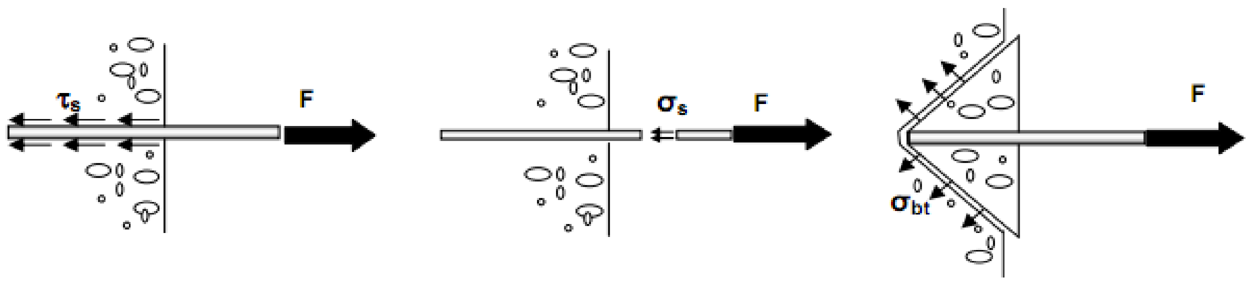
Chapter III
Bond

1 Introduction

Reinforced concrete results from an effective combination of materials with complementary characteristics: steel, for its ability to resist tensile stresses, and concrete, for its ability to resist compressive stresses. Concrete itself is a carefully formulated mixture of aggregates, cement, and water. Cement and steel, on the other hand, are produced through specific manufacturing processes.

Bond (Adhesion) is a phenomenon of tangential interaction at the steel–concrete interface due to friction and the strut action of the concrete. The rules to be followed relate to the Ultimate Limit State.

Consider a reinforcing bar anchored (or embedded) in a concrete mass/block. If a pull-out force is applied along the axis of the bar, three possible failure modes may occur:



Relative slipping/sliding of steel with respect to concrete (extraction of the bar within a concrete sleeve)

Failure of the steel in tension (perfect anchorage)

Concrete failure by pull-out of a concrete cone

Fig. III.1 : Pull-out test of a reinforcing bar embedded in a concrete block.

The steel–concrete bond is influenced by several factors, namely:

- Surface condition of the bars (plain or deformed);
- Shape of the bars;
- Grouping of reinforcement bars (bundling of reinforcement) ;
- Concrete strength;
- Transverse compression (clamping effect);
- Thickness of concrete.

In a reinforced concrete member, two types of reinforcement are distinguished:

- Longitudinal reinforcement (A_l) : Provided to resist axial forces (tension–compression), bending moments, torsion, and their combinations.

- Transverse reinforcement (A_t) : Provided to resist transverse forces, mainly shear forces.

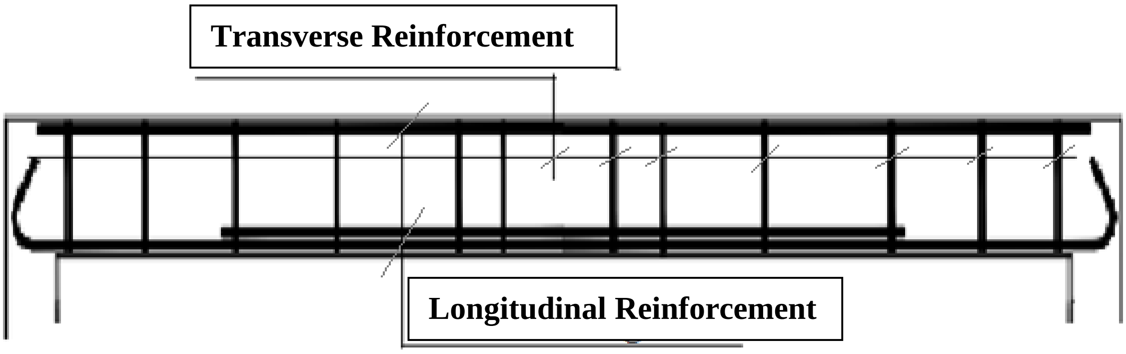


Fig III.2 Types of reinforcement.

2 Anchorage of Reinforcement Bars

Reinforcing bars must be properly anchored in concrete.

2.1. Straight Anchorage

A bar is considered to be properly anchored when the tensile force applied to it is entirely balanced by the bond stresses between steel and concrete within the anchorage zone.

The straight anchorage length l_S is defined as the length of a bar of diameter Ø capable of equilibrating, through a bond stress τ_{SU}, the force causing in this bar a tensile stress equal to the elastic limit (yield strength) of the steel fe (Fig. III.3).

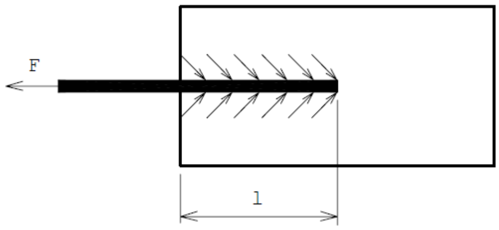


Fig. III.3 Straight anchorage

$$F = A.f_e = \frac{f_e.\pi.\phi^4}{4}$$

D'où $l_s = \frac{fe.\phi}{4.\tau_{su}}$

In the absence of calculations, the BAEL allows the following values to

be adopted on a fixed basis:

L_s= 40 Ø for FeE400 steel.

L_s= 50 Ø or FeE500, FeE215, and FeE235 steels.

For tensioned bars, the use of curved anchorage is recommended.

2.2. Curved Anchorage

When the dimensions of the structural element are insufficient to allow a straight anchorage of length l_s, a **curved anchorage** is used (e.g. at beam supports).

A curved anchorage consists of:

- Two straight segments **AB** and **CD**, of lengths $\mathbf{l_2}$ and $\mathbf{l_1}$ respectively ;
- A curved segment **BC**, with radius of curvature r and angle θ. (see Figure III.4).

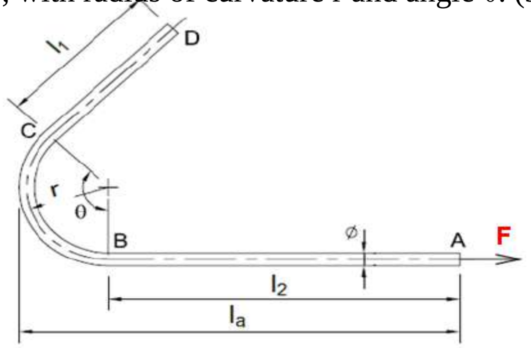


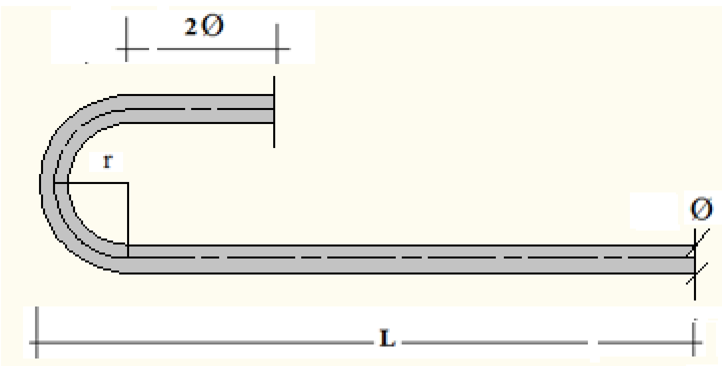
Fig. III.4 Curved anchorage of tensioned bars

l_a : anchorage length

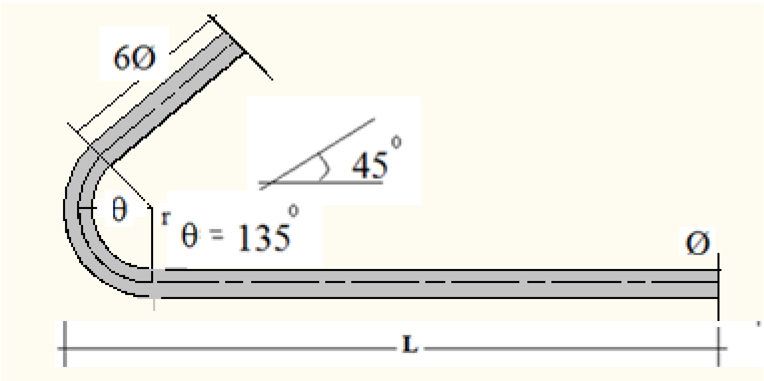
2.2.1 Types of Anchorage:

The most commonly used types

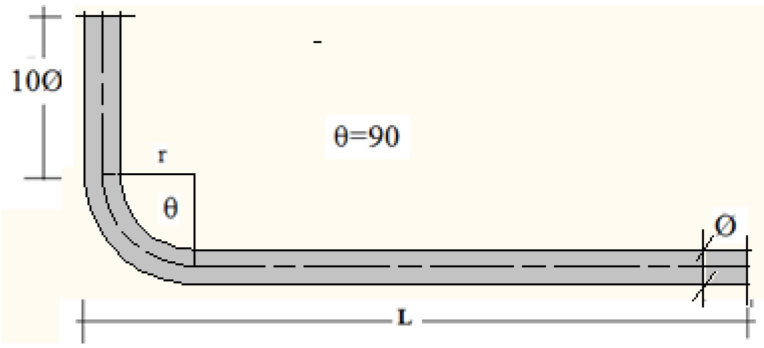
- *Standard hooks:*



- *45° hooks:*



- *Right-angle bends (90° hooks):*



In all cases, the following is adopted:

$L=0.6L_s$ for plain bars (RL)
$L=0.4L_s$ for deformed bars (HA)

L= the anchorage length in the support.

2.2.2. Radius of Curvature:

For fabrication reasons and to avoid crushing of concrete, the radius of curvature must meet the following conditions:

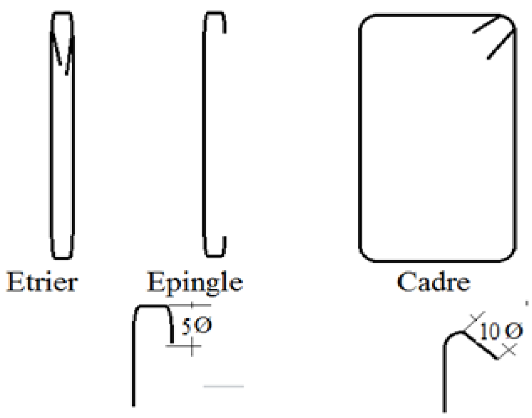
- $r \geq 3$ Ø for plain bars (R.L)
- $r \geq 5.5$ Ø for deformed bars (H.A)

For cantilevers, these conditions become:

- $r \geq 7$ Ø for plain bars (R.L)
- $r \geq 11$ Ø for deformed bars (H.A)

For **stirrups (étriers), U-shaped bars (épingles), and hoops or rectangular stirrups (cadres)**

- $r \geq 2$ Ø for plain bars (R.L)
- $r \geq 3$ Ø for deformed bars (H.A)



3. Lapping / Lap Splices:

Since reinforcement bars supplied commercially have limited lengths, it may be necessary, in long structural elements, to form longitudinal reinforcement using several bars.

To ensure continuity between the reinforcement bars, **lapping** (or a lap splice) is used, which means the bars are overlapped for a length Lr, called the lap length.

Lap Length:

a- Tensioned reinforcement:

For straight bars:

C ≤ 5 Ø : Lr = Ls

C > 5 Ø : Lr = Ls+t

Where C : concrete cover (of reinforcement).

For curved bars:

C ≤ 5 Ø : Lr = 0.6 Ls for plain bars (RL)

Lr = 0.4 Ls for deformed bars (HA)

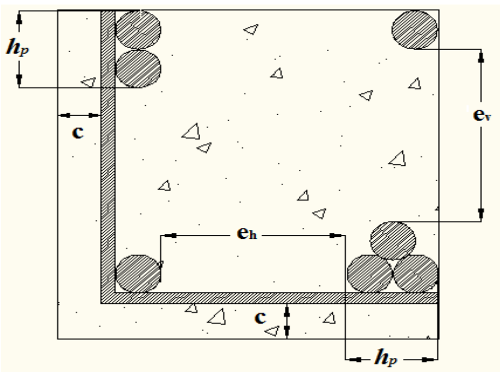
C > 5 Ø : Lr = 0.6 Ls+t for plain bars (RL)

Lr = 0.4 Ls +t for deformed bars (HA)

b- Compressed reinforcement:

Lap splices must be straight (no hooks): L'r= 0.6Ls

4. Reinforcement Detailing



4.1 Concrete Cover of reinforcement C (reinforcement cover):

Concrete cover is the distance from the outer surface of the reinforcement to the nearest concrete surface.

$C = max (C_1, C_2, C_g)$

C_1= max (Ǿ or h_p ;1 cm); where:

- Ø: Diameter of the bar.
- *h_p: Height of the bar bundle (group of bars), if used.*

C_2 = 5 cm very harmful cracking.

3 cm harmful cracking.

1 cm slightly harmful cracking.

C_g = maximum aggregate size used in the concrete.

4.2 Clear Spacing Between Reinforcement Bars

a- Horizontal spacing « e_h »:

$$e_h \geq \max (Ø \text{ or } h_p ; 1.5Cg)$$

b- Vertical spacing « e_v » :

$$e_v \geq \max (Ø \text{ or } h_p ; 1Cg)$$

5. Average Bond Stress

The average bond stress is equal to the ratio of the variation of axial force to the perimeter of the reinforcement:

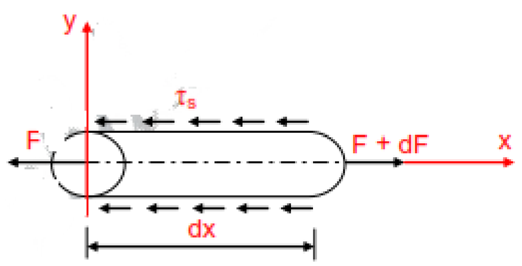


Fig. III.5 Equilibrium of a bar segment of length dx.

$$\tau_s = \frac{\frac{dF}{dx}}{\pi.\phi}$$

dF/dx : the variation per unit length of the axial force exerted on the reinforcement

4.1.2 Ultimate Bond Stress

To ensure proper anchoring, i.e. to prevent the reinforcement from slipping within the surrounding concrete sleeve, the bond stress must be limited to the following value:

$$\tau_{su} = 0,6.\psi_s^2.f_{tj}$$

ψ_s : anchorage coefficient relative to steel (plain or deformed bars)

Where : $\psi_s = \begin{cases} 1 & plain\ bars\ RL \\ 1,5 & deformed\ bars\ HA \end{cases}$

f_{tj} = 0,6+0,06.f_{cj} where : f_{tj} and f_{cj} in MPa

6. Flexural Bond Stress

In a beam subjected to bending with constant cross-section, t the **flexural bond stress** acting on a group of bars with total area $A_s$and useful perimeter $u_i$is given by:

$$\tau_{se} = \frac{V_u}{0,9.d.u_i}\frac{A_{si}}{A_s}$$

Où :

- A_s: total area of tension reinforcement.

- The useful perimeter u_i is taken as the minimum perimeter circumscribed around the cross-section of the bundle (group of bars)

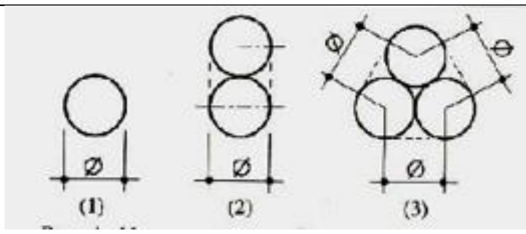
- It is recalled that V_u denotes the design value of the shear force at the ultimate limit state (ULS).

- The lever arm is fixed by convention at 0.9 times the effective depth d.

When all bars have the same diameter and are either isolated or grouped in identical bundles, the formula becomes:

$$\tau_{se} = \frac{V_u}{0,9.d.\Sigma u}$$

Where Σu denotes the sum of the useful perimeters of the bars or bundles.

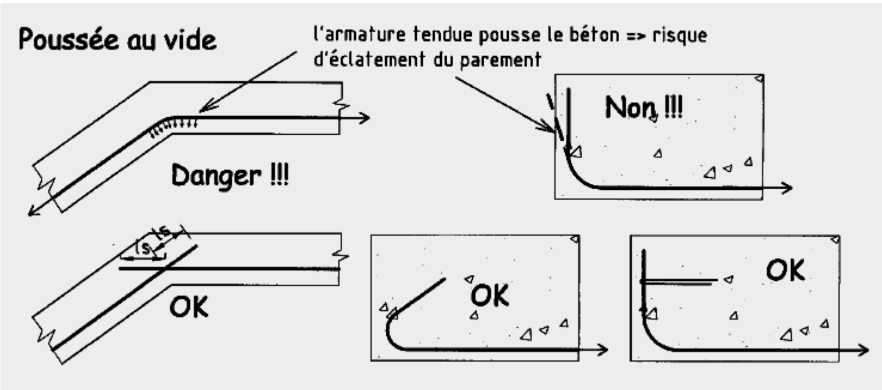
Table of effective perimeters	
1. Individual bars : π Ø	
2. Bundle of two bars: (π+2) Ø	
3. Bundle of three bars: (π+3) Ø	

The flexural bond stress τ_{se} must be less than the ultimate limit value:

$$\tau_{se,u} = \psi_s . f_{tj}$$

7. Bursting Forces (Concrete bursting Risk)

A construction method must be adopted that prevents any disorder caused by the bursting forces (or spalling forces) of the reinforcement. The arrangements shown in the following figure should be adopted:



The tension reinforcement pushes against the concrete=> risk of spalling of the facing

Chapter IV
Simple Tension

1. Introduction

A reinforced concrete member is subjected to simple tension when all the external forces acting to the left of a section *S* are reduced to a single normal force *N*, perpendicular to *S*, applied at the centroid (or center of gravity) of the section and directed to the left.

Such an element is called a tie member (tension member).

In this case:

- The centroid of the reinforcement coincides with the centroid of the concrete section.
- Tensile strength of the concrete is neglected.

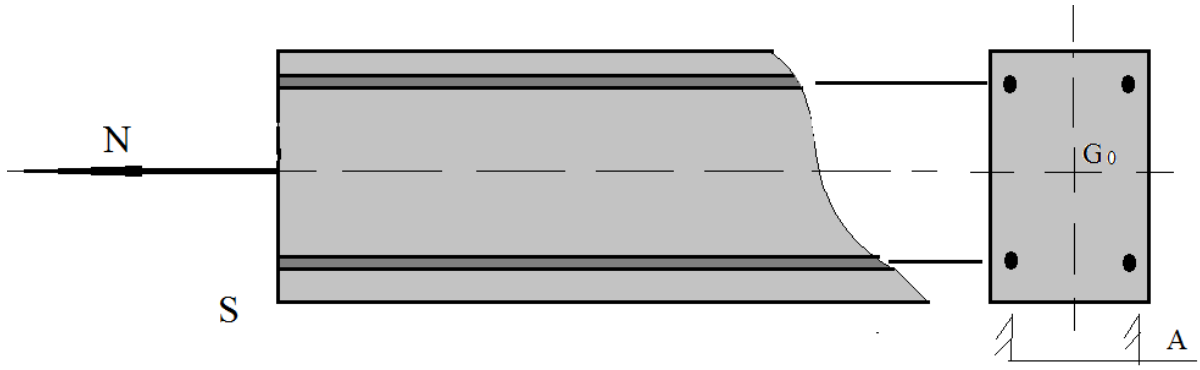


Fig IV.1: Tie member

2. Non-brittle failure condition

According to the design assumption, the concrete section **B** does not intervene in the calculation of steel (reinforcement), since concrete does not resist tension.

However, the load causing the concrete to crack must not lead to exceeding the yield strength of the steel.

$$B.f_{t28} \leq A.f_e \Rightarrow A_{min} = \frac{B.f_{t28}}{f_e}$$

Where :

B: cross-sectional area of concrete.

f_{t28} : tensile strength at 28 days $f_{t28} = 0.6 + 0.06\, f_{c28}$

fe : yield strength of the steel

3. Transverse reinforcement

S.E.T.: Transverse reinforcement plays only an assembly role. It is not necessary to connect the reinforcement bars located away from the corners by hooked ties or stirrups.

- Number of longitudinal bars n ≥ 4.
- Diameter of stirups: Ǿt
  - Slightly harmful cracking: Ǿt = 6mm.
  - Harmful cracking: Ǿt ≥ 6mm.
  - Very harmful cracking: Ǿt ≥ 8mm.
- Spacing between stirrups: St ≤ b' (where b'is the smallest dimension of the section.)

4. Remarks

a- If cracking is **slightly harmful**, the design is carried out **only at ULS (ELU)**.
b- If cracking is **harmful or very harmful**, the design is carried out **only at SLS (ELS).**

5. Calculation (design) of longitudinal reinforcement (ULS)

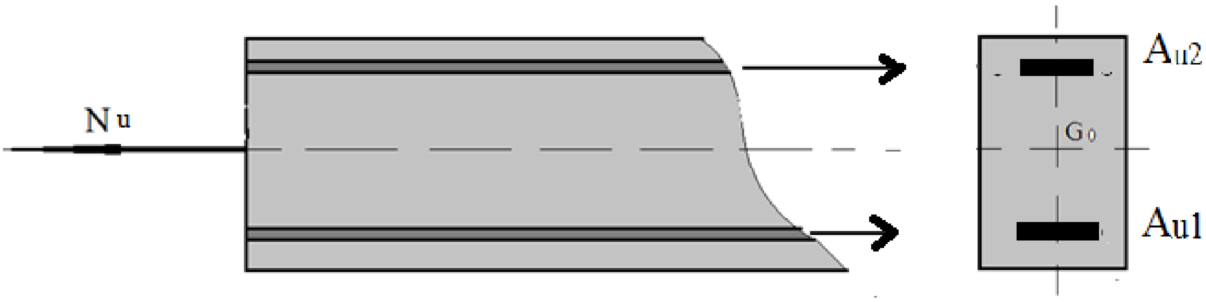
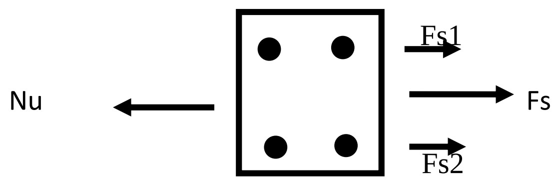


Fig. IV.2 : Reinforcement design at ULS – tensile concrete neglected

Equilibrium equation: ∑ Fexternal=∑ Finternal

Since tensile concrete is neglected, →∑ external forces are balanced by the reinforcement:

$F_{s1}=F_{s2}$; $F_s=F_{s1}+F_{s2}=(A_{u1}+Au2)\ \sigma_s = A_u\ \sigma_s$



$\sum$ Finternal = Fs and $\sum$ Fexternal = Nu

$N_u = A_u\ \sigma_s$ where $A_u = \frac{N_u}{\sigma_s}$

According to the three-pivot diagram:

Simple tension: Pivot A ⟶ $\varepsilon_s = \varepsilon_{su} = 10‰$ ⟶ $\sigma_s = f_e / \gamma_s$

6. Calculation (design) of longitudinal reinforcement (SLS)

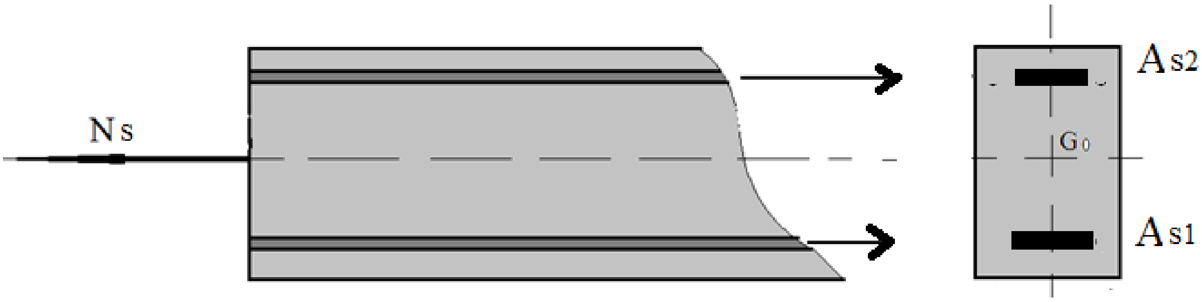


Fig. IV.3: Reinforcement design at SLS

$N_{ser} = N$

Aser = A

For economic reasons, σs is taken as the highest possible value, i.e., the allowable stress value.

$$A_{ser} = \frac{N_{ser}}{\bar{\sigma}_s}$$

$\bar{\sigma}_s$: The limit stress of the steel (elastic range), which depends on the type of cracking.

a- Slightly harmful cracking:

The limit stress of the steel is:

$$\sigma_s = \frac{f_e}{\gamma_s}$$

b- Harmful cracking:

The limit stress of the steel is:

$$\bar{\sigma}_s = \min\left(\frac{2}{3} f_e ; 110\sqrt{\eta . f_{tj}}\right)$$

c- Very harmful cracking:

The limit stress of the steel is:

$$\bar{\sigma}_s = \min\left(\frac{1}{2} f_e ; 90\sqrt{\eta . f_{tj}}\right)$$

With (for all cracking types):

- γ_s: safety factor
    - $\gamma_s = 1.15$(persistent or transient situation)
    - $\gamma_s = 1.0$(accidental situation)
- η: cracking coefficient
    - $\eta = 1.0$for plain bars (RL)
    - $\eta = 1.6$for deformed bars (HA)

7. Final reinforcement

- For **slightly harmful cracking**:

$$A = \max(A_u, A_{min})$$

- For **harmful or very harmful cracking**:

$$A = \max(A_{ser}, A_{min})$$

8. Dimensioning of the concrete section

SET: The concrete section **B** may be small, but the following must be checked:

a- The constructive provisions

b- Non-brittle failure condition: $A_{min} = \frac{B.f_{t28}}{f_e} \Rightarrow A \geq \frac{B.f_{t28}}{f_e} \Rightarrow B \leq \frac{A.f_e}{f_{t28}}$

9. Application

Consider a tie member with a section of **25 × 25 cm²**, belonging to an ordinary building, subjected to:

- N_g=-100 kN (Permanent load)
- N_q=-40 kN (Live load)
- Steel : FeE400 ; Concrete : F_{c28}=20MPa , $c_{g=}$25mm.

Design the reinforcement for:

- Slightly harmful cracking
- Harmful cracking

Solution:

Tie member ⟹ element subjected to simple tension.

1- Slightly harmful cracking → the design (or calculation) is done at ULS

a- Fundamental combination:

$1.35G_{max} + G_{min} + \gamma_{Qc}Q_c + \Sigma 1.3\psi_{0i}Q_i$

Nu= 1.35 (-100)+1.5 (-40)=-195 kN

Since N_u is obtained from the fundamental combination, therefore, persistent or transient situation:

So : $\begin{cases} \gamma_b = 1.5 \\ \gamma_s = 1.15 \end{cases}$

$\sigma_s = fe / \gamma_s = 400/1.15 = 348$ *Mpa*

$A_u = Nu/\sigma_s = 195000/(100\ 348) = 5.6\ cm^2$

Non-brittle failure condition: $A_{min} = \frac{B.f_{t28}}{f_e}$

$f_{t28} = 0.6 + 0.06.\ f_{c28} = 0.06+0.06x20 = 1.8$ Mpa

$$A_{min} = \frac{B.f_{t28}}{f_e} = \frac{(25x25)x1.8}{400} = 2.8cm^2$$

Finally : A= max (Au ; Amin) = 5.6 cm²

Applied Section: Aa ≥ A= 5.6 cm²

Number of longitudinal bars n ≥ 4 (tension)

- **Choice** : Ø ≤ b'/10 =25.10/10= 25 mm
- **i.e.,** 4 HA14 = 6.16 cm²

Note: The reinforcement bars must have a diameter ≥ 12 mm and be symmetrical across the concrete section.

Reinforcement cover: « c »

$c = max(c_1 ; c_2 ; c_g)$

$c_1 = max(\acute{Ø}_{lmax}; 1cm) = 1.4cm$

Slightly harmful cracking $c_2 = 1$ cm

- c = max (1.4 ;1 ;2.5) =2.5 cm

Horizontal spacing between longitudinal bars:

- $e_h \geq \max\left(\phi_{\max}, 1.5c_g\right) = 3.75cm$

Vertical spacing between longitudinal bars:

- $e_v \geq \max\left(\phi_{\max}, c_g\right) = 2.5cm$

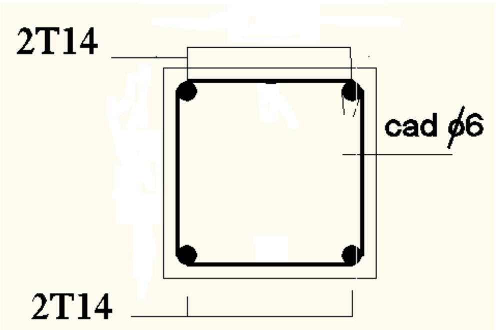
With 4 bars, spacing verification is not required.

Transverse reinforcement:

Slightly harmful cracking $\acute{Ø}_t = 6mm$

Spacing between stirrups: $st \leq b' = 25$ cm **i.e.,** st=25 cm

Reinforcement:



2- Harmful cracking → the design (or calculation) is done at SLS

a- Load Combination

$G_{max} + G_{\min} + Q_c + \Sigma\psi_{0i}Q_i$

Nser= (-100)+ (-40)=-140 kN

$$\bar{\sigma}_s = \min\left(\frac{2}{3}f_e;110\sqrt{\eta.f_{tj}}\right)$$

Steel HA → η=1.6

$\bar{\sigma}_s = 187MPa$

$$A_{ser} = \frac{N_{ser}}{\bar{\sigma}_s} = \frac{140x10}{187} = 7.48cm^2$$

$$A_{\min} = \frac{B.f_{t28}}{f_e} = \frac{(25x25)x1.8}{400} = 2.8cm^2$$

Finally: A= max (Aser; Amin) = 7.48 cm²

Applied section: Aa ≥ A= 7.48 cm²

Number of longitudinal bars n ≥ 4 (tension)

- **Choice**: Ø ≤ b'/10 =25.10/10= 25 mm
- **i.e.,** 4 HA16 = 8.04 cm²

Note: The reinforcement bars must have a diameter ≥ 12 mm and be symmetrical across the concrete section.

Reinforcement cover: « c »

c = max (c_1 ;c_2 ;c_g)

c_1=max ($Ø_{lmax}$; 1cm) =1.4cm

Slightly harmful cracking c_2=1 cm

- c = max (1.4 ;1 ;2.5) =2.5 cm

Horizontal spacing between longitudinal bars:

- $e_h \geq \max\left(\phi_{\max}, 1.5c_g\right) = 3.75cm$

Vertical spacing between longitudinal bars:

- $e_v \geq \max\left(\phi_{\max}, c_g\right) = 2.5cm$

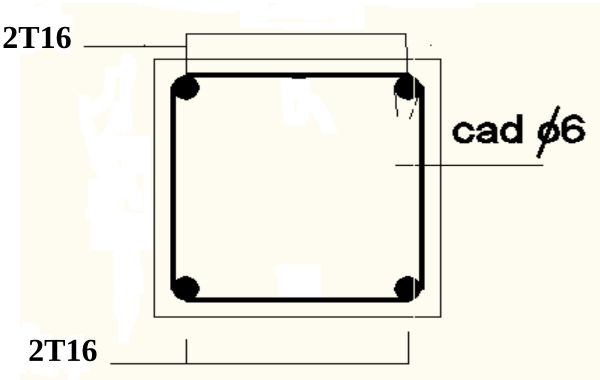
With 4 bars, spacing verification is not required.

Transverse reinforcement:

Harmful cracking $Ø_t \geq$ 6mm **i.e.,** $Ø_t$ = 6 mm

Spacing between stirrups: st ≤ b' = 25 cm **i.e.,** st=25 cm

Reinforcement :



Chapter V
Simple Compression

1. Introduction

A reinforced concrete member is subjected to simple compression when all the forces acting on it can be reduced, with respect to the centroid of the concrete section B′, to a single compressive force N'.

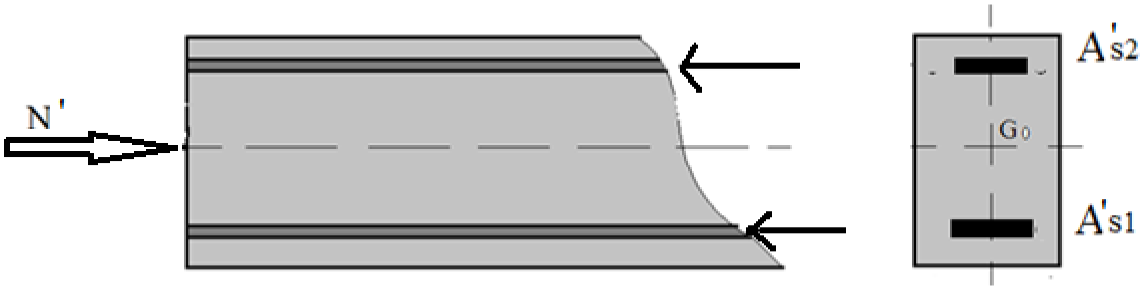


Fig. V.1 Member subjected to simple compression.

Structural elements commonly governed by this type of loading include:

- Building columns and walls;
- Bridge piers and abutments;
- Arches and shell structures.

2. Buckling Length and Slenderness of a Column

2.1 Buckling Length

Under the effect of a compressive force, columns may become unstable and buckle.

It is therefore necessary to consider, in the calculations, a fictitious length called the buckling length l_f, instead of the actual length (also called the free length) l_0.

The buckling length l_f depends on the connection type at the ends of the element (Fig. V.2).

a. Case of an isolated column

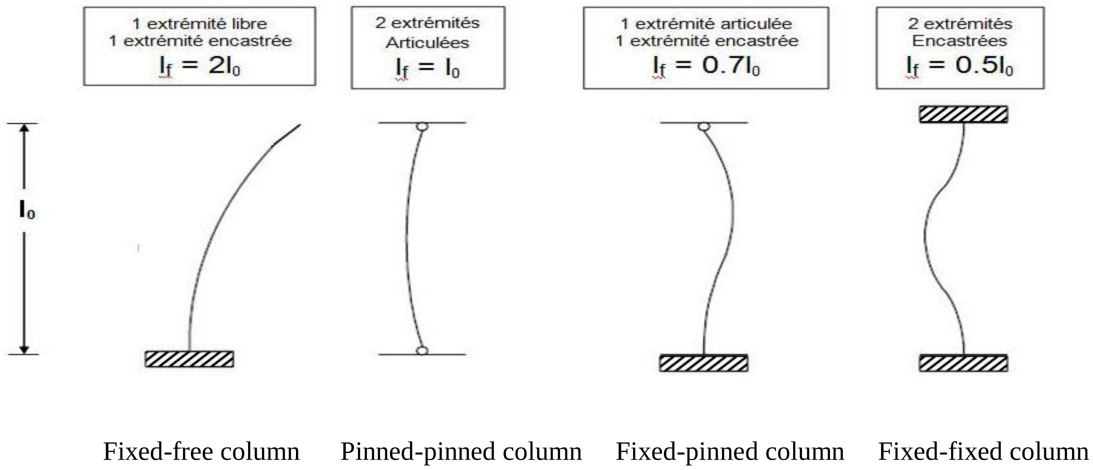
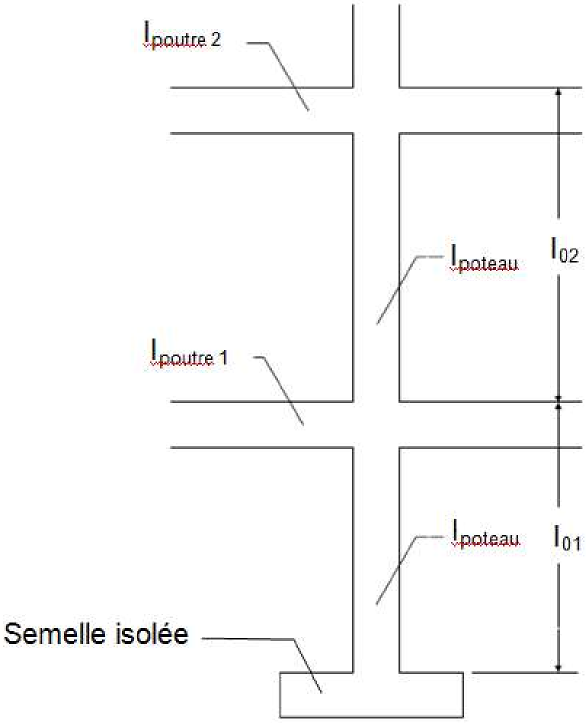


Fig V.2: Relationship between the free length l_0 and the buckling length l_f

- For columns in multi-storey buildings: $l_f = 0.7\ l_0$ (generally)
- For precast columns: $l_f = l_0$



Semelle isolée = individual or pad footing

Fig. V.3: Buckling length in a building

2.2 Slenderness

Slenderness is denoted by the symbol « **λ** » and is defined as the ratio of the buckling length l_f to the **minimum radius of gyration** i_{min}:

$$\lambda = \frac{l_f}{i_{min}} \quad \text{and} \quad i_{min} = \sqrt{\frac{I_{min}}{B}}$$

B: concrete sectional area

I_{min} : minimum moment of inertia of the cross-section with respect to the center of gravity of the section in the direction of the buckling considered.

Rectangular section (bxh) with h > b

$$I_{min} = \frac{h.b^3}{12} \ , \text{B = bxh}$$

$$i_{min} = \sqrt{\frac{h.b^3}{12.h.b}} = \sqrt{\frac{b^2}{12}} = \frac{b}{\sqrt{12}} \Rightarrow \lambda = \frac{l_f}{i_{min}} = \sqrt{12}\frac{l_f}{b} = 3,46\frac{l_f}{b}$$

$$\lambda = 3,46\frac{l_f}{b}$$

Circular section of diameter d

$$I_{\min} = \frac{\pi.d^4}{64}.et.B = \frac{\pi d^2}{4}$$

$$i_{\min} = \sqrt{\frac{\pi.d^4.4}{64.\pi.d^2}} = \sqrt{\frac{4.d^2}{64}} = \frac{d}{4} \Rightarrow \lambda = \frac{l_f}{i_{\min}} = 4\frac{l_f}{d}$$

$$\lambda = 4\frac{l_f}{d}$$

3. Reinforcement Detailing

3.1. Main longitudinal reinforcement:

- Longitudinal reinforcement must be placed **symmetrically**.

- The distance between the axes of two longitudinal bars must be at most equal to:

$$St = \min\left(15\phi_{l\ \min,}\ b' + 10cm, 40cm\right)$$

b': smallest side of the section

-Intermediate bars must be connected by transverse reinforcement (rectangular stirrups, stirrups, or hooked ties).

- In lap splice (or overlapping) zones, hooks should be avoided, as concrete may split.

- Deformed bars (HA) should be used

- For columns Ø ≥ 12 mm

- The centroid of the concrete section coincides with the centroid of steel.

-Longitudinal reinforcement must be distributed along the perimeter of the section:

*For square or rectangular sections: at least one bar must be placed in each corner n ≥ 4 and must be an even number

*For circular sections: at least 6 bars must be uniformly distributed n ≥ 6

3.2. Non-brittle failure condition (minimum reinforcement):

$$A_{min} = \max(4\% p; 0,2\% B)$$

P: perimeter

B: section

For a rectangular section: $A_{min} = \max\left(\frac{8.(b+h)}{100}; \frac{0,2.bh}{100}\right)$

Maximum reinforcement

Amax =5% B $\qquad A_{min} \leq A_{app} \leq A_{max}$

3.3. Transverse reinforcement:

Transverse reinforcement consists of **plain round bars (RL)** or **deformed bars (HA)**, depending on the shear design (or calculation).

Øt ≥ 6 mm for harmful cracking

Øt ≥ 8 mm for very harmful cracking

Øt = Øl max /3

$Ø_{lmax}$: maximum diameter of longitudinal bars applied.

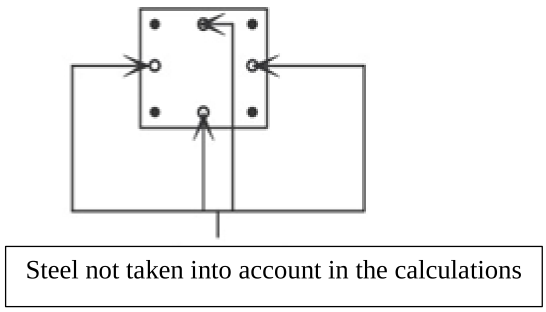
Øt : diameter of transverse reinforcement

4. Verification of Columns:

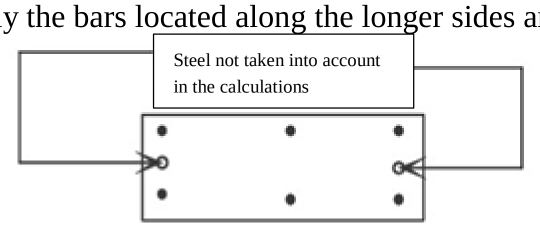
The calculation is carried out at **ULS (ELUR)**.

If λ ≥ 35 :

- Square section: only the bars located at the **corners** are considered.



- **Rectangular section**: only the bars located along the longer sides are considered.



5. Dimensioning and Calculation (or Design) of Sections (B′, A′)

The dimensioning of the concrete section **B′** and the calculation of reinforcement **A′** are performed at the:

Ultimate Limit State of Buckling "ULS-B" (F.C – A.C)

Using simplified empirical rules, the calculation of concrete members subjected to buckling (cross-sectional instability) is based on the following formula:

$$N_U' \leq \alpha \left(\frac{B_r'.f_{cj}}{0.9\theta\gamma_b} + A'\frac{f_e}{\gamma_s} \right)$$

N'$_u$: applied centered compressive force.

α : reduction coefficient depending on slenderness λ

For $\lambda \leq 50$ ⟶ $\alpha = 0.85/[1+0.2(\lambda/35)^2]$

For $50 < \lambda \leq 70$ ⟶ $\alpha = 0.6(50/\lambda)^2$

Reduced section B_r'

B'r (reduced section): BAEL rules penalize columns with small sections, which are susceptible to imperfections in execution. Therefore, these rules specify a reduced section for calculating the sections and maximum ultimate load.

B'_r=(h-2cm)(b-2cm)

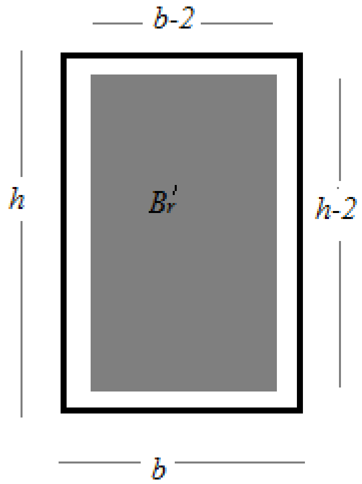


Fig. V.4 : Reduced section ***B'$_r$***.

Where :

$\theta = 1$ for loads applied longer than 24 hours

$\theta = 0.9$ for loads applied between 1 and 24 hours

$\theta = 0.85$ for loads applied for less than 1 hour

- **Safety factor for concrete γ_b:**

= 1.5 for Fundamental Combinations (F.C) (permanent + variable loads) in persistent or transient situations.

=1.15 for Accidental Combinations (A.C) (permanent + accidental + variable loads) in accidental situations.

- **Safety factor for steel** γ_s:

= 1.15 for Fundamental Combinations (F.C) (permanent + variable loads) in persistent or transient design situations.
= 1.0 for Accidental Combinations (A.C) (permanent + accidental + variable loads) in accidental design situations.

where

$$A_U' \geq \left(\frac{N_u'}{\alpha} - \frac{B_r' . f_{cj}}{0.9\theta\gamma_b} \right) . \frac{\gamma_s}{f_e}$$

where

$$B_r' \geq \frac{0.9\theta\gamma_b}{f_{cj}} \left(\frac{N_u'}{\alpha} - A' \frac{f_e}{\gamma_s} \right)$$

we take ; A'/Br' = 1% so : A' = 0.01 Br'

Rectangular section with $b \leq h$, for all reinforcement to participate in resistance of the column we take $\lambda \leq 35$

$\lambda = 3.46\ l_f/b \leq 35$ ⟶ $b \geq l_f/10$

Br' = (h-2) (b-2) ⟶ h = [Br'/(b-2)] + 2 cm

If $h \leq b$ ⟶ adopt a **square column** of side *b*.

Circular section, for all reinforcement to participate in resistance of the column we take $\lambda \leq 35$

$\lambda = 4\ l_f/b \leq 35$ ⟶ $D \geq l_f/9$

6. Application

Design the reinforcement of a **25 × 25 cm² column** belonging to a typical multi-storey building, subjected to:

- N_q=+550 kN (due to operating (or live) loads)
- Steel: FeE400; Concrete: F_{c28}=25MPa, Nominal cover $c_{g=}$25mm, $l_f = 0.7\ l_0 = 2.57$ m
- Harmful cracking, Nq; centered load

Solution:

centered load ⟹ Centered compression → calculation at **ULS (ELUR)**

Slenderness: $\lambda = 3.46\ l_f/b = 3.46 \times 257/25 = 35.57$

$\lambda \leq 70$: The simplified formula may be used.

$$N_U' \leq \alpha\left(\frac{B_r'.f_{cj}}{0.9\theta\gamma_b} + A'\frac{f_e}{\gamma_s}\right)$$

where $$A_U' \geq \left(\frac{N_u'}{\alpha} - \frac{B_r'.f_{cj}}{0.9\theta\gamma_b}\right).\frac{\gamma_s}{f_e}$$

Calculation of Nu

$N_G = \rho\ V$

V : volume

$Lf = 0.7\ l_0$ where $l_0 = l_f/0.7 = 2.57/0.7 = 3.67$ m

$N_G = 25\ [0.25 \times 0.25 \times 3.67] = +5.73$ KN

a- Fundamental combination (CF)

$$1.35G_{max} + G_{min} + \gamma_{Qc}Q_c + \Sigma 1.3\psi_{0i}Q_i$$

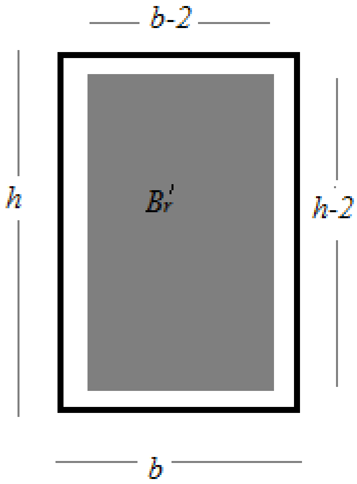
Nu= 1.35 (+5.73)+1.5 (+550)=+832.74 kN

Nu is given by CF ⟶ $\begin{cases} \gamma_b = 1.5 \\ \gamma_s = 1.15 \end{cases}$

$\lambda \leq 50$ → $\alpha = 0.85/[1+0.2(\lambda/35)^2]$

$\alpha = 0.7$

Calulation of the Reduced section :



$$Br = (25-2)(25-2) = 529cm^2$$

Where :

$\theta = 1$ for loads applied longer than 24 hours

$\theta = 0.9$ for loads applied between 1 and 24 hours

$\theta = 0.85$ for loads applied for less than 1 hour

therefore, we take : $\theta = 1$

$$A_U' \geq \left(\frac{N_u'}{\alpha} - \frac{B_r'.f_{cj}}{0.9\theta\gamma_b}\right).\frac{\gamma_s}{f_e}$$

$$A_U' \geq \left(\frac{832,74.10^3}{0.7} - \frac{529.25.10^2}{0,9.1.1,5}\right).\frac{1,15}{400.10^2}$$

$A_u \geq 6,04\ cm^2$

Non-brittle failure condition:

$$A_{\min} = \max\left(\frac{8.(b+h)}{100};\frac{0,2.bh}{100}\right)$$

$$A_{\min} = \max(1;1,25)$$

$$A_{\min} = 1,25$$

Finally : A= max (Au ; Amin) = 6,04 cm²

Applied section: Aa ≥ A= 6,04 cm²

The number of longitudinal bars n ≥ 4 and even

λ ≥ 35, therefore necessarily 4 bars

Choice: Ø ≤ b'/10 = 25 x 10/10 = 25 mm and Ø ≥ 12 mm

i.e., 4 HA14 = 6.16 cm²

- **Note:** the reinforcement bars must be ≥ 12 mm in diameter and symmetrical across the concrete section.

Transverse reinforcement:

Øt ≥ Ølmax/3 = 4,67 mm

Øt ≥ 6mm ; harmful cracking

Let: Øt = 6 mm

Spacing « St »

$$St = \min\left(15\phi_{l\ \min},\ b' + 10cm, 40cm\right)$$

$$St = \min\left(21cm,\ 35cm, 40cm\right)$$

Let: St = 20 cm

Reinforcement detailing:

Reinforcement Cover: « c »

C = max (c1; c2; cg)

C_1=max ($Ø_{lmax}$; 1cm)=1.4cm

harmful cracking C2=3 cm

c = max (1.4 ;3 ;2.5)=3 cm

Horizontal spacing between longitudinal bars:

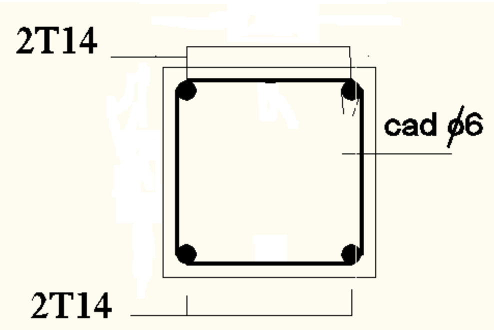
$$e_h \geq \max\left(\phi_{\max}, 1.5c_g\right) = 3.75cm$$

Vertical spacing between longitudinal bars:

$e_v \geq \max\left(\phi_{max}, c_g\right) = 2.5cm$

With 4 bars, spacing verification is not required.

Reinforcement:



Chapter VI
Simple Bending

6-I. RECTANGULAR SECTION

1. GENERAL DEFINITION

For a reinforced concrete member (beam) subjected to simple bending, the internal forces to the left of a cross-section S can be reduced to:

- A bending moment M_f
- A shear force V

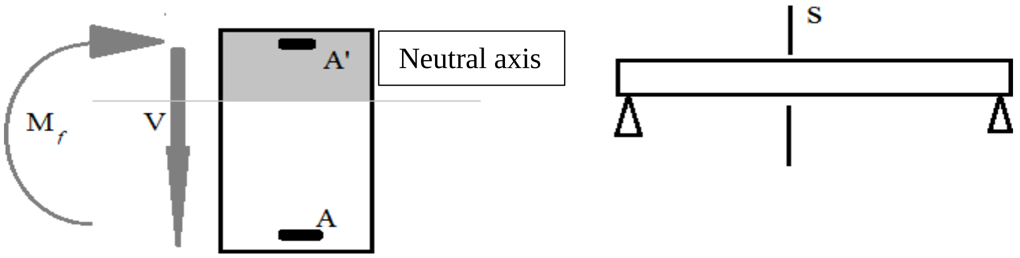


Fig. 6.1 Types of actions due to simple bending

In this chapter, we will study only the effects of the bending moment M_f.

ULTIMATE LIMIT STATE OF RESISTANCE

According to the three-pivot diagram:

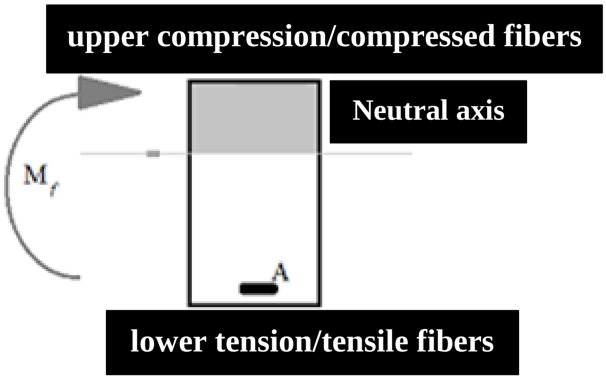
Simple bending: Pivot A, region 1b

Pivot B, regions 2a and 2b

1.1 Rectangular section without compression reinforcement

By assumption (hypothesis) :

$M_f>0$



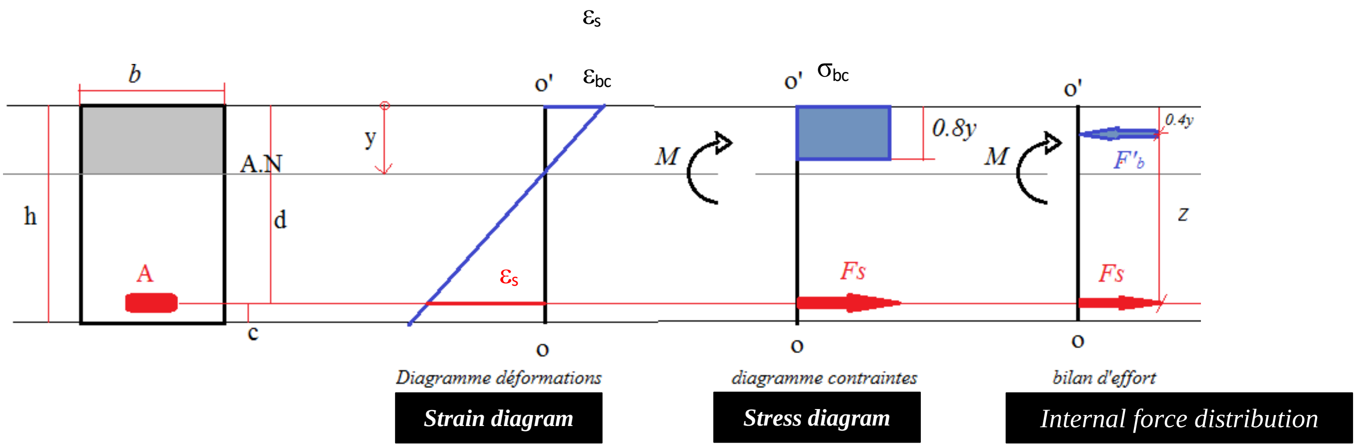


Fig.6.2 Strain, stress, and internal force distribution at the ULS.

1.1.1 **Notations :**

d=0.9h : effective depth.

c=0.1h: distance between the centroid of tensile reinforcement and the most tensile fiber.

y: distance from the neutral axis to the most compressed fiber

A: total area of tensile reinforcement

$\sigma_{bcu} = \frac{0.85}{\theta}\frac{f_{cj}}{\gamma_b}$ stress in the compressed zone

σ_s : tensile stress in the tensile reinforcement

1.1.2 **Equilibrium equations:**

$$\sum F = 0, \sum M = 0$$

Let :

$$\sum F = 0 \Rightarrow F_b^{'} = F_s$$

$$F_s = A\sigma_s, F_B^{'} = 0.8by\sigma_{bc} \Rightarrow 0.8by\sigma_{bc} - A\sigma_s = 0$$

$$\sum M = 0$$

$$a)\sum M /_{F_s} = 0 \Rightarrow M - F_S z = 0 \qquad \Rightarrow M - A\sigma_s z = 0$$

$$b)\sum M /_{F_b^{'}} = 0 \Rightarrow M - F_B^{'} z = 0 \qquad \Rightarrow M - 0.8by\sigma_{bc} z = 0$$

$$z = d - 0.4y \qquad \Rightarrow M - 0.8by\sigma_{bc}(d - 0.4y) = 0$$

$$y = \alpha d, \quad z = \beta d$$

Simple bending $\rightarrow$ regions **1b,** $\mu = \frac{M}{\sigma_{BC} b d^2}$ **2a, 2b**

$$\Rightarrow 0 \le y \le d \Rightarrow 0 \le \alpha \le 1$$

$$z = d - 0.4y \qquad \Rightarrow z = d\left(1 - 0.4\alpha\right)$$

with,

$z = \beta d \quad \Rightarrow \beta = 1 - 0.4\alpha$

so,

$$M - 0.8\, by\sigma_{bc}\,(d - 0.4y) = 0$$

$$M - 0.8\, by\sigma_{bc} d\,(1 - 0.4\alpha) = 0$$

$$M - 0.8 by\sigma_{bc}\alpha d^2(1 - 0.4\alpha) = 0$$

$$\frac{M}{b\sigma_{bc} bd^2} - 0.8\alpha(1 - 0.4\alpha) = 0$$

As $\mu = \dfrac{M}{b\sigma_{bc} bd^2} \Rightarrow \mu = 0.8\alpha(1 - 0.4\alpha)$

$$\mu - 0.8\alpha + 0.32\alpha^2 = 0$$

As $0 \le \alpha \le 1 \Rightarrow \alpha$ *positive root*

$$\alpha = 1.25(1 - \sqrt{1 - 2\mu})$$

Remark $\alpha \uparrow \quad \mu \uparrow \quad :$

1.1.3 Reference moments of the section (M_{AB}-M_{BC})

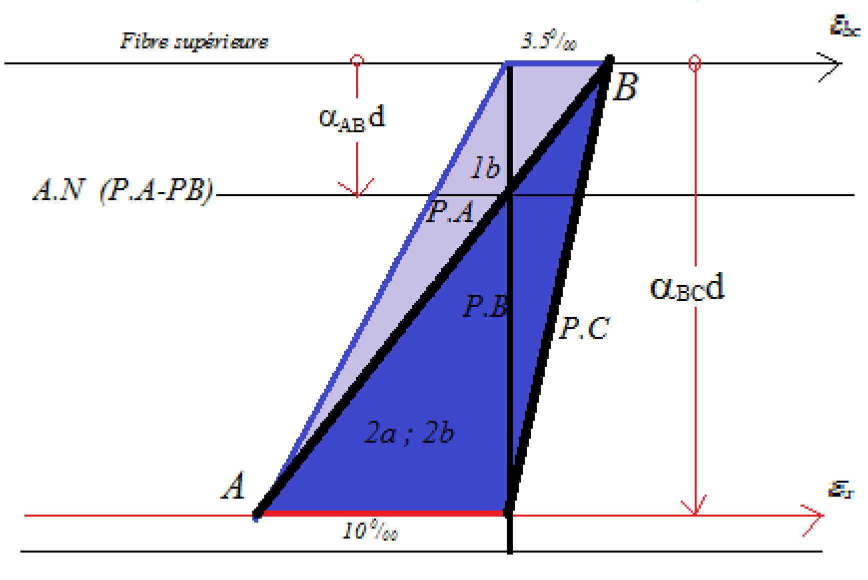


Fig.6.3 Reference moments at the ULS.

M_{AB} : ultimate moment representing the boundary between pivot A and pivot B, causing:

- Steel $\varepsilon_s = \varepsilon_{su} = 10\ ^0/_{00}$
- Concrete $\varepsilon_{bc} = \varepsilon_{bcu} = 3.5\ ^0/_{00}$

M_{BC} : ultimate moment representing the boundary between pivot B and pivot C, causing:

- Steel $\varepsilon_s = 0^0/_{00}$ (y = d)
- Concrete $\varepsilon_{bc} = \varepsilon_{bcu} = 3.5\ ^0/_{00}$

a) For $M=M_{AB}$: Ultimate moment at the boundary between pivot A and pivot B

We have $\mu = \dfrac{M}{b\sigma_{bc}bd^2} \Rightarrow \qquad M = \mu\sigma_{bc}bd^2$

1) *for* $M = M_{AB} \Rightarrow \mu_{AB} = \dfrac{M_{AB}}{b\sigma_{bc}bd^2} \Rightarrow M_{AB} = \mu_{AB}\sigma_{bc}bd^2$
we have

$$\mu = 0.8\alpha(1-0.4\alpha) \Rightarrow \mu_{AB} = 0.8\alpha_{AB}(1-0.4\alpha_{AB})$$

$$\Delta \equiv \Delta \Rightarrow \frac{3.5}{10} = \frac{\alpha_{AB}d}{d(1-\alpha_{AB})} \Rightarrow \alpha_{AB} = 0.2593 \qquad \mu_{AB} = 0.186$$

$$M_{AB} = 0.186\sigma_{bc}bd^2$$

if	$0 \le y \le \sigma_{AB}d = 0.2593d$	
$c-a-d$	$0 \le \mu \le 0.186$	*We are in the Pivot A*
$c-a-d$	$M \le M_{AB}$	

with $\varepsilon_s = 10\ ‰_{00}, \sigma_s = \dfrac{f_e}{\gamma_s}$

b) For $M = M_{BC}$: Limit moment between regions 2 and 3 (P.B-P.C)

Limit case for section design: *y=d* $\alpha_{BC} = 1$

$$\mu = 0.8\alpha(1-0.4\alpha) \Rightarrow \mu_{BC} = 0.8\alpha_{BC}(1-0.4\alpha_{bc})$$

$$\alpha_{BC} = 1 \qquad , \mu_{BC} = 0.48$$

$$M_{BC} = 0.48\sigma_{bc}bd^2$$

if	$0.2593d \le y \le d$	
$c-a-d$	$0.186 \le \mu \le 0.48$	*We are in the Pivot B*
$c-a-d$	$M \le M_{Bc}$	

with $\varepsilon_{bc} = 3.5^0/_{00}, \varepsilon_s < 10^0/_{00}$

c)- For M >M_{BC} : Limit moment region 3 :

if	$y \rangle d$	*We are in the Pivot C*
$c-a-d$	$\mu \rangle 0.48$	
$c-a-d$	$M \rangle M_{Bc}$	Case of compound bending

1.1.4 **Relative strains in steel**

$a) - For \quad \mu \leq \mu_{AB} \Rightarrow$ Pivot A

$$\Rightarrow \varepsilon_s = 10‰ \qquad \sigma = \frac{f_e}{\gamma_s}$$

b) - For $\mu_{AB} \langle \mu \leq \mu_{AB} \Rightarrow$ Pivot B

$$\Delta \equiv \Delta \qquad \frac{\varepsilon_s}{\varepsilon_{bc}} = \frac{d-y}{y} = \frac{d(1-\alpha)}{\alpha d} = \frac{1-\alpha}{\alpha}$$

$$\varepsilon_s = \varepsilon_{bc}\frac{1-\alpha}{\alpha} = 3.5‰ \frac{1-\alpha}{\alpha}$$

Remark $\alpha \uparrow \quad \varepsilon_s \downarrow$

Si $\varepsilon_s \langle \varepsilon_{sl} \Rightarrow \qquad \sigma_s = \varepsilon_s E_s$

Si $\varepsilon_s \geq \varepsilon_{sl} \Rightarrow \qquad \sigma_s = \dfrac{f_e}{\gamma_s}$

1.1.5 Resistant moment M_l :

The resistant moment is the moment obtained when the elongation of the tensile reinforcement is:

$$\varepsilon_s = \varepsilon_{sl} \Rightarrow \quad \sigma_s = \frac{f_e}{\gamma_s}$$

Thus, the resistant moment « M_l » is the limit moment between regions 2a and 2b.

we have $$M = \mu\sigma_{bc}bd^2 \Rightarrow \qquad M_l = \mu_l\sigma_{bc}bd^2$$

$$\mu_l = 0.8\alpha_l(1-0.4\alpha_l)$$

$$\Delta \equiv \Delta \Rightarrow \frac{3.5‰}{\varepsilon_{sl}} = \frac{\alpha_l d}{d(1-\alpha_l)} \Rightarrow \varepsilon_{sl} = 3.5‰\frac{(1-\alpha_l)}{\alpha_l}$$

$$\alpha_l = \frac{3.5}{1000\varepsilon_{sl}+3.5}$$

With $$\varepsilon_{sl} = \frac{f_e}{\gamma_s E_s} \quad Et \quad \beta_l = 1-0.4\alpha_l$$

then:

if $M \le M_l$, $\mu \le \mu_l$ $\Rightarrow$ Compression reinforcement is not necessary, as the tension reinforcement works at its ultimate capacity and the compressed concrete alone resists the compressive stresses: A'=0

if $M \rangle M_l$, $\mu \rangle \mu_l$ $\Rightarrow$ The tension reinforcement does not work at its ultimate capacity $\sigma_s = f_e/\gamma_s$. Therefore, to make the tension reinforcement work at its ultimate capacity ($\varepsilon_s = \varepsilon_{sl}$; $\sigma_s = f_e/\gamma_s$), it is necessary to provide compression reinforcement A'. Therefore, A' is necessary.

1.1.6 Calculation of tensile (tension) reinforcement A_u

$$\sum M /_{F_b'} = 0 \quad \Rightarrow M - F_s z$$

$$M = F_s z = \sigma_s A_u d\left(1-0.4\alpha\right) = \sigma_s A_u d\beta$$

$$A_u = \frac{M}{\sigma_s \beta d}$$

2.1.6 Non-brittleness condition:

(Non-brittle failure condition)

$$A_{min} = 0.23bd\,\frac{f_{t28}}{f_e}$$

2.1.7 Summary: Calculation flowchart at the ULS for a rectangular section without compression reinforcement

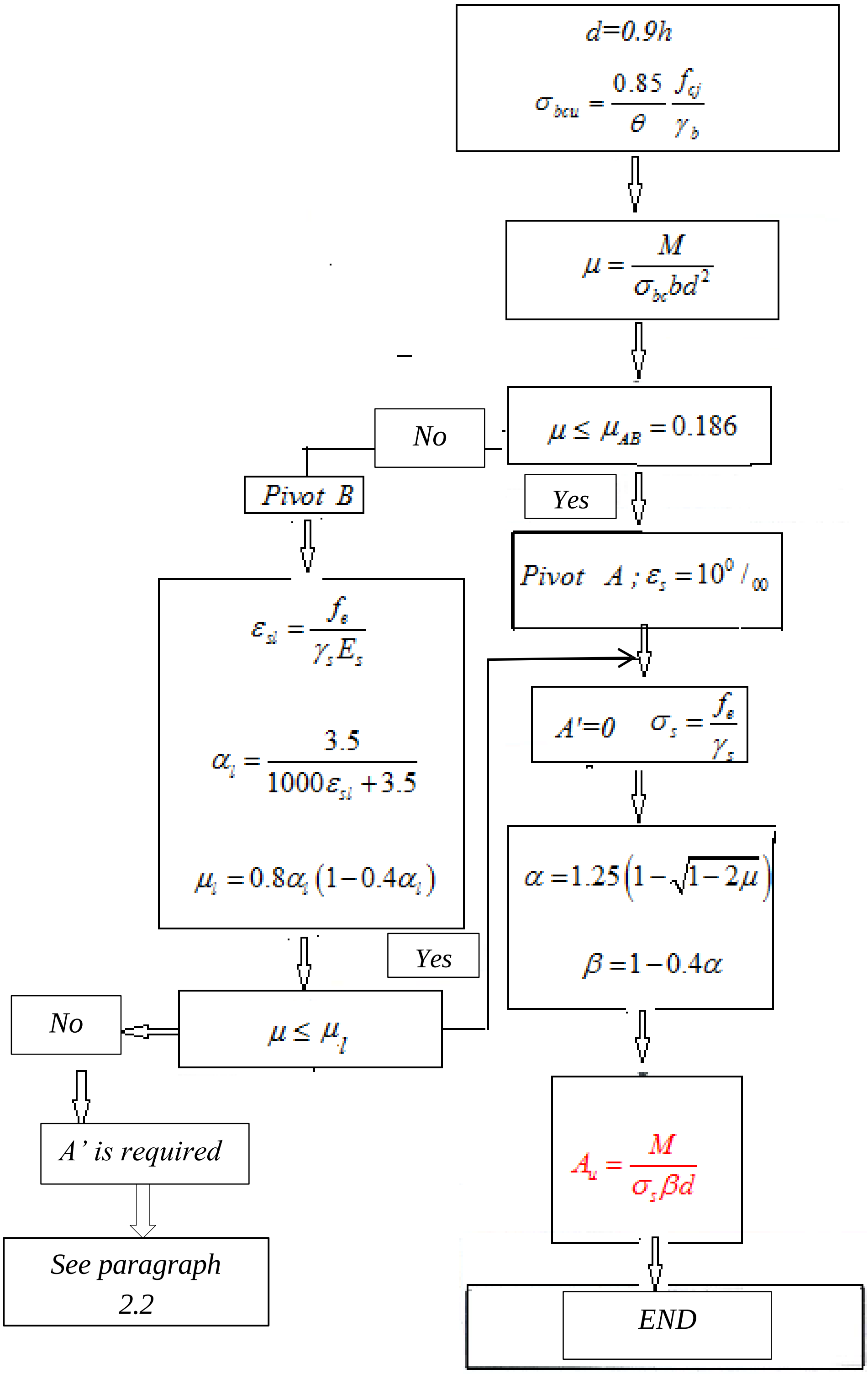
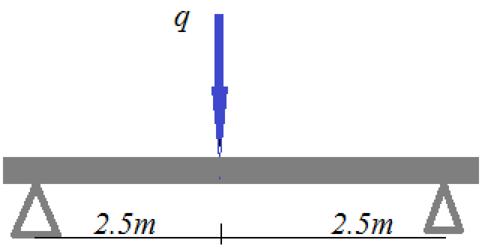


Fig.6.4 Calculation flowchart for a rectangular section without compression reinforcement at the ULS

APPLICATION EXAMPLE

Consider a reinforced concrete beam with a **30 × 60 cm²** cross-section, subjected to a **concentrated working load** q = 200 kN, with ; FeE400; f_{c28} = 25MPa.

Determine the area of tensile reinforcement at the Ultimate Limit State of Resistance (ULS).



$$g = 0.3*0.6*25 = 4.5kN/ml$$

$$Mg = \frac{gl^2}{8} = 14.06kNm$$

$$Mq = q\frac{l}{4} = 250kNm$$

$$C.F \quad , M_u = 1.35Mg + 1.5Mq = 394kNm$$

$$\sigma_{bc} = \frac{0.85 f_{c28}}{\theta\gamma_b} \qquad \theta = 1 \qquad \gamma_b = 1.5, \Rightarrow \sigma_{bc} = 14.2MPa \quad ,$$

$$\mu = \frac{M_u}{\sigma_{bc} bd^2} = 0.317$$

$$\mu_{AB} = 0.186 \leq \mu = 0.317 \leq 0.48 \Rightarrow Pivot \quad B$$

$$\Rightarrow \varepsilon_{bc} = 3.5\ ^0\!/_{00}$$

Verification of the need for compression reinforcement

$$\varepsilon_{sl} = \frac{f_e}{\gamma_s E_s} = 1.739\,\%_{0}$$

$$\alpha_l = \frac{3.5}{1000\varepsilon_{sl} + 3.5} = 0.668$$

$$\mu_l = 0.8\alpha_l\left(1 - 0.4\alpha_l\right) = 0.392$$

$$\mu = 0.317 \langle \mu_l \qquad \text{(région 2a)} \qquad \Rightarrow \varepsilon_s \rangle \varepsilon_{sl} \Rightarrow \sigma_s = \frac{f_e}{\gamma_s} = 348MPa$$

Therefore, **compression reinforcement is not required**: A'=0

$$\alpha = 1.25(1-\sqrt{1-2\mu}) = 0.494$$

$$\beta = 1 - 0.4\alpha = 0.802$$

$$A_u = \frac{M}{\sigma_s \beta d} = 26.14\ cm^2$$

Minimum reinforcement:

$$f_{t28} = 0.6 + 0.06 f_{c28} = 2.1 MPa$$

$$A_{min} = 0.23bd\frac{f_{t28}}{f_e} = 1.94 cm^2$$

Final reinforcement:

$$A_f = \max\left(A_u, A_{min}\right) = 26.14 cm^2$$

Adopted reinforcement:

(Subject to ELS reinforcement verification – see paragraph 3)

A_{app}=3x3T20=9T20=28.26cm^2

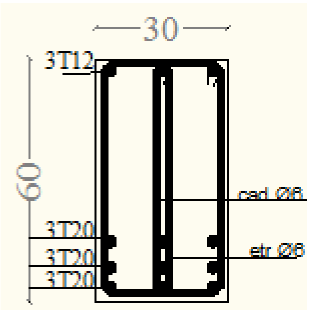
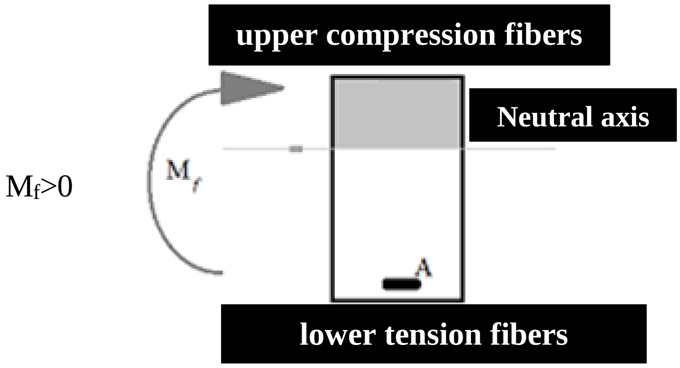


Fig.6.5 Reinforcement detailing of the beam.

1.2 Rectangular Section with Compression Reinforcement

by assumption (hypothesis):

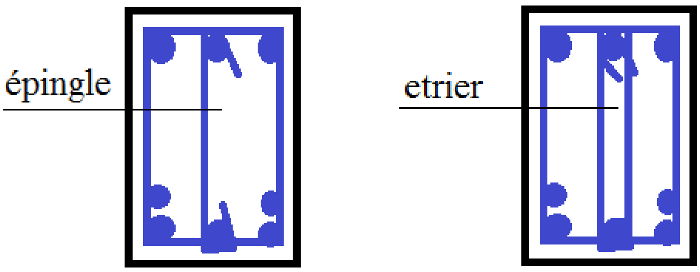


In this case:

$$\left.\begin{array}{l}\mu \rangle \mu_l \\ M \rangle M_l\end{array}\right] \Rightarrow A' \quad \text{is required} \quad \left.\right] \Rightarrow \begin{array}{l} region2b \\ \varepsilon_{bc} = 3.5‰ \end{array}$$

1.2.1 **Practical Arrangements (Dispositions):**

If compression reinforcement is placed outside the corners of the section, the **BAEL code** requires the use of **stirrups (**étriers) **or hooked tie bars (**épingles), spaced at a maximum of 15 Ǿ. (Ǿ: minimum diameter of the compression reinforcement).



1.2.2 **Internal force distribution :**

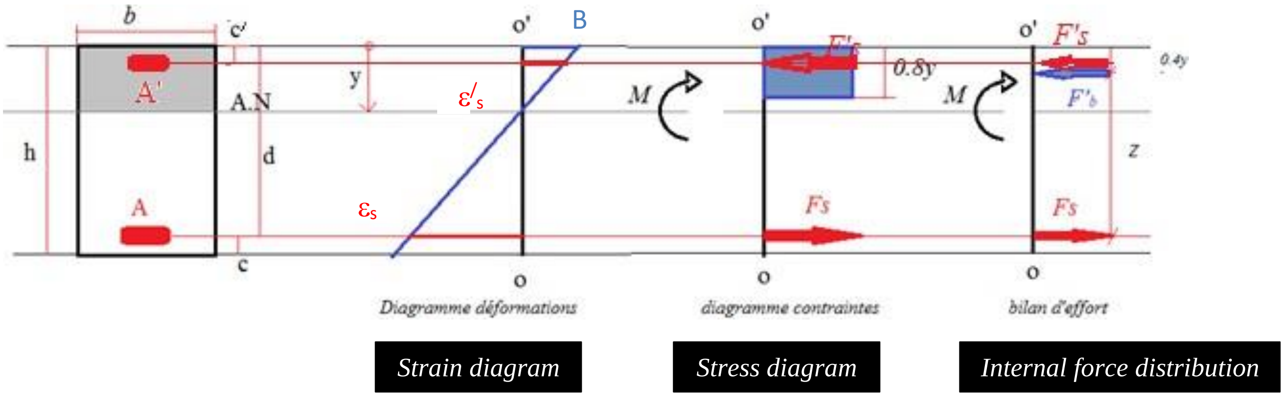


Fig.6.4 Strain, stress, and internal force distribution at the ULS.

A’: total area of compression reinforcement.

C’: distance between the centroid of compression reinforcement and the most compressed concrete fibre.

ε’$_S$: relative shortening strain of compression reinforcement.

F’s: internal compressive force in compression reinforcement.

1.2.3 **Determination of Reinforcement (A; A′):**

Method 01: This method consists of splitting the concrete section subjected to the bending moment **M** into two parts:

- The concrete section balances part of the applied moment, corresponding to the **limiting moment** M_l. This moment is resisted by part of the tensile reinforcement **A₁**.
- The remaining moment M_2=M-M_l is balanced by A_2 et A'

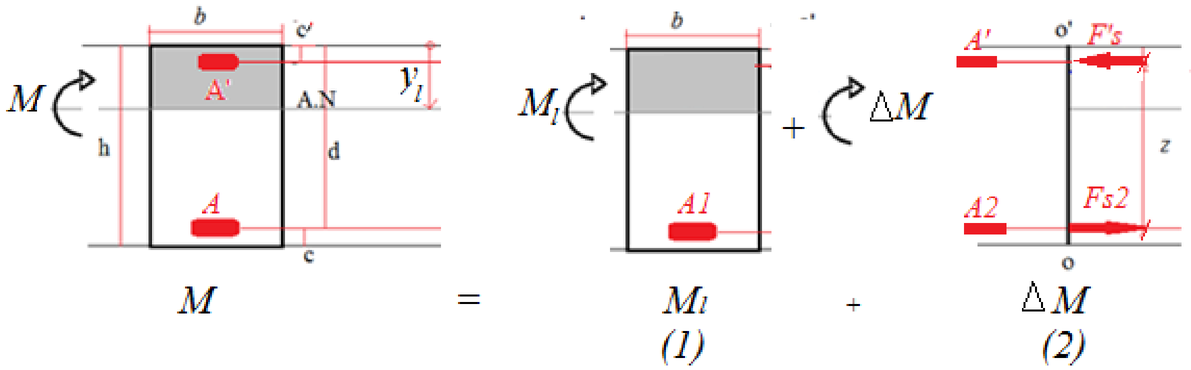


Fig.6.5 Distribution of moments across the different sections
(Concrete and Reinforcement) at the ELU

Section 1 :

$$M_l = \mu_l \sigma_{bc} b d^2 \Rightarrow A_1 = \frac{M_l}{\sigma_s \beta_l d}$$

$$\alpha_l = 1.25\left(1-\sqrt{1-2\mu_l}\right), \beta_l = 1-0.4\alpha_l$$

$$\varepsilon_s = \varepsilon_{sl} = \frac{f_e}{\gamma_s E_s} \Rightarrow \sigma_s = \frac{f_e}{\gamma_s}$$

Et $\alpha_l = \dfrac{3.5}{1000\varepsilon_{sl} + 3.5}$

Section 2 :

The remaining section is subjected to: $\Delta M = M - Ml$

$$\sum M /_{A2} = 0$$

$$\Delta M - F_s^{/}\left(d - c^{/}\right) = 0$$

$$\Delta M - A^{/}\sigma_s^{/}\left(d - c^{/}\right) = 0$$

$$A' = \frac{\Delta M}{\sigma_S \left(d - c'\right)}$$

$$\sigma_s^{'} = ?$$

$$\Delta \equiv \Delta \Rightarrow \frac{\varepsilon_s^{'}}{\varepsilon_{bc}} = \frac{y_l - c'}{y_l} = \frac{\alpha_l d - c'}{\alpha_l d}$$

$$\Rightarrow \varepsilon_s^{'} = \varepsilon_{bc}\frac{\alpha_l d - c'}{\alpha_l d} \qquad \varepsilon_{bc} = 3.5\text{‰}$$

$$\sigma_s^{'} = \begin{cases} Si \quad \varepsilon_s^{'} \geq \varepsilon_{sl}^{'} \Rightarrow \sigma_s = \dfrac{f_e}{\gamma_s} \\ si \quad \varepsilon_s^{'} \langle \varepsilon_{sl}^{'} \Rightarrow \sigma_s = \varepsilon_s^{'} E_s \end{cases}$$

$$\sum M /_{A^{'}} = 0$$

$$\Delta M - F_{s2}\left(d - c'\right) = 0$$

$$\Delta M - A_2 \sigma_s \left(d - c'\right) = 0$$

$$A_2 = \frac{\Delta M}{\sigma_S \left(d - c'\right)}, with \qquad \sigma_s = \frac{f_e}{\gamma_s}$$

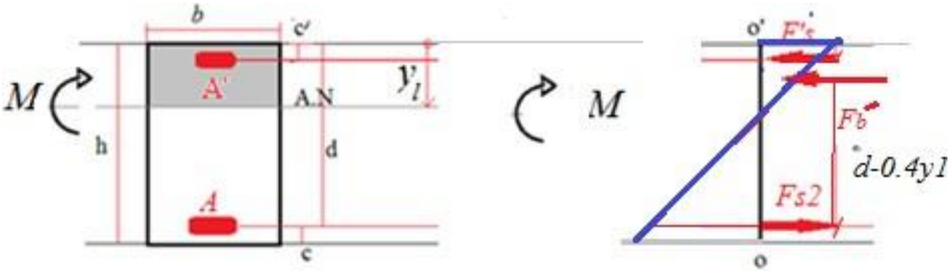
Reinforcement at ELU:

$$A_u = A_1 + A_2$$

$$A^{/} = A^{/}$$

Method 2: The total bending moment M is balanced by the equilibrium of internal forces in the concrete, the tensile reinforcement, and the compressive reinforcement.

The neutral axis corresponds to the limiting moment M_l $(y=y_l)$



$$\sum F=0,\quad \sum M=0$$

$$a-\sum F=0 \Rightarrow F_s^{/}+F_b^{/}+F_s=0$$

With

$$\left.\begin{aligned} F_s^{'} &= A'\sigma_s^{'} \\ F_b^{'} &= 0.8by_l\sigma_{bc} \\ F_s &= A\sigma_s \end{aligned}\right\} \quad A^{'}\sigma_s^{'}+0.8by_l\sigma_{bc}-A\sigma_s=0$$

$$b-\sum M/_A=0 \Rightarrow M-A^{'}\sigma_s^{'}(d-c')-0.8by_l\sigma_{bc}(d-0.4y_l)=0$$

$$\Rightarrow A'=\frac{M-0.8by_l\sigma_{bc}(d-0.4y_l)}{\sigma_s^{'}(d-c')}$$

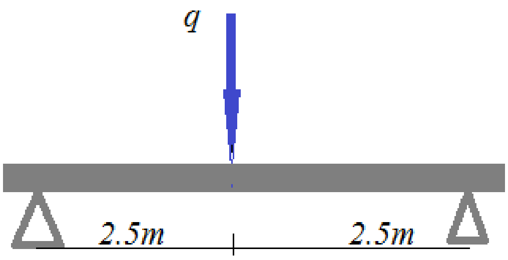
$$\sum F=0 \Rightarrow \qquad \Rightarrow A=\frac{A'\sigma_s^{'}-0.8by_l\sigma_{bc}}{\sigma_s}$$

APPLICATION EXAMPLE

Rectangular Section with Compression Reinforcement

Consider a reinforced concrete beam with a **30 × 60 cm²** section, subjected to a concentrated working load q=264 kN, with: **FeE400**, **fc28 = 25 MPa**, slightly harmful cracking, **c′ = 4 cm**.

Determine the tensile reinforcement area at the **ULS**.



$$g = 0.3*0.6*25 = 4.5kN/ml$$

$$Mg = \frac{gl^2}{8} = 14.06kNm$$

$$Mg = q\frac{1}{4} = 330kNm$$

$$C.F \quad M_u = 1.35Mg - 1.5Mq = 514.98kNm$$

$$\sigma_{bc} = \frac{0.85 f_{c28}}{\theta\gamma_b}, \quad \theta = 1, \quad \gamma_b = 1.5 \Rightarrow \sigma_{bc} = 14.2MPa$$

$$\mu = \frac{M_u}{\sigma_{bc} b d^2} = 0.414$$

$$\mu_{AB} = 0.186 \le \mu = 0.414 \le 0.48 \Rightarrow \quad Pivot\ B$$

$$\Rightarrow \varepsilon_{bc} = 3.5\,^0/_{00}$$

Verification of the need for compression reinforcement

$$\varepsilon_{sl} = \frac{f_e}{\gamma_s E_s} = 1.739\text{‰}$$

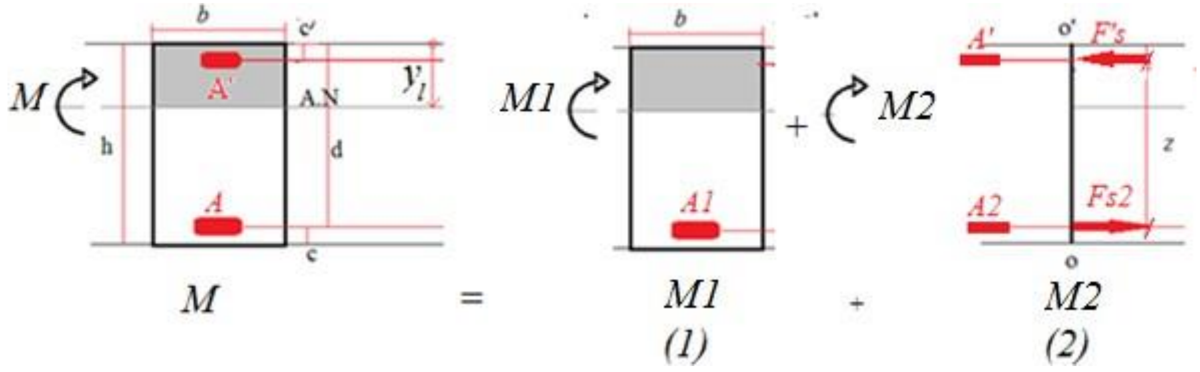
$$\alpha_l = \frac{3.5}{1000\varepsilon_{sl} + 3.5} = 0.668$$

$$\mu_l = 0.8\alpha_l(1 - 0.4\alpha_l) = 0.392$$

$$\mu = 0.414 > \mu_l = 0.392 \quad (\text{région 2b}) \Rightarrow$$

Therefore, **compression reinforcement is required** $A' \exists$

Method 1:



$M_1 = M_l$

$M_l = \mu_l \sigma_{bc} b d^2 = 486948.7Nm$

$\Rightarrow A_1 = \dfrac{M_l}{\sigma_s \beta_l d}$

$\varepsilon_s = \varepsilon_{sl} = \dfrac{f_e}{\gamma_s E_s} \Rightarrow \sigma_s = \dfrac{f_e}{\gamma_s}$=348MPa

$$A_l = \frac{M_l}{\sigma_s \beta_l d} = 35.35cm^2$$

Section 2:

$M_2 = M - M_l = 28032Nm$

$\Rightarrow A_2 = \dfrac{M_2}{\sigma_s \left(d - c^/\right)}$

$\Rightarrow A_2 = 1.55cm^2$

Compression reinforcement

$$\Rightarrow A^/ = \frac{M_2}{\sigma_S^/ \left(d - c^/\right)}$$

$\sigma = fct\left(\varepsilon_s^{'}\right) = ?$

$\Delta \equiv \Delta \Rightarrow \dfrac{\varepsilon_S^{'}}{\varepsilon_{sl}} = \dfrac{y_l - c'}{d - y_l} = \varepsilon_{sl} \dfrac{y_l - c'}{d - y_l}$

$y_l = \alpha_l d \Rightarrow \varepsilon_s^{'} = 3.11‰$

$\varepsilon_s^{'} \rangle \varepsilon_{sl} = 1.739‰ \Rightarrow \sigma_s^{'} = \dfrac{f_e}{\gamma_s} = 348MPa$

$$\Rightarrow A^{/} = \frac{M_2}{\sigma_S^{/}\left(d - c^{/}\right)} = 1.6cm^2$$

Reinforcement at ELU:

$$A_u = A_1 + A_2 = 37cm^2$$

$$A^{/} = A^{/} = 1.6cm^2$$

Minimum reinforcement (tensile reinforcement) :

$f_{t28} = 0.6 + 0.06 f_{c28} = 2.1MPa$

$A_{\min} = 0.23bd \frac{f_{c28}}{f_e} = 1.94cm^2$

Final reinforcement

$A = \max\left(A_u, A_{\min}\right) = 37cm^2$

$A^{/} = 1.6cm^2$

Applied reinforcement (subject to SLS verification):

A_{app}=3x4T20=12T20

A'_{app}=4T12

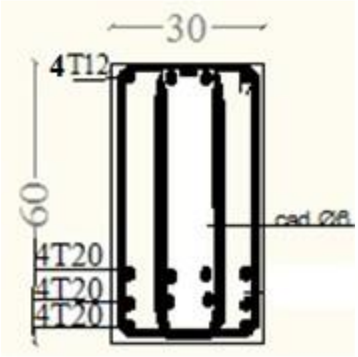
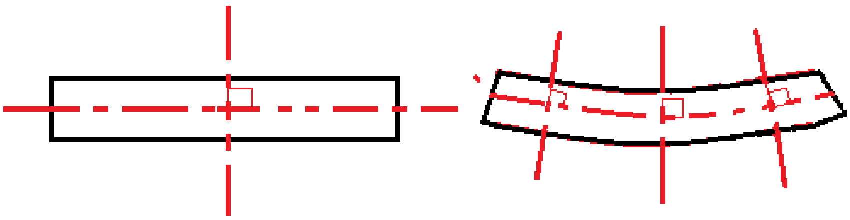


Fig.6.6 **Reinforcement detailing of the beam.**

1.3 SERVICEABILITY LIMIT STATE (ELS)

Calculation hypotheses

H1: Preservation of plane sections: (Navier's hypothesis: small deformations) Cross-sections remain plane after deformation



H2: The tensile strength of concrete is neglected due to cracking.

H3: **Compatibility of steel–concrete deformations**: There is no relative slip between the steel reinforcement and the concrete: $\varepsilon_b = \varepsilon_a$

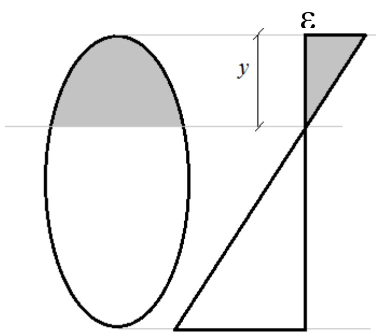
H4: Concrete and steel are considered as linear elastic materials; Hooke's law applies: $\sigma = E\varepsilon$

$$\text{Concrete: } \sigma_b = E_b \varepsilon_b$$

$$\text{Steel: } \sigma_s = E_s \varepsilon_s$$

Hence, strains are proportional to their distance from the neutral axis.

$\varepsilon = ky$



H5: By convention, the ratio of the longitudinal modulus of elasticity of steel to that of concrete (a ratio called the equivalence coefficient n) is taken as:

$$n = \frac{E_s}{E_b} = 15$$

H6 : The areas of steel reinforcement are not deducted from the area of compressed concrete in the calculations.

H7: A group of reinforcing bars may be replaced by a single equivalent bar having the same area and the same centroid as the group of bars.

From these hypotheses, the following results:

a- The stress diagram and the strain diagram are made up of straight lines (stresses are proportional to strains and strain is proportional to the distance to the neutral axis).

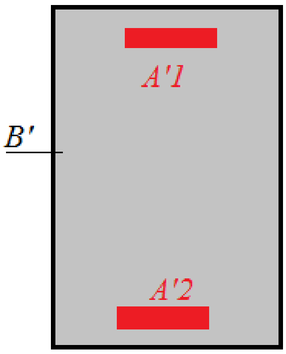
b- By homogenising the reinforced-concrete section, the formulas of Strength of Materials (RDM) are applicable.

This homogenization is achieved by replacing a steel section of area A with a concrete section of area nA=15A having the same centroid as the steel section considered.

Let:

- **B_0**: homogenised concrete section
- **B′**: compressed concrete section

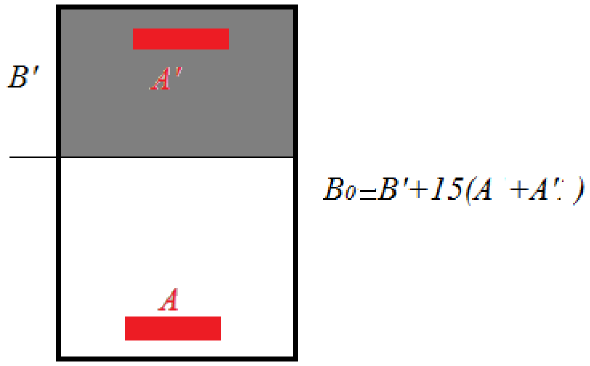
1. **Fully compressed section (FCS)**



$$B_0 = B' + 15(A'1 + A'2)$$

$$B_0 = B' + 15(A') \ ; \ A' = A'1 + A'2$$

2. **Partially compressed section (PCS)**

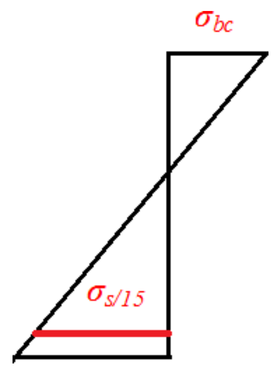


For a fully tensioned section: the section is homogeneous since only the reinforcement is active (*A = constant*).

c- **Stress in steel :**

The stress in a steel fibre is equal to 15 times the stress in a concrete fibre having the same centroid:

$$\sigma_s = n\ \sigma_{bc} = 15\ \sigma_{bc}$$



1.4 Allowable stresses (verifications to be performed)

1.4.1 Allowable stress in compressed concrete:

Limiting the concrete stress is intended to prevent the formation of cracks parallel to the direction of compressive stresses.

$$\sigma_{bc} \leq \overline{\sigma_{bc}} \quad with \quad \overline{\sigma_{bc}} = 0.6 f_{c28}$$

1.4.2 Maximum stresses in tensile reinforcement : (crack control verification)

In order to reduce the risk of crack formation and limit crack width, tensile reinforcement stresses are limited to the limiting stress that depends on the crack category:

- ***Slightly harmful cracking (slightly prejudicial)***
  In this case, the limiting steel stress is:

$$\overline{\sigma_s} = \frac{f_e}{\gamma_s}$$

2. ***Harmful cracking (prejudicial)***
   In this case, the limiting steel stress is:

$$\overline{\sigma_s} = \min\left(\frac{2}{3} f_e, 110\sqrt{\eta f_{t28}}\right)$$

For this cracking category, transverse reinforcement (closed-loop stirrups, stirrups, hooked tie bars) must have a diameter ≥ 6 mm, and the spacing between longitudinal bars *a* must be ≤ 4 times their diameter (if Ø_l > 20 mm).
The minimum diameter of transverse reinforcement for this type of cracking is 6 mm.

- ***Very harmful cracking (very prejudicial)***
  In this case, the limiting steel stress is:

$$\overline{\sigma_s} = \min\left(\frac{2}{3} f_e, 90\sqrt{\eta f_{t28}}\right)$$

For this cracking category, transverse reinforcement (hoops, stirrups, hooked ties) must have a diameter ≥ 8 mm, and the spacing between longitudinal bars *a* must be ≤ 3 times their diameter (if Ø_l > 20 mm).

With, for all cracking categories:

γ_s : safety factor : = 1.15

η : cracking coefficient: = 1.0 for pain bars (R.L)

= 1.6 for deformed bars (H.A)

2.1.2 Verifications to be carried out:

First method:

This method consists of verifying stresses in concrete and reinforcement.

- **Slightly harmful cracking**

$$\sigma_{bc} \leq \overline{\sigma_{bc}} = 0.6\, f_{c28}$$

This verification is unnecessary for a fully tensioned section (FTS).

- **Harmful or very harmful cracking**

$$\sigma_{bc} \leq \overline{\sigma_{bc}} = 0.6\, f_{c28} ; \left(\sigma_s , \sigma_s^{'}\right) \leq \overline{\sigma_s}$$

These stresses in concrete and reinforcement are obtained by considering the reinforcement applied after a calculation in the ultimate limit state of resistance, while verifying the minimum reinforcement.

In the case where the aforementioned inequalities are verified, the reinforcements calculated/designed at the ELU (applied) are suitable for the serviceability limit state (ELS).

Otherwise, the reinforcement calculated/designed at ELU (applied) is not suitable for SLS, and recalculation at ELS is required.

The second method:

This method consists of:

- Calculating the reinforcement at the ultimate limit state (ELU).
- Calculating the reinforcement at the serviceability limit state (ELS).
- Calculating the minimum reinforcement.
- Determining the maximum section of the aforementioned reinforcement.

3.3. Calculation of reinforcement for rectangular sections:

3.3.1Rectangular section without compression reinforcement:

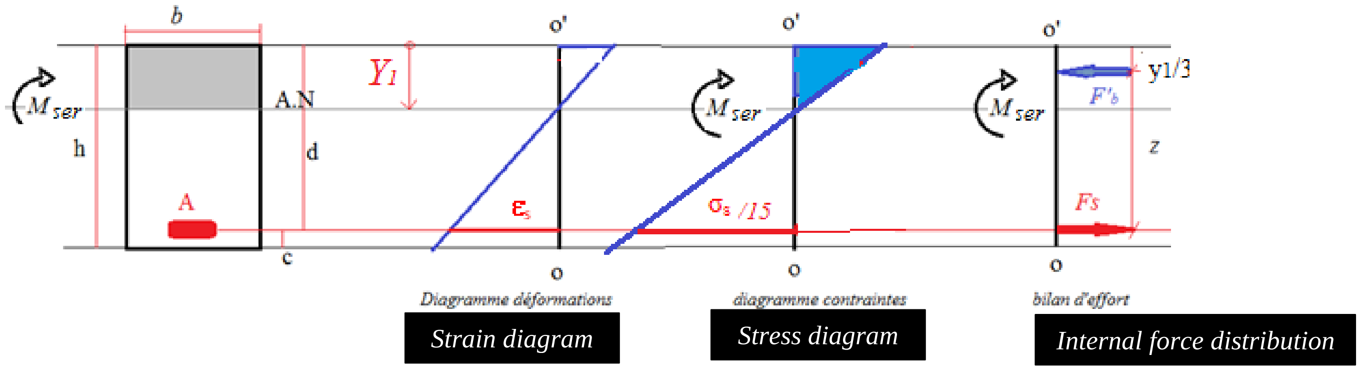


Fig.6.7 Strain, stress, and internal force distributions at the SLS.

$$F_b^{'}=\frac{1}{2}\sigma_{bc}y_l b$$

Let : $k_1=\frac{\sigma_s}{\sigma_{bc}}, y_1=\alpha_1 d$

$$\Delta\equiv\Delta\Rightarrow\alpha_1=\frac{15}{15+k_1}, z=d-\frac{y_1}{3}=d\left(1-\frac{\alpha_1}{3}\right)$$

let : $z=\beta_1 d \Rightarrow \beta_1=1-\frac{\alpha_1}{3}$

$$\sum M/_A=0$$

$$M_{ser}-F_b^{'}z=0\rightarrow M_{ser}-\frac{1}{2}\sigma_{bc}y_1\beta_1 d=0$$

$$M_{ser}=\frac{\alpha_1\beta_1}{2}\sigma_{bc}\beta d^2$$

Let : $\frac{\alpha_1\beta_1}{2}=\mu_1^{'}$ $\qquad M_{ser}=\mu_1^{'}\sigma_{bc}bd^2$

For the resisting moment of the concrete section, it is sufficient to impose the limiting stress, that's to say:

$\sigma_{bc}=\overline{\sigma_{bc}}=0.6f_{c28}$ $\qquad M_{serR}=\mu_1^{'}\overline{\sigma_{bc}}bd^2$

$A'=0\rightarrow$ $\qquad \sigma_{bc}\leq\overline{\sigma_{bc}}=0.6f_{c28}$

Expressing *Mser* as a function of σ_s :

$$M_{ser} = \mu_1^{'} \sigma_{bc} b d^2$$

On a $\frac{\sigma_s}{\sigma_{bc}} = k_1 \rightarrow M_{ser} = \frac{\mu_1^{'}}{k_1} \sigma_s b d^2$

Let : $\mu_1 = \frac{\mu_1^{'}}{k} \rightarrow M_{ser} = \mu_1 \sigma_s b d^2$ $\qquad \mu_1 = \frac{M_{ser}}{\sigma_s b d^2}$

$$\sum M /_{F_b^{'}} = 0$$

$$M_{ser} - F_s z = 0$$

For a limiting case $\rightarrow \sigma_s = \sigma_s$ (*fct de la fissuration*)

$F_s = \overline{\sigma_s} A_{ser}$,*with* $\qquad z = \beta_1 d \rightarrow$

$M_{ser} - \overline{\sigma_s} A_{ser} \beta_1 d = 0 \rightarrow \qquad A_{ser} = \frac{M_{ser}}{\overline{\sigma_s} \beta_1 d}$

The determination of coefficients α_1, β_1 *et* k_1 :

$$\sum M /_A = 0$$

$$M_{ser} - \frac{1}{2} b y_1 \sigma_{bc} \left(d - \frac{y_1}{3} \right) = 0$$

$$M_{ser} - \frac{1}{2} b \alpha_1 d^2 \sigma_{bc} \left(1 - \frac{\alpha_1}{3} \right) = \frac{1}{6} \alpha_1 d^2 \sigma_{bc} \left(3 - \alpha_1 \right) = 0$$

Similar triangles

$$\frac{\sigma_{bc}}{\sigma_s / 15} = \frac{y_1}{d - y_1} \rightarrow \sigma_{bc} = \frac{\sigma_s}{15} \left(\frac{\alpha_1}{1 - \alpha_1} \right)$$

For the calculation of reinforcemet $\rightarrow \sigma_s = \overline{\sigma_s}$ $\qquad$ (*limiting case*)

For a limiting case $\rightarrow \sigma_s = \overline{\sigma_s}$ $\qquad$ (*fct de la fissuration*)

$$M_{ser} - \frac{1}{90} b d^2 \overline{\sigma_s} \alpha_1^2 \left(\frac{3 - \alpha_1}{1 - \alpha_1} \right) = 0$$

After development :

$$\alpha_1^3 - 3\alpha_1^2 - 90\mu_1 \alpha_1 + 90\mu_1 = 0$$

With :

$$\mu_1 = \frac{M_{ser}}{b d^2 \overline{\sigma_s}} et \; 0 \le \alpha_1 \le 1$$

Cubic equation: Approximate method

$$\alpha_1 = 1 + 2\sqrt{\lambda}\cos\left(240^0 + \frac{\varphi^0}{3}\right)$$

With $\lambda = 1 + 30\mu_1$

$$\cos\varphi^0 = \frac{1}{\sqrt[3]{\lambda^2}}$$

De la $\Rightarrow \left\{k_1 = 15\frac{1-\alpha_1}{\alpha_1}\right.$, $\sigma_{bc} = \frac{\overline{\sigma_s}}{k_1}$

$$\beta_1 = 1 - \frac{\alpha_1}{3} \quad , \quad A_{ser} = \frac{M_{ser}}{\overline{\sigma_s}\beta_1 d} si\ \sigma_{bc} \le \overline{\sigma_{bc}}$$

3.3.2- *Rectangular section with compression reinforcement:*

$$\left(Si\ \sigma_{bc} \le \overline{\sigma_{bc}} = 0.6 f_{c28} \quad A^{/} \text{ is required}\right)$$

BAEL rules recommend that the moment resisted by compression reinforcement should be at most equal to 40% of the applied moment (also valid at ULS).

Calculation steps : limiting case

$$\sigma_{bc} = \frac{\overline{\sigma_s}}{15} \rangle \overline{\sigma_{bc}}$$

Limiting case : $\begin{cases} \sigma_s = \overline{\sigma_s} \\ \sigma_{bc} = \overline{\sigma_{bc}} = 0.6 f_{c\,28} \end{cases}$

$$k_1 = \frac{\overline{\sigma_s}}{\overline{\sigma_{bc}}}, \qquad \alpha_1 = \frac{15}{15 + k_1}, \qquad \Delta \cong \rightarrow \sigma_s^{'} = \frac{15(y_1 - c')}{y_1}\overline{\sigma_{bc}}$$

Method 1:

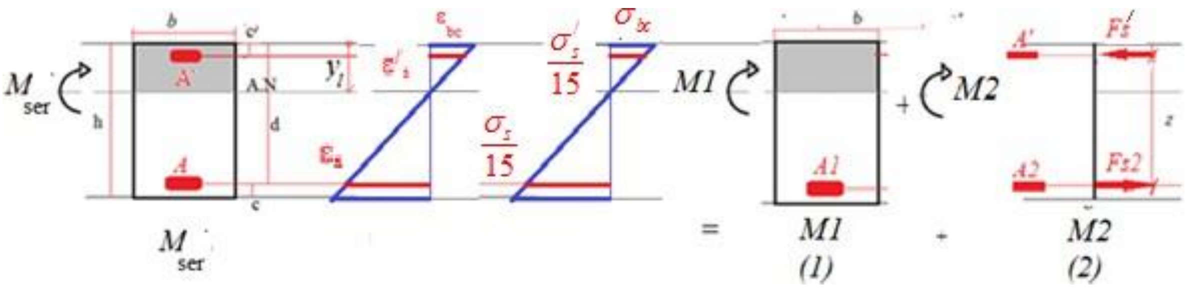


Fig.6.8 Distribution of moments on the different sections (Concrete and Reinforcement) at the SLS.

Section (1) :

$M_1 = M_r$ (resisting moment)

$$M_r = \mu_1' \overline{\sigma_{bc}} bd^2, \mu_1' = \frac{\alpha_1 \beta_1}{2}$$

$$k_1 = \frac{\overline{\sigma_s}}{\overline{\sigma_{bc}}}, \quad \alpha_1 = \frac{15}{15 + k_1}, \quad \beta_1 = 1 - \frac{\alpha_1}{3}$$

after equilibrium

$$A_1 = \frac{M_r}{\overline{\sigma_s} \beta_1 d}$$

Section (2) :

$$M_2 = M_{ser} - M_1 \leq 0.40 M_{ser}$$

$$\sum M /_{A2} = 0$$

$$M_2 - F_s^{/} \left(d - c^{/}\right) = 0$$

$$\Delta M - A^{/} \sigma_s^{/} \left(d - c^{/}\right) = 0$$

$$A^{/} = \frac{\Delta M}{\sigma_s^{/} \left(d - c^{/}\right)}$$

With

$$\begin{cases} \sigma_s' = 15 \dfrac{y_1 - c'}{y_1} \overline{\sigma_{bc}} \\ y_1 = \alpha_1 d \partial \end{cases}$$

$$\sum M /_A = 0$$

$$\Delta M - F_{S2} \left(d - c'\right) = 0$$

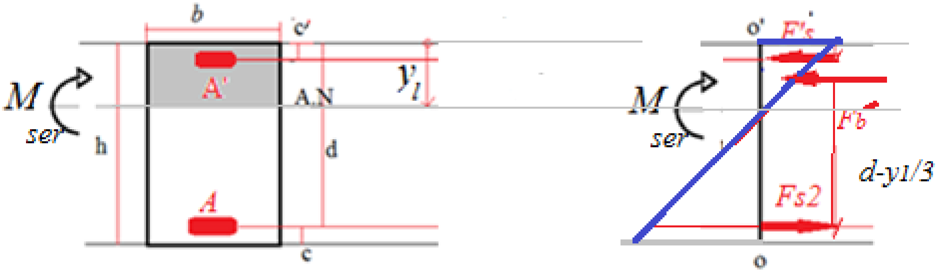
$$\Delta M - A_2 \overline{\sigma_S} \left(d - c'\right) = 0$$

$$A_2 = \frac{\Delta M}{\overline{\sigma_s} \left(d - c'\right)}$$

Finally, the calculated reinforcement will be:

$$\begin{cases} A_{ser} = A_1 + A_2 \\ A'_{ser} = A^{/} \end{cases}$$

Method 2:



$$\sum F = 0, \quad \sum M = 0$$

$$a - \sum F = 0 \Rightarrow F_s^{/} + F_b^{/} + F_s = 0$$

with

$$\begin{array}{l} F_S^{'} = A_{ser}^{'} \sigma_s^{'} \\ F_b^{'} = \frac{1}{2} b y_l \overline{\sigma_{bc}} \\ F_S = A_{ser} \overline{\sigma_s} \end{array} \Bigg| \; A_{ser}^{'} \sigma_s^{'} + \frac{1}{2} b y_l \overline{\sigma_{bc}} - A_{ser} \overline{\sigma_s} = 0$$

$$b - \sum M \; /_A = 0 \Rightarrow M_{ser} - A_{ser}^{'} \sigma_s^{'} (d - c') - \frac{1}{2} b y_l \overline{\sigma_{bc}} \left(d - \frac{y_1}{3} \right) = 0$$

$$\Rightarrow A_{ser}^{'} = \frac{M_{ser} \; \frac{1}{2} b y_l \overline{\sigma_{bc}} \left(d - \frac{y_1}{3} \right)}{\sigma_s^{'} (d - c')}$$

With

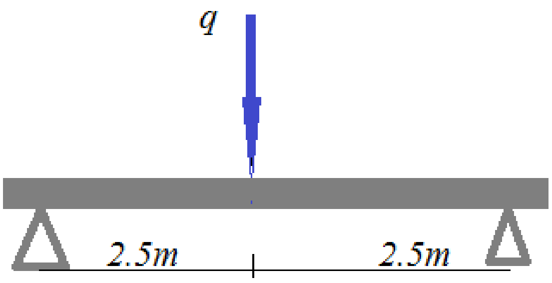
$$\begin{cases} \sigma_s^{'} = 15 \frac{y_1 - c'}{y_1} \overline{\sigma_{bc}} \\ y_1 = \alpha_1 d \, \partial \end{cases}$$

$$\sum F = 0 \Rightarrow \qquad \Rightarrow A_{ser} = \frac{A_{ser}^{'} \sigma_s^{'} + \frac{1}{2} b y_l \overline{\sigma_{bc}}}{\overline{\sigma_s}}$$

APPLICATION EXAMPLE

Rectangular section with compression reinforcement

Consider a reinforced concrete beam with a cross-section of 30x60 cm^2 γ_b=25kN/m^3 subjected to a **concentrated working load** q=264 kN, with; FeE400; f_{c28}=25MPa; harmful cracking; C'=4cm. Determine the **reinforcement areas**.



a) **Ultimate Limit State (ULS)**

$$g = 0.3*0.6*25 = 4.5kN/ml$$

$$Mg = \frac{gl^2}{8} = 14.06kNm$$

$$Mg = q\frac{1}{4} = 330kNm$$

$$C.F \quad M_u = 1.35Mg - 1.5Mq = 514.98kNm$$

$$\sigma_{bc} = \frac{0.85 f_{c28}}{\theta\gamma_b}, \quad \theta = 1, \quad \gamma_b = 1.5 \Rightarrow \sigma_{bc} = 14.2MPa$$

$$\mu = \frac{M_u}{\sigma_{bc} b d^2} = 0.414$$

$$\mu_{AB} = 0.186 \le \mu = 0.414 \le 0.48 \Rightarrow \quad Pivot\ B$$

$$\Rightarrow \varepsilon_{bc} = 3.5^0/_{00}$$

Verification of the need for compression reinforcement

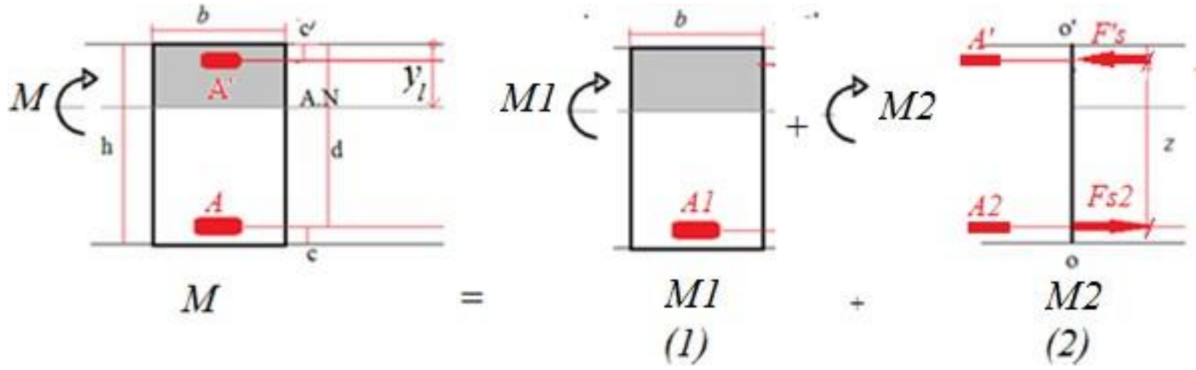
$$\varepsilon_{sl} = \frac{f_e}{\gamma_s E_s} = 1.739‰$$

$$\alpha_l = \frac{3.5}{1000\varepsilon_{sl} + 3.5} = 0.668$$

$$\mu_l = 0.8\alpha_l(1 - 0.4\alpha_l) = 0.392$$

$\mu = 0.414 > \mu_l = 0.392 \quad (\text{région 2b}) \Rightarrow$

compression reinforcement is required $\quad A' \exists$



$$M_1 = M_l$$

$$M_l = \mu_l \sigma_{bc} b d^2 = 486948.7 Nm$$

$$\Rightarrow A_1 = \frac{M_l}{\sigma_s \beta_l d}$$

$$\varepsilon_s = \varepsilon_{sl} = \frac{f_e}{\gamma_s E_s} \Rightarrow \sigma_s = \frac{f_e}{\gamma_s} = 348 \text{MPa}$$

$$A_l = \frac{M_l}{\sigma_s \beta_l d} = 35.35 cm^2$$

Section 2:

$$M_2 = M - M_l = 28032 Nm$$

$$\Rightarrow A_2 = \frac{M_2}{\sigma_s (d - c')} = 1.55 cm^2$$

Compression reinforcement:

$$\Rightarrow A' = \frac{M_2}{\sigma_s' (d - c')}$$

$$\sigma_s' = fct(\varepsilon_s') = ?$$

$$\Delta \equiv \Delta \Rightarrow \frac{\varepsilon_s'}{\varepsilon_{sl}} = \frac{y_l - c'}{d - y_l} = \varepsilon_{sl} \frac{y_l - c'}{d - y_l}$$

$$y_l = \alpha_l d \Rightarrow \varepsilon_s' = 3.11‰$$

$$\varepsilon_s' \rangle \varepsilon_{sl} = 1.739 ‰ \Rightarrow \sigma_s' = \frac{f_e}{y_s} = 348 MPa$$

$$\Rightarrow A' = \frac{M_2}{\sigma_s' (d - c')} = 1.6 cm^2$$

Reinforcement at ULS

$$A_u = A_1 + A_2 = 37 cm^2$$

$$A' = A' = 1.6 cm^2$$

c) Serviceability Limit State ELS

$$Mg = \frac{gl^2}{8} = 14.06kNm$$

$$Mg = q\frac{l}{4} = 330kNm$$

ELS $\quad M_{ser} = Mg + Mq = 344.06kNm$

$$\mu_l = \frac{M_{ser}}{\overline{\sigma_s} bd^2}$$

Harmful cracking

$$\overline{\sigma_s} = \min\left(\frac{2}{3}f_e, 110\sqrt{\eta f_{t28}}\right)$$

$f_{t28} = 2.1MPa, \qquad \overline{\sigma_s} = 201.6MPa$

$$\mu_l = \frac{M_{ser}}{\overline{\sigma_s} bd^2} = 0.01951$$

$$\lambda = 1 + 30\mu_1 = 1.585$$

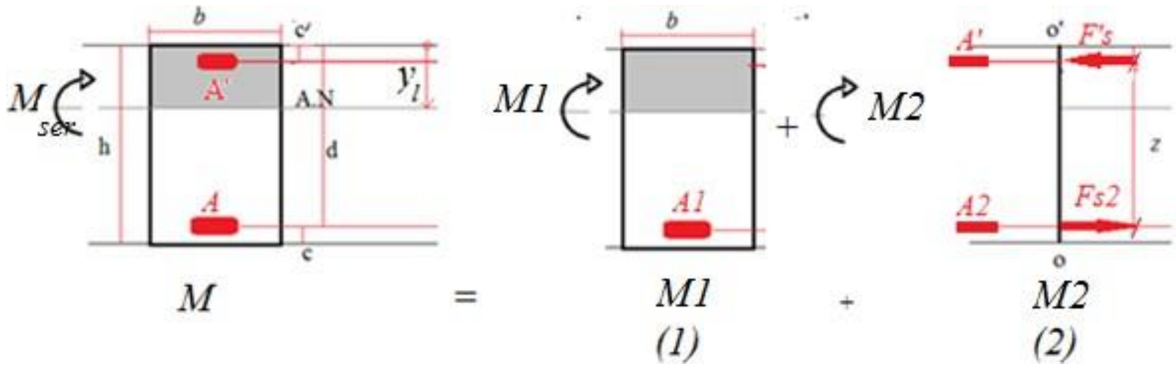
$$\cos\varphi = \frac{1}{\sqrt{\lambda^3}} = 0.501 \rightarrow \varphi = 59.93^0$$

$$\alpha_1 = 1 + 2\sqrt{\lambda}\cos\left(240 + \frac{\varphi}{3}\right) = 0.562$$

$$k_1 = 15\frac{1-\alpha_1}{\alpha_1} = 11.7$$

$$\sigma_{bc} = \frac{\overline{\sigma_s}}{k_1} = 17.2MPa \rangle \overline{\sigma_{bc}} = 0.6f_{c28} = 15MPa$$

$\Rightarrow A'$ is required



Section 1 :

$$\sigma_s = \overline{\sigma_s} = 201.6MPa$$

$$\sigma_{bc} = \overline{\sigma_{bc}} = 15MPa$$

$$k_1 = \frac{\overline{\sigma_s}}{\overline{\sigma_{bc}}} = \frac{201.6}{15} = 13.44MPa$$

$$\alpha_1 = \frac{15}{15+k_1} = \frac{15}{15+13.44} = 0.527$$

$$\beta_1 = 1 - \frac{\alpha_1}{3} = 1 - \frac{0.527}{3} = 0.824$$

$$\mu_1^{'} = \frac{\alpha_1 \beta_1}{2} = \frac{0.527*0.824}{2} = 0.217$$

$$y_1 = \alpha_1 d = 0.527*54 = 28.46cm$$

$$\sigma_s^{'} = 15\frac{(y_1 - c')}{y_1}\overline{\sigma_{bc}} = 185.46MPa$$

$M_1 = M_r$ (resisting moment)

$$M_r = \mu_1^{'}\overline{\sigma_{bc}}bd^2 = 0.217*30*54^2*15 = 284747.4Nm$$

$$A_1 = \frac{M_r}{\overline{\sigma_s}\beta_1 d} = \frac{284747.4}{201.6*0.824*54} = 31.75cm^2$$

Section 2:

$M_2 = M_{ser} - M = 59312Nm \langle 0.4M_{ser}$ C.V

$$\Rightarrow A^{'} = \frac{M_2}{\sigma_s^{'}(d-c')} = \frac{59312}{185.46*(54-4)} = 6.4cm^2$$

$$\Rightarrow A_2 = \frac{M_2}{\overline{\sigma_s}(d-c')} = \frac{59312}{201.6*(54-4)} = 5.9cm^2$$

Reinforcement at ELS

$A_{ser} = A_1 + A_2 + 37.75cm^2$

$A_{min} = A' = 6.4cm^2$

Minimum reinforcement (tension reinforcement):

$f_{t28} = 0.6 + 0.06 f_{c28} = 2.1MPa$

$A_{min} = 0.23bd \dfrac{f_{t28}}{f_e} = 1.96cm^2$

Final reinforcement :

$$\text{ELU} \rightarrow A_u + A_u^{'} = 36.9 + 1.55 = 38.45cm^2$$
$$\text{ELS} \rightarrow A_{ser} + A_{ser}^{'} = 37.75 + 6.4 = 44.15cm^2$$
$$A_f = \max\left(A_u + A_u^{'}, A_{ser} + A_{ser}^{'}\right) = 44.15cm^2$$

So SLS is dominant with:

$A_{ser} = A_1 + A_2 + 37.75cm^2 \rangle A_{min} = A' = 1.96cm^2$

$A_{ser} = A' = 6.4cm^2$

Applied reinforcement :

$$A_{app} = 3*4T\,20 = 12T\,20$$
$$A_{app}^{'} = 2T\,16 = 2T\,14$$

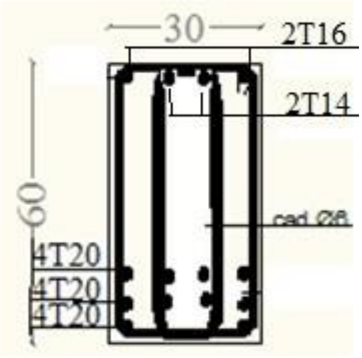


Fig.6.9 Schéma de ferraillage de la poutre.

3.4 Determination of Stresses :

The study is carried out for the **general case of a rectangular section with compression reinforcement** A’. If compression reinforcement is not required, **A′ is taken equal to zero**, and the calculation of **σ′** is not necessary.

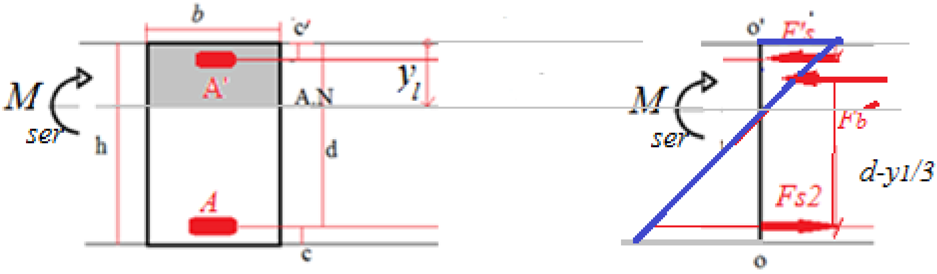


Fig.6.10 Internal force equilibrium at the SLS.

We have :

$\sigma_{bc} = E_b \varepsilon_{bc}$, Posons $\varepsilon_{bc} = k y_1$

$\sigma_{bc} = E_b k y_1$, with $K = kE_b$

$$\sigma_{bc} = K y_1$$

$\Delta \equiv \Delta$

$$\frac{\sigma_{bc}}{\sigma_s / 15} = \frac{y_1}{d - y_1}$$

$\sigma_s y_1 = 15\sigma_{bc} (d - y_1)$

$\sigma_s y_1 = 15K y_1 (d - y_1)$

$$\sigma_s = 15K (d - y_1)$$

$\Delta \equiv \Delta$

$$\frac{\sigma_{bc}}{\sigma_s' / 15} = \frac{y_1}{y_1 - c'}$$

$\sigma_s' y_1 = 15\sigma_{bc} (y_1 - c')$

$\sigma_s' y_1 = 15K y_1 (y_1 - c')$

$$\sigma_s' = 15K (y_1 - c')$$

$$\begin{cases} \sigma_{bc} = K y_1 \\ \sigma_s = 15K (d - y_1) \quad \text{with} \quad [y_1] cm, [\sigma] MPa \\ \sigma_s' = 15K (y_1 - c') \end{cases}$$

3.4.1 Position of the Neutral Axis:

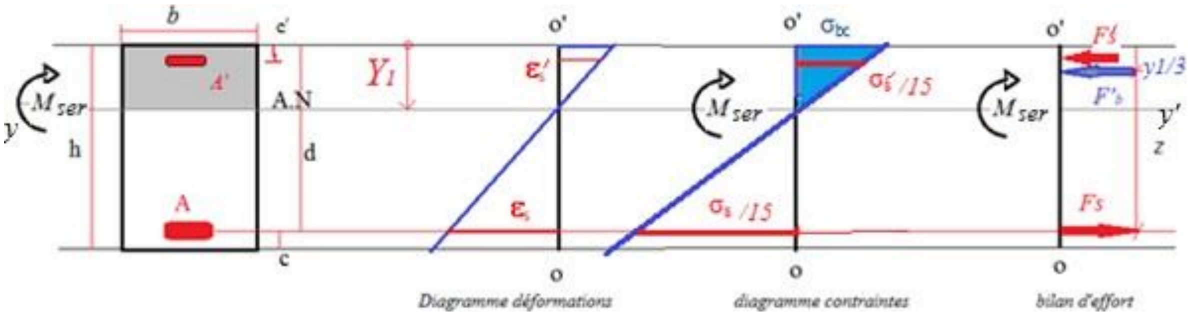


Fig.6.11 Position of the neutral axis at the ELS.

d = 0.9h: effective depth.

c = 0.1h: distance between the centroid of tensile reinforcement and the most tensioned fiber.

c' = 0.1h: distance between the centroid of compression reinforcement and the most compressed fiber.

Y_1: distance from the neutral axis to the most compressed fiber. The position of the neutral axis can be obtained by stating that the statical moment with respect to the neutral axis is zero (Syy'=0).

A: total area of tensile reinforcement

A': total area of compression reinforcement.

F'_b: compressive force in concrete.

F_c: Tensile force in tension reinforcement

F'_c : compressive force in compression reinforcement

The position of the neutral axis (A.N) can be obtained by the static moment with respect to the neutral axis (Syy'=0).

$$S_{yy'} = 0$$

$$by_1 \frac{y_1}{2} + 15A'(y_1 - c') - 15A(d - y_1) = 0$$

$$y_1 = \text{ positive root}$$

Determination of the Coefficient K:

$$\sum M /_{yy'} = 0$$

$$M_{ser} - F_S^{'}(y_1 - c') - F_b^{'}\left(y_1 - \frac{y_1}{3}\right) - F_s(d - y_1) = 0$$

$$M_{ser} - A'\sigma_s^{'}(y_1 - c') - \frac{1}{2}by_1\sigma_{bc}\left(y_1 - \frac{y_1}{3}\right) - A\sigma_S(d - y_1) = 0$$

With $\begin{cases} \sigma_{bc} = Ky_1 \\ \sigma_s = 15K(d - y_1) \\ \sigma_s^{'} = 15K(y_1 - c') \end{cases}$

$$M_{ser} - A'15K(y_1 - c')^2 - \frac{1}{3}bKy_1^3 - A15K(d - y_1)^2 = 0$$

$$\frac{M_{ser}}{K} = \frac{1}{3}by_1^3 + 15A(d - y_1)^2 + 15A'(y_1 - c')^2$$

$$\frac{M_{ser}}{K} = I_{yy'} \qquad K = \frac{M_{ser}}{I_{yy'}}$$

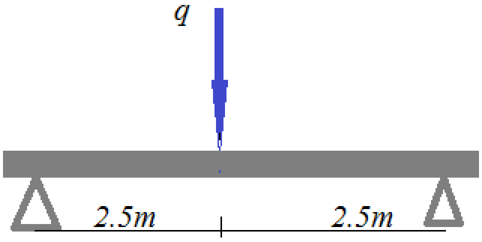
$$[M_{ser}] = Nm, [I_{yy'}] = cm^4$$

For the applied reinforcement to be suitable for the serviceability limit state (SLS), the following conditions must be met:

$$\begin{cases} 1)\sigma_{bc} = Ky_1 \le \overline{\sigma_{bc}} \\ 2)\sigma_s = 15K(d - y_1) \le \overline{\sigma_s} \\ 3)\sigma_s^{'} = 15K(y_1 - c') \le \overline{\sigma_s} \end{cases}$$

If any of these inequalities is not verified, the reinforcement must be recalculated at the SLS.

APPLICATION EXAMPLE



Consider a reinforced concrete beam with a cross-section of 30x60 cm^2 subjected to a concentrated working load q=264 kN, with; FeE400; f_{c28}=25MPa; harmful cracking, c'=4 cm. Following the **ELU design**, verify stresses at the **ELS**

ELS :

$$\begin{cases} g = 0.3*0.6*25 = 4.5kN\ /ml \\ Mg = \dfrac{gl^2}{8} = 14.06kNm \\ Mq = q\dfrac{l}{4} = 330kNm \end{cases}$$

$$\begin{cases} A_u = 36.90cm^2 \rightarrow soit\ 8T\ 25 = 39.25cm^2 \\ A_u^{'} = 1.56cm^2 \rightarrow soit\ 2T\ 12 = 2.26cm^2 \end{cases}$$

ELS : Streesses verification

Mser = 344.06kNm

The following conditions must be verified

$$\begin{cases} 1)\sigma_{bc} = Ky_1 = 0.546*29.86 = 16.30MPa \rangle \overline{\sigma_{bc}} = 15MPa \\ 2)\sigma_s = 15K(d - y_1) = 15*0.546(54 - 29.86) = 190.6 \langle \overline{\sigma_s} \\ 3)\sigma_s^{'} = 15K(y_1 - c') = 15*0.546(29.86 - 4) = 211.8 \rangle \overline{\sigma_s} \end{cases}$$

1) Calculation of the position of the neutral axis y_1

a) calcul de y_1

$I_{yy'} = 630276cm^4$ $S_{yy'} = 0$

$$by_1\frac{y_1}{2} + 15A'(y_1 - c') - 15A(d - y_1) = 0$$

$$15y_1^2 + 15*2.26(y_1 - 5) - 15*39.25(d - y_1) = 0$$

$$15y_1^2 + 622.25y_1 - 31962 = 0$$

$$\Delta = 2304915 \qquad y_1 = \frac{-622.25 + \sqrt{2304915}}{30} = 29.86cm$$

b) calcul de K

$$A\min = \frac{I_{y_0y_0^{'}}}{0.81hV^{'}}\frac{f_{t28}}{f_e}$$

$$I_{yy'} = \frac{1}{3}30*29.86^3 + 15*39.25(54 - 29.86)^2 + 15*2.26(29.86 - 4)^2$$

$$K = \frac{[M_{ser}]Nm}{[I_{yy'}]cm^4} = \frac{344060}{630276.6} = 0.5459$$

$$K = \frac{[M_{ser}]Nm}{[I_{yy'}]cm^4} = \frac{344060}{630276.6} = 0.5459$$

$$\begin{cases} 1)\sigma_{bc} = Ky_1 = 0.546*29.86 = 16.30MPa \rangle \overline{\sigma_{bc}} = 15MPa \\ 2)\sigma_s = 15K(d - y_1) = 15*0.546(54 - 29.86) = 190.6 \langle \overline{\sigma_s} \\ 3)\sigma_s^{'} = 15K(y_1 - c') = 15*0.546(29.86 - 4) = 211.8 \rangle \overline{\sigma_s} \end{cases}$$

Conclusion :

The reinforcement designed at the ELU does not satisfy the ELS requirements.
The reinforcement areas must be recalculated at the ELS.

6.II. T-SECTION

1. GENERAL DEFINITIONS :

1.1. Generalities:

The **T-section** is frequently encountered in **building floors**, **bridge decks**, **retaining walls**, etc. The use of T-sections in construction allows a **significant saving in concrete**, (reduction in the concrete section which is subjected to tensile stresses (tension zone).

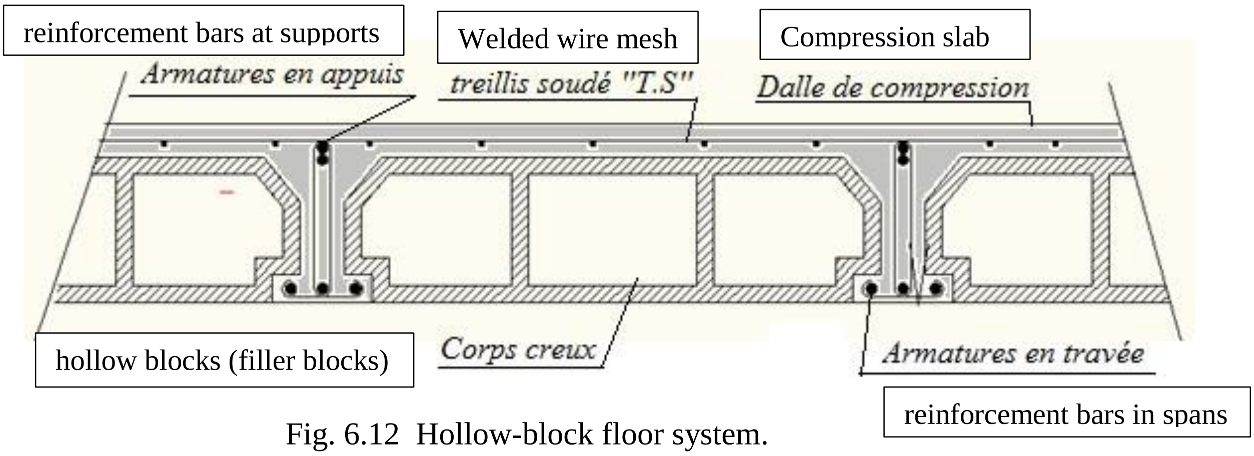


Fig. 6.12 Hollow-block floor system.

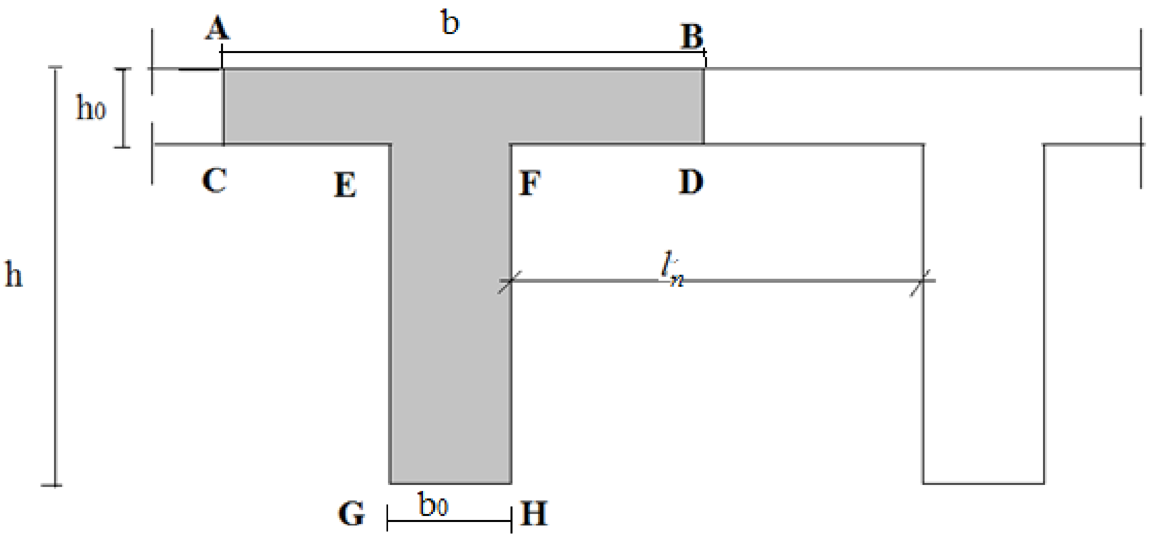


Fig. 6.13 T Section.

Technical terminology:

Part ABCD: compression flange

Part EFGH: rib or web

For all that follows, the **bending moment is considered positive**.
If the bending moment is **negative**, the calculation becomes the same as calculating a rectangular section "b0h"

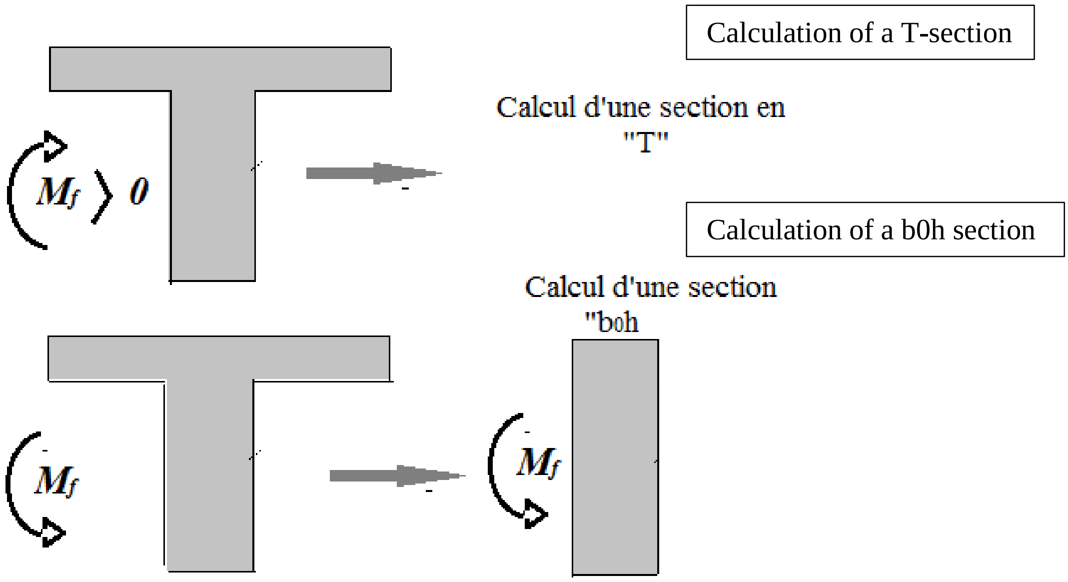
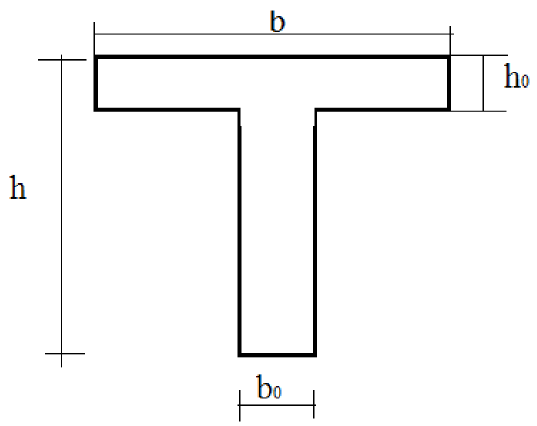


Fig. 6.14 Cross-section of the beam.

Notations

- **h**: total depth (height)
- **b**: flange width
- **h₀**: flange thickness (height)
- **b₀**: rib/web width



1-2 Effective width of the compression flange:

$$\frac{b-b_0}{2} \leq \min\left(\frac{l}{10};\frac{l_n}{2}\right)$$

l : span length (minimum length between the inner faces of supporting beams)

l_n : distance between the inner faces of two adjacent ribs

2. ULTIMATE LIMIT STATE (ELU) CALCULATION

2.1. Position of the neutral axis:

For **positive bending moment** (calculation of a T section), two cases are distinguished:

a) Neutral axis within the compression flange:

In this case, the compression flange alone can resist the compressive stresses produced by the bending moment, hence: $h_0 \geq 0.8$ y.

As a limit case of this case $h_0 = 0.8$ y.

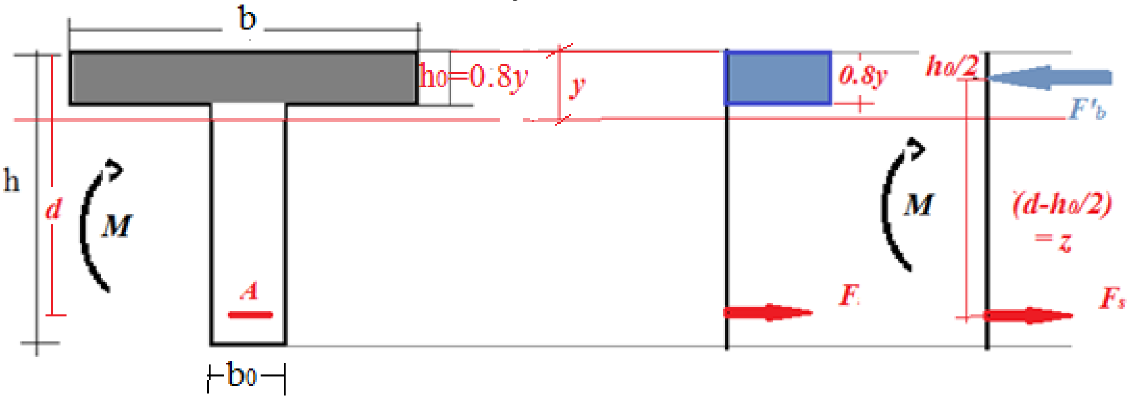
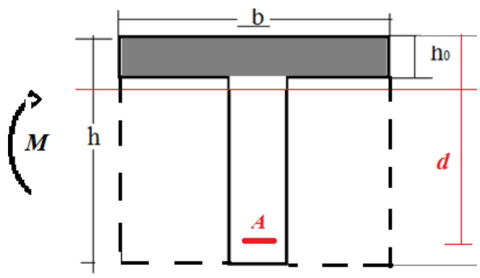


Fig. 6.15 Neutral axis in the flange (h_0=0.8y).

Resisting moment of the flange « M_T » :

$$\left.\begin{aligned} M_{bt} &= M_T = F_b' z \\ F_b' &= 0.8 y b \sigma_{bc} = h_0 b \sigma_{bc} \end{aligned}\right\} \Rightarrow M_T = b h_0 \sigma_{bc}\left(d - \frac{h_0}{2}\right)$$

If $M_u \leq M_T$: **neutral axis in the flange:** Therefore, the compression flange area « bh_0 » can resist the compressive stresses due to the ultimate moment on its own. In this case, it involves calculating a rectangular section with section **« bh ».**



b) Neutral axis within the web:

if $M_u > M_T$ **neutral axis in the web**: the compression flange cannot resist the compressive stresses produced by the bending moment, hence $h_0 < 0.8$ y.

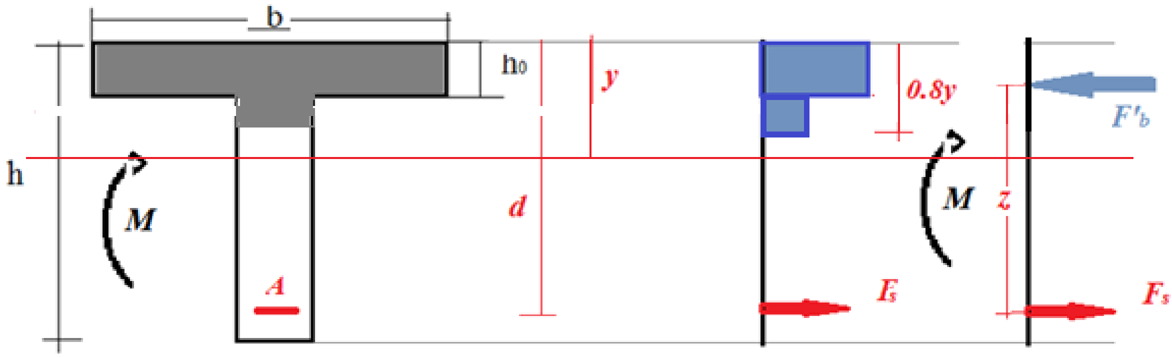


Fig. 6.16 Neutral axis in the web ($h_0 < 0.8$y).

In this case, the reinforcement calculation must take into account, in addition to the compression flange, the contribution of a portion of the rib to the beam's resistance. Therefore, it involves calculating a T-section.

2.2. T-section without compression reinforcement: Calculating a T-section involves decomposing the T-shaped cross-section into two sections, namely:

- Rectangular section with dimensions b0h (the rib).

- Rectangular section with dimensions bh0 (the flange).

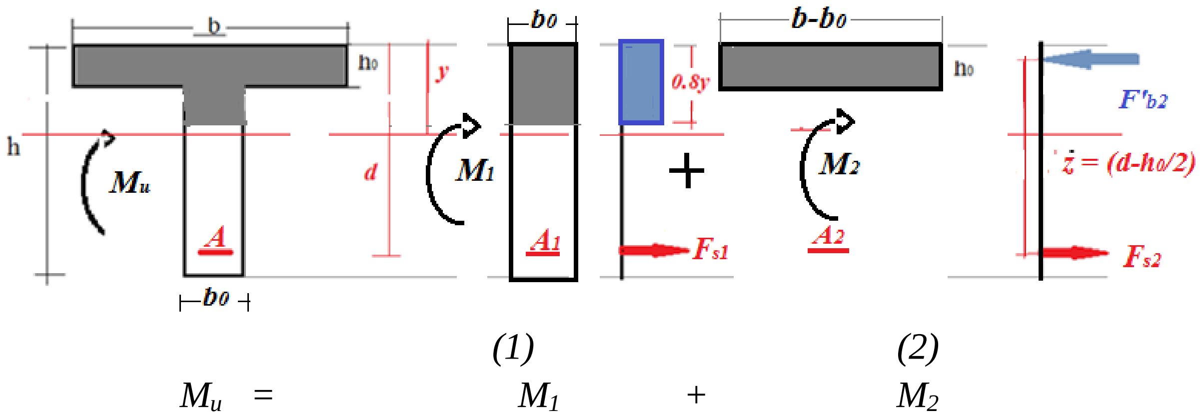


Fig.6.17 Decomposition of the T-section at ULS without compression reinforcement.

Calculation of M_1 et M_2 :

Section (2

$$\sum M / A_2 = 0 \rightarrow M_2 - F'_{b2}\left(d - \frac{h_0}{2}\right)$$

$$M_2 = (b - b_0) h_0 \sigma_{bc}\left(d - \frac{h_0}{2}\right)$$

$$\sum M / F'_{b2} \rightarrow M_2 - F_s\left(d - \frac{h_0}{2}\right) = 0$$

$$M_2 - A_2\sigma_s\left(d - \frac{h_0}{2}\right) = 0$$

$$\rightarrow (b - b_0) h_0 \sigma_{bc}\left(d - \frac{h_0}{2}\right) = A_2\sigma_s\left(d - \frac{h_0}{2}\right) = 0$$

$$A_2 = \frac{(b - b_0) h_0 \sigma_{bc}}{\sigma_s}$$

$$\sigma_s = \frac{f_e}{\gamma_s}$$

Section (1) :

$M_1 = M_n$ (bending moment acting on the web) $= M_u - M_2$

The reinforcement calculation is equivalent to the calculation of a rectangular section (b0h) subjected to $M_1 = M_n$.

Pivot B :

$$si \quad \left.\begin{matrix} M_n \leq M_l \\ \mu_n \leq \mu_l \end{matrix}\right] \Rightarrow$$

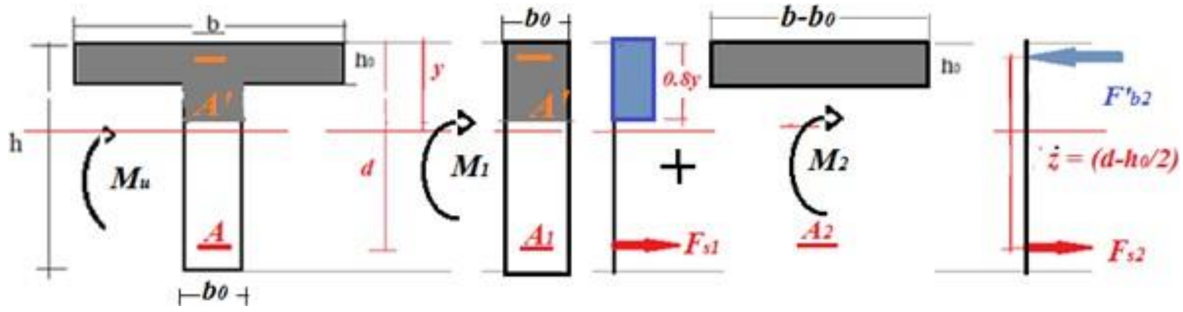
Compression reinforcement is not necessary, because tension reinforcement works at its ultimate capacity and the compressed concrete alone resists the compressive stresses: A'=0

$$\mu_n = \frac{M_n}{b\sigma_{bc} b_0 d^2} \Rightarrow \begin{cases} \mu_n \leq \mu_{AB} = 0.186 \rightarrow \ PivotA \\ A' = 0, \sigma_s = \dfrac{f_e}{\gamma_s} \\ A_u = \dfrac{M_n}{\sigma_s \beta d} \\ \mu_{AB} \langle \mu_n \leq \mu_{bc} = 0.48 \rightarrow \ PivotB \ \rightarrow A_u = \dfrac{M_n}{\sigma_s \beta d} \end{cases}$$

$$\text{if} \quad \left.\begin{matrix} M_n \rangle M_l \\ \mu_n \rangle \mu_l \end{matrix}\right] \Rightarrow$$

The tension reinforcement does not work at its ultimate capacity $\sigma_s = f_e/\gamma_s$. Therefore, to make the tension reinforcement work at its ultimate capacity ($\varepsilon_s = \varepsilon_{sl}$; $\sigma_s = f_e/\gamma_s$), it is necessary to provide compression reinforcement A'. Therefore A' is necessary.

2.3. T-section with compression reinforcement:



(1) (2)

M_u = M_1 + M_2

Fig.6.18 Decomposition of the T-section at ULS with compression reinforcement.

Section (2)

$$\sum M / A_2 = 0 \rightarrow M_2 - F'_{b2}\left(d - \frac{h_0}{2}\right)$$

$$M_2 = (b - b_0) h_0 \sigma_{bc}\left(d - \frac{h_0}{2}\right)$$

$$\sum M / F'_{b2} \rightarrow M_2 - F_s\left(d - \frac{h_0}{2}\right) = 0$$

$$M_2 - A_2\sigma_s\left(d - \frac{h_0}{2}\right) = 0$$

$$\rightarrow (b - b_0) h_0 \sigma_{bc}\left(d - \frac{h_0}{2}\right) = A_2\sigma_s\left(d - \frac{h_0}{2}\right) = 0$$

$$A_2 = \frac{(b - b_0) h_0 \sigma_{bc}}{\sigma_s}$$

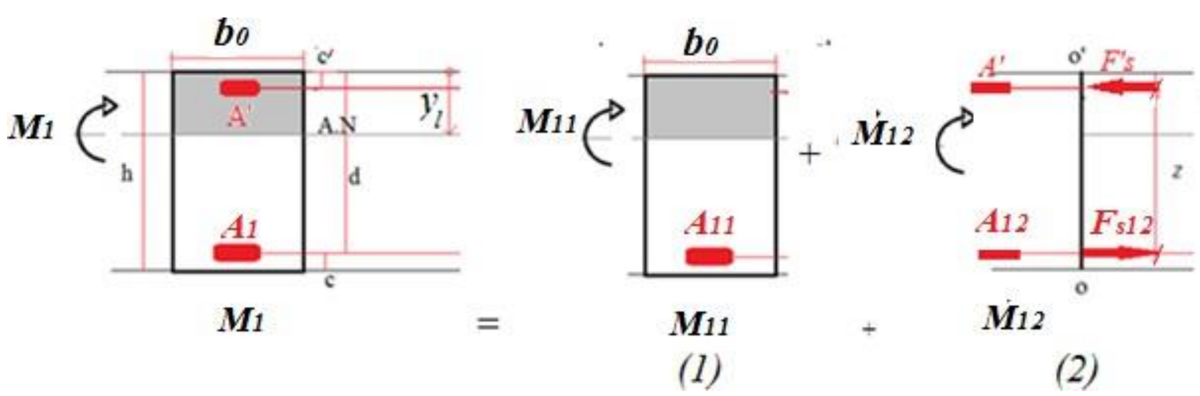
$$\sigma_s = \frac{f_e}{\gamma_s}$$

Section (1)

$M_1 = M_n = M_u - M_2$

$$\left.\mu_1 = \mu_n = \mu_n = \frac{M_1}{b\sigma_{bc} b_0 d^2} \rangle \mu_l \right] \Rightarrow A' \quad \text{est nécessaire}$$

Reinforcement calculation: Decomposition method « Method 1 »



Section 1 :

$M_{11} = M_l = \mu_l \sigma_{bc} b_0 d^2$

$\alpha_1 = 1.25\left(1 - \sqrt{1 - 2\mu_l}\right), \beta_l = 1 - 0.4\alpha_l$

$\varepsilon_s = \varepsilon_{sl} = \frac{f_e}{\gamma_s E_s} \Rightarrow \sigma_s = \frac{f_e}{\gamma_s}$

$\alpha_l = \frac{3.5}{1000\varepsilon_{sl} + 3.5}$

$$\Rightarrow A_{11} = \frac{M_{11}}{\sigma_s \beta_l d}$$

Section 2 :

The remaining section is subjected to: $M_{12} = M_1 - M_{11}$

$\sum M /_{A12} = 0$

$M_{12} - F_s^/\left(d - c^/\right) = 0$

$M_{12} - A^/ \sigma_s^/\left(d - c^/\right) = 0$

$$A^/ = \frac{M_{12}}{\sigma_s^/\left(d - c^/\right)}$$

$\sigma_s^{'}$?

$$\Delta \equiv \Delta \Rightarrow \frac{\varepsilon_s^{'}}{\varepsilon_{bc}} = \frac{y_l - c'}{y_l} = \frac{\alpha_l d - c'}{\alpha_l d}$$

$$\Rightarrow \varepsilon_s^{'} = \varepsilon_{bc} \frac{\alpha_l d - c'}{\alpha_l d}, \qquad \varepsilon_{bc} = 3.5‰$$

$$\sigma_s^{'} = \begin{cases} \varepsilon_s^{'} \geq \varepsilon_{sl}^{'} \Rightarrow \sigma_s = \dfrac{f_e}{\gamma_s} \\ \varepsilon_s^{'} < \varepsilon_{sl}^{'} \Rightarrow \sigma_s = \varepsilon_s^{'} E_s \end{cases}$$

$$\sum M /_{A'} = 0$$

$$M_{12} - F_{s12}\left(d - c'\right) = 0$$

$$M_{12} - A_{12}\sigma_s\left(d - c'\right) = 0$$

$$A_{12} = \frac{M_{12}}{\sigma_s\left(d - c'\right)}$$

$$with \quad \sigma_s = \frac{f_e}{\gamma_s}$$

Final reinforcement at ULS is :

$$A_u = A_{11} + A_{12} + A_2$$

$$A_u^{'} = A'$$

3. CALCULATION AT SERVICEABILITY LIMIT STATE (ELS)

As with the ELUR, for everything that follows, the bending moment will be considered positive. If the bending moment is negative, the calculation is equivalent to the calculation of a rectangular section

"b0h"

3.1. T-section without compression reinforcement: Reinforcement calculations are recommended for **harmful or very harmful cracking**.

3.1.1. Position of the neutral axis:

For a positive bending moment (calculation of a T-section), two cases are distinguished.

a) *Neutral axis in the flange*:

In this case, the compression flange alone can resist the compressive stresses produced by the bending moment, hence $y_1 \leq h_0$.

As a limit case for this situation, $h_0 = y_1$.

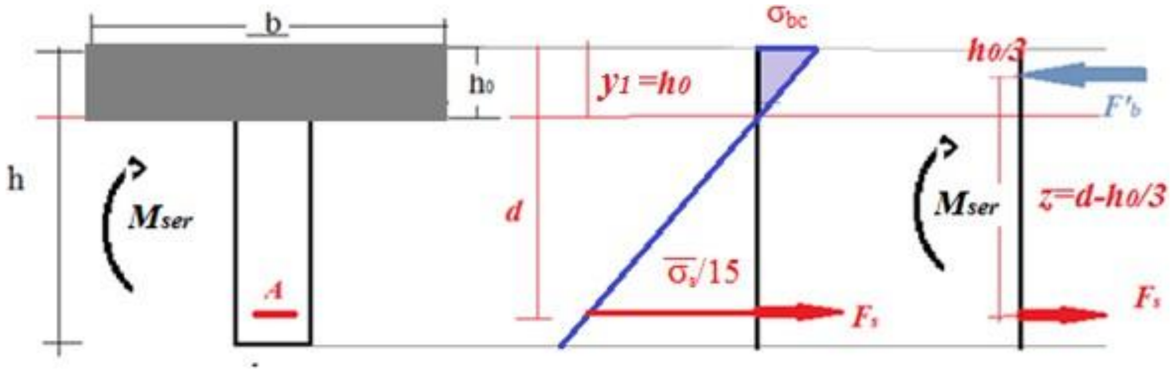


Fig. 6.19 neutral axis in the flange ($h_0=y_1$).

Resisting moment of the flange « M_T » :

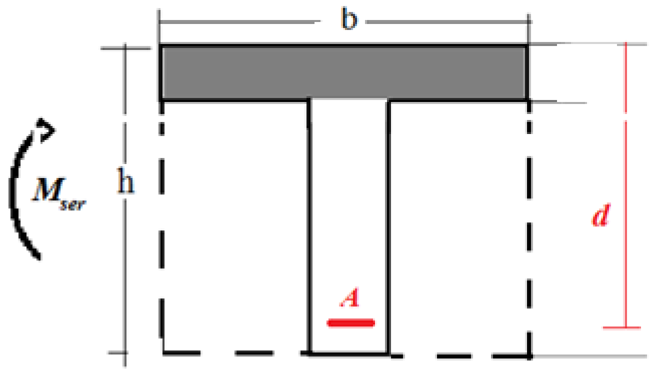
$$\left.\begin{aligned} M_T &= F_b' z \\ F_b' &= \frac{y b \sigma_{bc}}{2} = \frac{h_0 b \sigma_{bc}}{2} \end{aligned}\right\} \Rightarrow M_T = \frac{b h_0 \sigma_{bc}}{2}\left(d - \frac{h_0}{3}\right)$$

$$\Delta \equiv \Delta$$

$$\frac{\sigma_{bc}}{\sigma_s' / 15} = \frac{h_0}{(d-h_0)} \rightarrow \qquad \sigma_{bc} = \frac{\overline{\sigma_s}}{15}\frac{h_0}{(d-h_0)}$$

$$M_T = \frac{b h_0^2}{30}\frac{\left(d - \frac{h_0}{3}\right)}{(d-h_0)}\overline{\sigma_s}$$

neutral axis in the flange « $M_{ser} \leq M_T$ » : Therefore, the compression flange area "bh0" can resist the compressive stresses due to the service moment on its own. In this case, the calculation is equivalent to calculating a rectangular section with section "bh".



b) **Neutral axis within the web**:

If $M_{ser} > M_T$, **neutral axis in the web**. the compression flange cannot resist alone the compressive stresses produced by the bending moment, hence, $y_1 > h_0$.

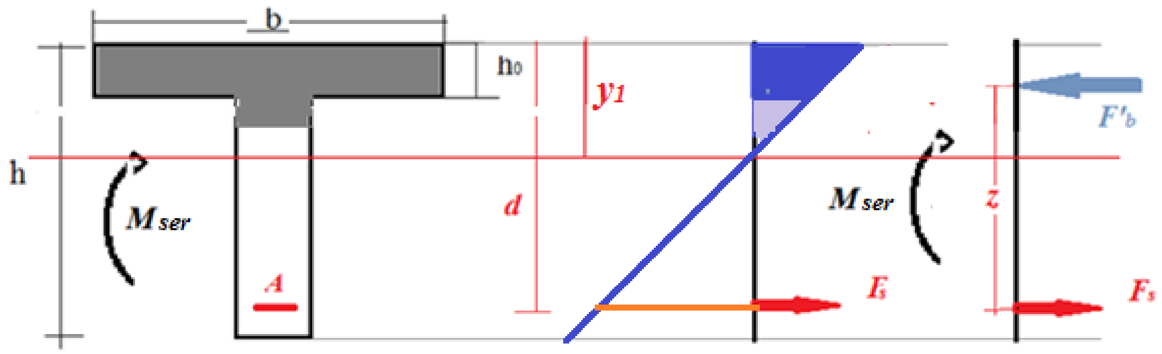


Fig. 6.20 **Neutral axis in the web** ($h_0 < y_1$).

In this case, the reinforcement calculation must take into account, in addition to the compression flange, the contribution of a portion of the rib to the beam's resistance. Therefore, it involves calculating a T-section.

Given the complexity of the calculation, the reinforcement calculation in this case is based on the application of the approximate method. By assimilating the resultant of the compressive stresses of the table and the compressed rib to the distance $z = d - h0/2$ from the center of gravity of the tension reinforcement.

$$z = d - \frac{h_0}{2}$$

$$\sum M /_{F_b'} = 0 \rightarrow \qquad A_{ser} = \frac{M_{ser}}{z\overline{\sigma_s}}$$

Check for compression reinforcement:

$$\sigma_{bc} = \left[\frac{A_{ser} d}{bh_0} + \frac{h_0}{30} \right] \frac{\overline{\sigma_s}}{z}$$

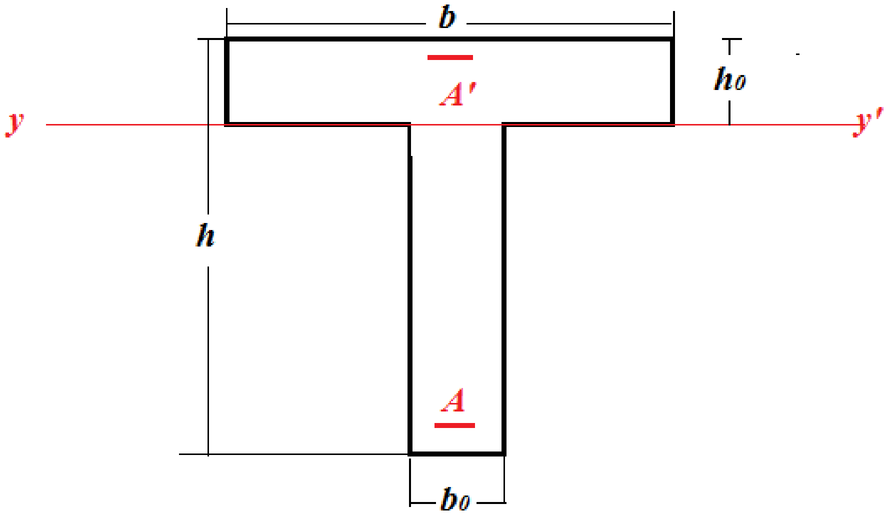
Si $\sigma_{bc} \leq \overline{\sigma_{bc}} = 0.6 f_{c28} \rightarrow A'$

Si $\sigma_{bc} \rangle \overline{\sigma_{bc}} A' \neq 0$

3.2.Stress verification:

The analytical development is considered for a general case of a T-section with compression reinforcement. If A' is not necessary, A' is replaced by 0 in the equations.

Position of neutral axis: To determine the position of the neutral axis (flange or rib), the limit case, $y_1 = h_0$ is considered for the calculation of the static moment $S_{yy'}$.



$$S_{yy} = b\frac{h_0^2}{2} + 15A'(h_0 - c') - 15A(d - h_0)$$

Case 1 : if $S_{yy'} \geq 0$: neutral axis in the flange ($y_0 \leq h_0$).

$$S_{yy} = b\frac{y_1^2}{2} + 15A'(y_1 - c') - 15A(d - y_1) = 0 \rightarrow y_1$$

Determination of coefficient K

$$I_{yy'} = \frac{1}{3}by_1^3 + 15A(d - y_1)^2 + 15A(y_1 - c')^2$$

$$[M_{ser}] = Nm, [I_{yy}'] = cm^4 \qquad K = \frac{M_{ser}}{I_{yy'}}$$

Case 2 : if $S_{yy'} < 0$: neutral axis in the rib ($y_0 > h_0$)

$$S_{yy'} = (b - b_0)h_0\left(y_1 - \frac{h_0}{2}\right) + b_0 y_1^2 + 15A'(y_1 - c') - 15A'(d - y_1) = 0 \rightarrow y_1$$

Determination of coefficient K

$$I_{yy'} = (b - b_0)h_0\left(y_1 - \frac{h_0}{2}\right)^2 + \frac{b_0 y_1^3}{3} + 15A(d - y_1)^2 + 15A'(y_1 - c')^2$$

$$[M_{ser}] = Nm, [I_{yy}'] = cm^4 \qquad K = \frac{M_{ser}}{I_{yy'}}$$

For the applied reinforcement to be suitable for the ELS, it is necessary to have:

$$\begin{cases} 1)\sigma_{bc} = Ky_1 \leq \overline{\sigma_{bc}} \\ 2)\sigma_s = 15K(d - y_1) \leq \overline{\sigma_s} \\ 3)\sigma_s' = 15K(y_1 - c') \leq \overline{\sigma_s} \end{cases}$$

If one of the inequalities is not verified, the reinforcement must be recalculated at the ELS.

3.2. Non-brittleness condition

Whether the T-section is treated as rectangular or T-shaped, the **minimum reinforcement** is:

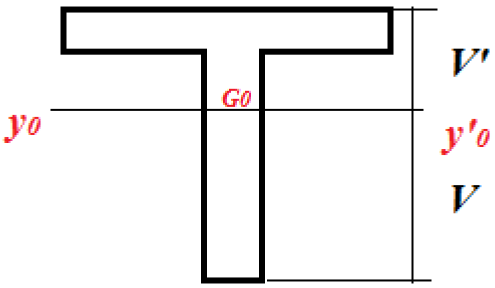
a- For M > 0

$$A\min = \frac{I_{y_0 y_0'}}{0.81hV}\frac{f_{t28}}{f_e}$$

b- For M < 0

$$A\min = \frac{I_{y_0 y_0'}}{0.81hV'}\frac{f_{t28}}{f_e}$$

with $I_{y_0 y'_0}$: moment of inertia of the concrete T-section about its centroid



G_0 : centroid of the concrete section alone.

APPLICATION EXAMPLE

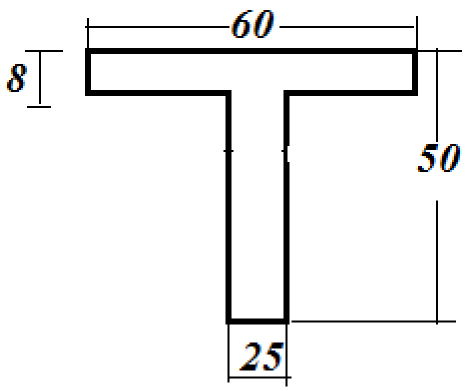
Consider a reinforced concrete beam
with a T-shaped section,
subjected to:

Mu=300kNm; fundamental combination

Mser=201kNm

with; FeE400; f_{c28}=25MPa; cg=25 mm.

Harmful cracking. Determine the reinforcement area



Solution :

1- ELU :

Position of the neutral axis

$$\left.\begin{array}{l} M_T = bh_0\sigma_{bc}\left(d-\frac{h_0}{2}\right) = 279.kNm \\ M_u = 300kNm \rangle M_T = 279.5kNm \end{array}\right\}$$ Axe neutre dans la nervure

$$M_u = M_n + M_2 = M_1 + M_2$$

$$M_2 = (b-b_0)h_0\sigma_{bc}\left(d-\frac{h_0}{2}\right) = 163.02kNm$$

$\underline{Section\ 1}$

$$M_1 = M_u - M_2 = 136.93kNm$$

$$\mu_1 = \frac{M_1}{\sigma_{bc}b_0d^2} = 0.190$$

$$\mu_{AB} = 0.186 \leq \mu = 0.190 \leq 0.48 \quad \Rightarrow \quad Pivot \quad B$$

$$\Rightarrow \varepsilon_{bc} = 3.5^0/_{00}$$

Check for compression reinforcement:

$$\varepsilon_{sl} = \frac{f_e}{\gamma_s E_s} = 1.73910^{-3}$$

$$\alpha_1 = \frac{3.5}{1000\varepsilon_{sl} + 3.5} = 0.668$$

$$\mu_l = 0.8\alpha_l (1 - 0.4\alpha_l) = 0.392$$

$\mu = 0.190 \langle \mu_1$ (region 2a) $\Rightarrow \varepsilon_s \rangle \varepsilon_{sl} \Rightarrow \sigma_s = \frac{f_e}{\gamma_s} = 348MPa$

therefore, compression reinforcement is not necessary: A'=0

$$\alpha_1 = 1.25\left(1 - \sqrt{1 - 2\mu_1}\right)$$

$$\beta_1 = 1 - 0.4\alpha_1 = 0.894$$

$$A_1 = \frac{M_1}{\sigma_s \beta_1 d} = 9.8cm^2$$

Section 2

$$A_2 = \frac{(b - b_0) h_0 \sigma_{bc}}{\sigma_s} = 11.41cm^2$$

Final reinforcement at ELU is:

$$A_u = A_1 + A_2 = 21.21cm^2$$

$$A' = 0$$

2- <u>ELS :</u>

<u>Position of the neutral axis:</u>

Resisting Moment of the flange

$$M_T = \frac{b h_0^2}{30} \frac{\left(d - \frac{h_0}{3}\right)}{(d - h_0)} \overline{\sigma_s}$$

s

$$\overline{\sigma_s} = \min\left(\frac{2}{3} f_e ; 110\sqrt{\eta f_{t28}}\right) = 201.6MPa$$

$$M_T = 29.52kN \langle M_{ser} = 201kNm$$

Therefore, neutral axis in the web. Use of the approximate method for the calculation of reinforcement.

$$z = d - \frac{h_0}{2} = 41cm \qquad A_{ser} = \frac{M_{ser}}{z\overline{\sigma_s}} = 24.32cm^2$$

Check for compression reinforcement

$$\sigma_{bc} = \left[\frac{A_{ser}d}{bh_0} + \frac{h_0}{30}\right]\frac{\overline{\sigma_s}}{z} = 12.52MPa$$

$$\sigma_{bc} = 12.52MPa \leq \overline{\sigma_{bc}} = 0.6f_{c28} = 15MPa \rightarrow A' \quad \text{not necessary}$$

3- Minimum reinforcement :

$$A\min = \frac{I_{y_0y_0'}}{0.81hV}\frac{f_{t28}}{f_e}$$

$$V = 28.43cm$$
$$V' = 21.57cm$$

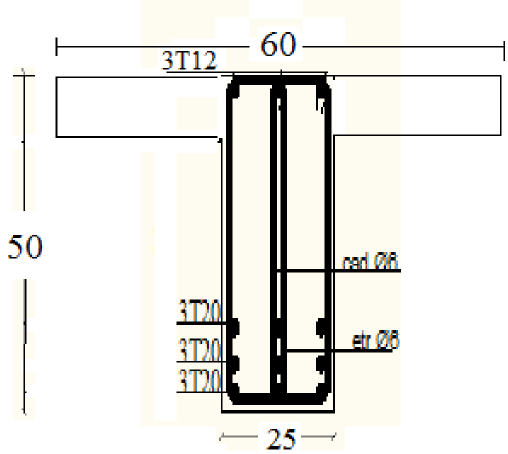
$$I_{y_0y_0'} = b_0\frac{V^3}{3} - (b - b_0)\frac{(V' - h_0)^3}{3} = 363053.5cm^4$$

$$A\min = \frac{I_{y_0y_0'}}{0.81hV}\frac{f_{t28}}{f_e} = 1.32cm^2$$

4-Final reinforcement:

$$A = max(A_u; A_{ser}; A_{min}) = 24.32cm^2 => 9T20$$

$$A^{/} = 0 => 3T12 \text{ of assembly}$$



3.3. T-Section with Compression Reinforcement

$$\sigma_{bc} \rangle \overline{\sigma_{bc}} = 0.6 f_{c28} \rightarrow A^{/} \text{ is required}$$

If compression reinforcement is placed outside the corners of the section, BAEL rules require stirrups or hooked ties spaced at most every 15 Ǿ (where Ǿ is the diameter of the compression bars)

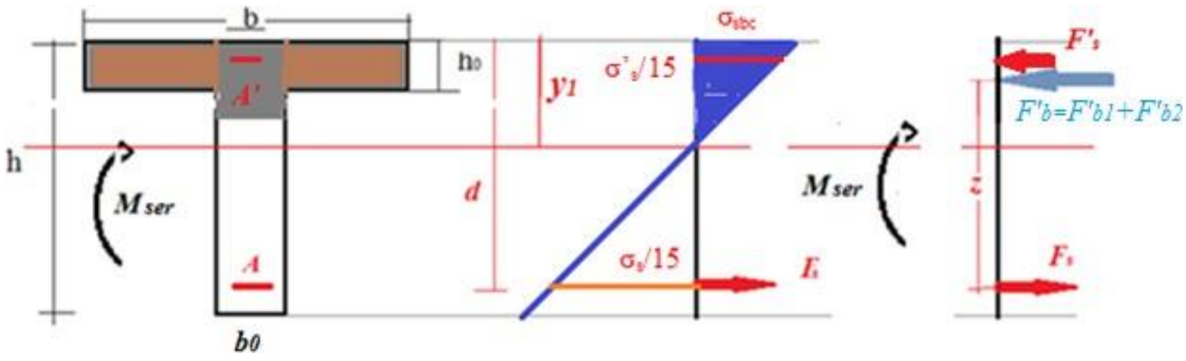


Fig.6.21 Internal force distribution of a T-section at SLS with compression reinforcement

F'_{b1} : compressive force in the web (b_0 h)

$$F'_{b1} = \frac{1}{2}\overline{\sigma_{bc}} b_0 y_1; \qquad \text{(appliqué à)} \quad \frac{y_1}{3}$$

F'_{b2} : compressive force in the reduced flange (b-b_0)h_0

$$F'_{b2} = \frac{h_0}{2}\left(\overline{\sigma_{bc}} + \overline{\sigma_{bcl}}\right)(b - b_0); \qquad \text{(applied at } a\text{)} \quad \frac{y_1}{3}$$

with;

$$\sigma_{bc} = \overline{\sigma_{bc}}; \sigma_s = \overline{\sigma_s}; \sigma'_s = 15\left(\frac{y_1 - c'}{y_1}\right)\overline{\sigma_{bc}}$$

$$k_1 = \frac{\overline{\sigma_s}}{\overline{\sigma_{bc}}}; \alpha_1 = \frac{15}{15 + k_1}; y_1 = \alpha_1 d$$

F'_b: Resultant Compression Force (F'_{b1} + F'_{b12}) applied at distance « z » from the tensile reinforcement.

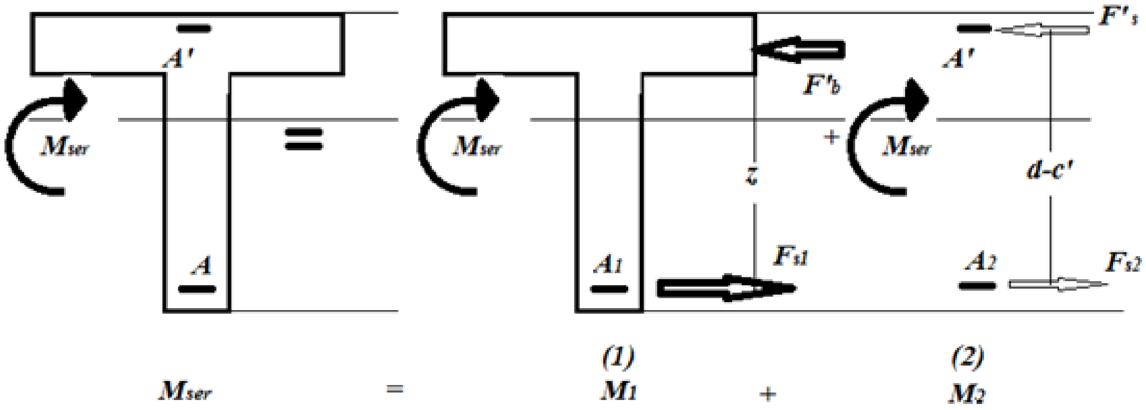


Fig.6.22 Decomposition of the T-section at ELS with compression reinforcement.

with:

$$z = d - \frac{h_0}{2} + \frac{bh_0^3 - b_0(y_1 - h_0)^2(2y_1 + h_0)}{6\left[by_1^2 - (b - b_0)(y_1 - h_0)^2\right]}$$

<u>Section (1) :</u> Balanced by M_1 :

$$\sum M / A_1 = 0 \rightarrow M_1 - F_b^{'} z = 0$$

$$M_1 = F_b^{'} z$$

$$F_b^{'} = \frac{1}{2}\left[b_0 y_1 + h_0(b - b_0)\frac{2y_1 - h_0}{y_1}\right]\overline{\sigma_{bc}}$$

$$\sum M / F_b^{'} = 0 \rightarrow M_1 - F_{sl} z = 0$$

$$M_1 = A_1 \overline{\sigma_s} z$$

$$A_1 = \frac{M_1}{\overline{\sigma_s} z} = \frac{F_b^{'}}{\overline{\sigma_s}}$$

<u>Section (2) :</u> Balanced by M2:

$$M_2 = M_{ser} - M_1$$
$$\sum M /_{A2} = 0$$
$$M_2 - F_s^{'}(d - c') = 0$$
$$M_2 - A^{'}\sigma_s^{'}(d - c') = 0$$
$$\sigma_s^{'} = 15\left(\frac{y_1 - c'}{y_1}\right)\overline{\sigma_{bc}} \qquad A' = \frac{M_2}{\sigma_s^{'}(d - c')}$$

$$\sum M /_{A'} = 0$$
$$M_2 - F_{s2}(d - c') = 0$$
$$M_2 - A_2\sigma_s(d - c') = 0$$
$$A_2 = \frac{M_2}{\overline{\sigma_s}(d - c')}$$

Final reinforcement at SLS:

$$A_{ser} = A_1 + A_2$$
$$A_{ser}^{/} = A^{/}$$

APPLICATION EXAMPLE

Consider a reinforced concrete beam with a T-shaped section, subjected to:

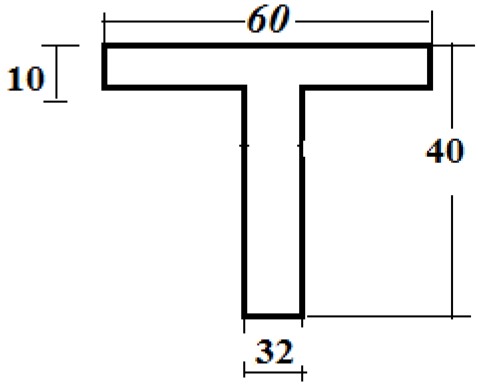
Mu=283kNm; fundamental combination

Mser=172kNm

with; FeE500; $E_a=2.10^5$ MPa f_{c28}=20MPa;

cg=25 mm ; c=c'= 5 cm . Fissuration préjudiciable

Harmful cracking. Determine the required reinforcement area



Solution :

1- **ELU**

$$\sigma_{bc} = 11.33MPa$$

$$\left.\begin{array}{l} M_T = bh_0\sigma_{bc}\left(d - \frac{h_0}{2}\right) = 210738Nm \\ M_u = 283kNm \rangle M_T = 210.7kNm \end{array}\right\} \textit{neutral axis in the web}$$

$$M_u = M_n + M_2 = M_1 + M_2$$

$$M_2 = (b - b_0)h_0\sigma_{bc}\left(d - \frac{h_0}{2}\right) = 98344.4Nm$$

$$\sigma_s = \frac{f_e}{\gamma_s} = 435MPa \qquad \Rightarrow A_2 = \frac{M_2}{\sigma_s\left(d - \frac{h_0}{2}\right)} = 7.29cm^2$$

Section 1

$$M_1 = M_u - M_2 = 184656Nm$$

$$\mu_1 = \frac{M_1}{\sigma_{bc}b_0d^2} = 0.416$$

$$\mu_{AB} = 0.186 \le \mu = 0.416 \le 0.48 \Rightarrow Pivot \quad B$$

$$\Rightarrow \varepsilon_{bc} = 3.510^{-3}$$

Compression reinforcement verification:

$$\varepsilon_{sl} = \frac{f_e}{\gamma_s E_s} = 2.17410^{-3}$$

$$\alpha_l = \frac{3.5}{1000\varepsilon_{sl} + 3.5} = 0.617; \qquad \beta_l = 1 - 0.4\alpha_l = 0.753$$

$$\mu_l = 0.8\alpha_l\left(1 - 0.4\alpha_l\right) = 0.372$$

$$\mu_l = 0.416 \rangle \mu_l \qquad \text{(region 2b)} \qquad \Rightarrow A' \neq 0; \sigma_s = \frac{f_e}{\gamma_s} = 435MPa$$

Section 11 :

$$M_{11} = M_1 = \mu_l \sigma_{bc} b_0 d^2 = 165219Nm$$

$$\Rightarrow A_{11} = \frac{M_{11}}{\sigma_s \beta_l d} = 14.41cm^2$$

Section 12:

‰ The remaining section is subjected to:

$$M_{12} = M_1 - M_{11} = 19437Nm$$

$$A' = \frac{M_{12}}{\sigma_s^{'}\left(d - c'\right)}$$

$$\varepsilon_s^{'} = \varepsilon_{bc}\frac{\alpha_l d - c'}{\alpha_l d} = 3.5\frac{0.617*35 - 5}{0.617*35}10^{-3} = 2.689.10^{-3}$$

$$\sigma_s^{'} = \left\{\varepsilon_s^{'} \geq \varepsilon_{sl}^{'} \Rightarrow \sigma_s = \frac{f_e}{\gamma_s} = 435MPa\right.$$

$$A' = \frac{M_{12}}{\sigma_s^{'}\left(d - c'\right)} = 1.5cm^2$$

$$A_{12} = \frac{M_{12}}{\sigma_s\left(d - c'\right)} = 1.5cm^2$$

With $\sigma_s = \frac{f_e}{\gamma_s} = 435MPa$

final reinforcement at ULS is:

$$A_u = A_{11} + A_{12} + A_2 = 14.41 + 1.5 + 7.29 = 23.2cm^2$$

$$A_u^{'} = A' = 1.5cm^2$$

2- ELS :

Position of the neutral axis: Flange resisting moment

$f_{t28} = 0.6 + 0.6 f_{c28} = 1.8 MPa$

$$M_T = \frac{bh_0^2}{30}\frac{\left(d - \frac{h_0}{3}\right)}{\left(d - h_0\right)}\overline{\sigma_s}$$

$$\overline{\sigma_s} = \min\left(\frac{2}{3} f_e ; 110\sqrt{\eta f_{t28}}\right) = 186.7 MPa$$

$$M_T = 46919 Nm \langle M_{ser}$$

therefore, neutral axis in the web. Approximate method used.

$$z = d - \frac{h_0}{2} = 30 cm$$

$$A_{ser} = \frac{M_{ser}}{z\overline{\sigma_s}} = 30.7 cm^2$$

Compression reinforcement verification :

$$\sigma_{bc} = \left[\frac{A_{ser} d}{bh_0} + \frac{h_0}{30}\right]\frac{\overline{\sigma_s}}{z} = 13.2 MPa$$

$$\sigma_{bc} = 13.2 MPa \rangle \overline{\sigma_{bc}} = 0.6 f_{c28} = 12 MPa \rightarrow A^{/} \text{ is required}$$

Limit Case:

$$\overline{\sigma_{bc}} = \sigma_{bc} = 12 MPa \quad \alpha_1 = \frac{15}{15 + k_1} = 0.491; \quad \overline{\sigma_s} = 186.7 MPa; \quad y_1 = \alpha_1 d = 17.2$$

$$k_1 = \frac{\overline{\sigma_s}}{\overline{\sigma_{bc}}} = 15.56; \qquad \sigma_s^{'} = 15\left(\frac{y_1 - c'}{y_1}\right)\overline{\sigma_{bc}} = 127.7 MPa$$

$$z = d - \frac{h_0}{2} + \frac{bh_0^3 - b_0\left(y_1 - h_0\right)^2\left(2y_1 + h_0\right)}{6\left[by_1^2 - \left(b - b_0\right)\left(y_1 - h_0\right)^2\right]} = 29.85 cm$$

Section (1) : Balanced by M_1 :

$$F_b^{'} = \frac{1}{2}\left[b_0 y_1 + h_0\left(b - b_0\right)\frac{2y_1 - h_0}{y_1}\right]\overline{\sigma_{bc}} = 568565N$$

$$A_1 = \frac{F_b^{'}}{\overline{\sigma_s}} = 30.45cm^2$$

Section (2) : Balanced by M_2:

$$M_2 = M_{ser} - M_1 = 2283Nm$$

$$A' = \frac{M_2}{\sigma_S^{'}\left(d - c'\right)} = 0.6cm^2$$

$$A_2 = \frac{M_2}{\overline{\sigma_s}\left(d - c'\right)} = 0.41cm$$

Final reinforcement at SLS is :

$$A_{ser} = A_1 + A_2 = 30.9cm^2$$

$$A_{ser}^{'} = A' = 0.6cm^2$$

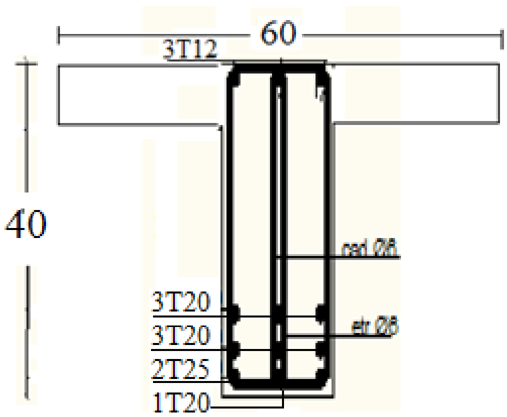
Minimum reinforcement: According to the DTU associated with BAEL 91 (revised 1999), for a positive bending moment, the following approximate method is applicable.

$$A_{\min} = 0.23b_0 d\frac{f_{t28}}{f_e} = 0.93cm^2$$

Final reinforcement:

$$\max\left(A + A'; A + A_{ser}^{'}\right) = A + A' = 31.5cm^2$$

$$A_f = A_{ser} = 30.9cm^2 \rangle A_{\min} \Rightarrow 2HA25 + 7HA20 = 31.78cm^2$$



$$A_f^{'} = A_{ser}^{'} = 0.6cm^2 \Rightarrow 3HA12 = 3.39cm^2$$

CONCLUSION

This handout applies to the calculation of reinforced concrete according to the B.A.E.L 91 rules and certain rules defined by Eurocode 2. Its objective is to present the theoretical analytical development for calculating the basic elements of reinforced concrete structures (tie rods, columns, beams, etc.) based on well-defined behavior laws in ultimate limit states of strength (ELU) and serviceability limit states (ELS).

Each chapter presents the assumptions and equilibrium equations to be able to design and reinforce the concrete cross-section. All of this is illustrated with application examples.

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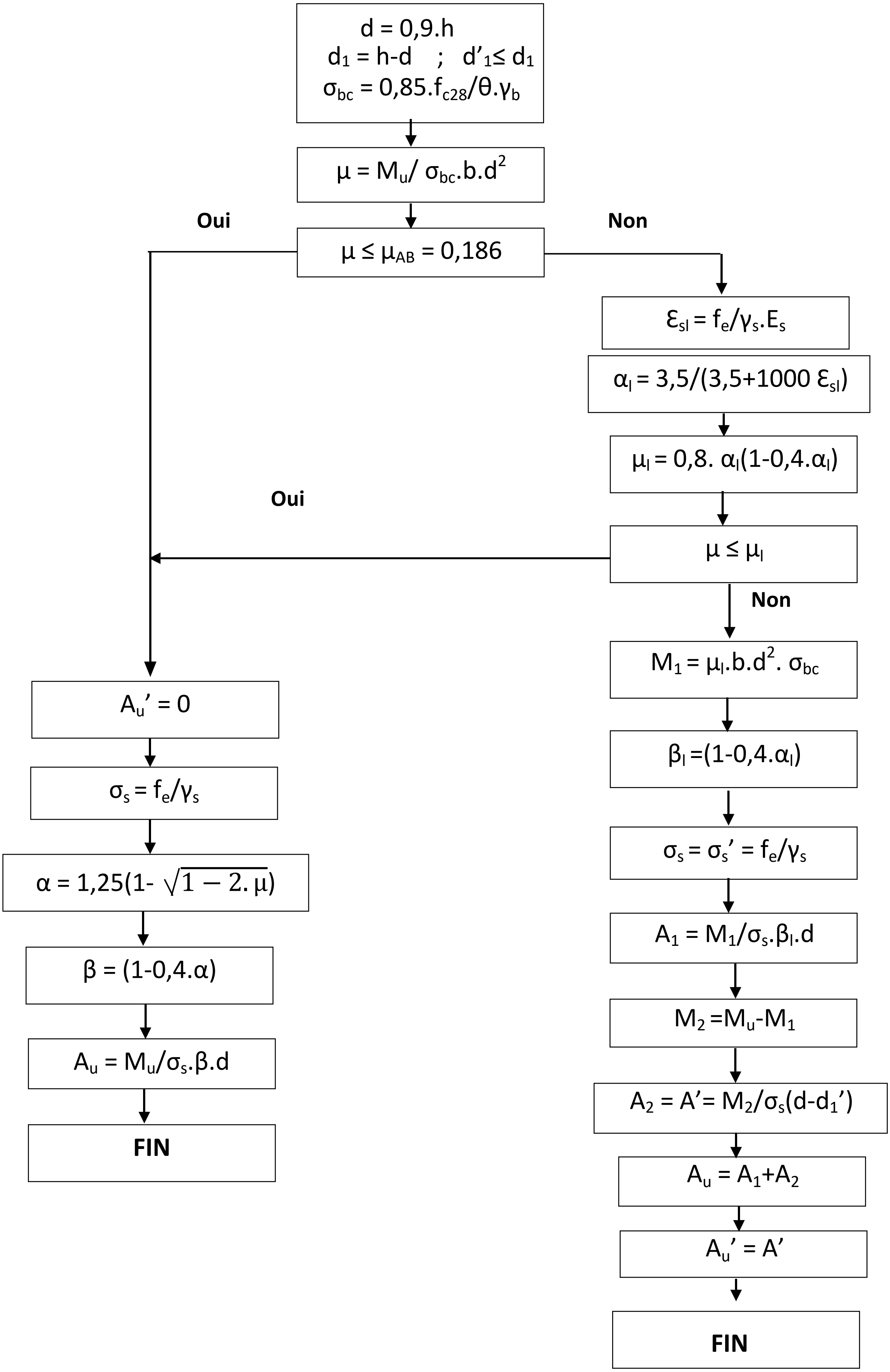
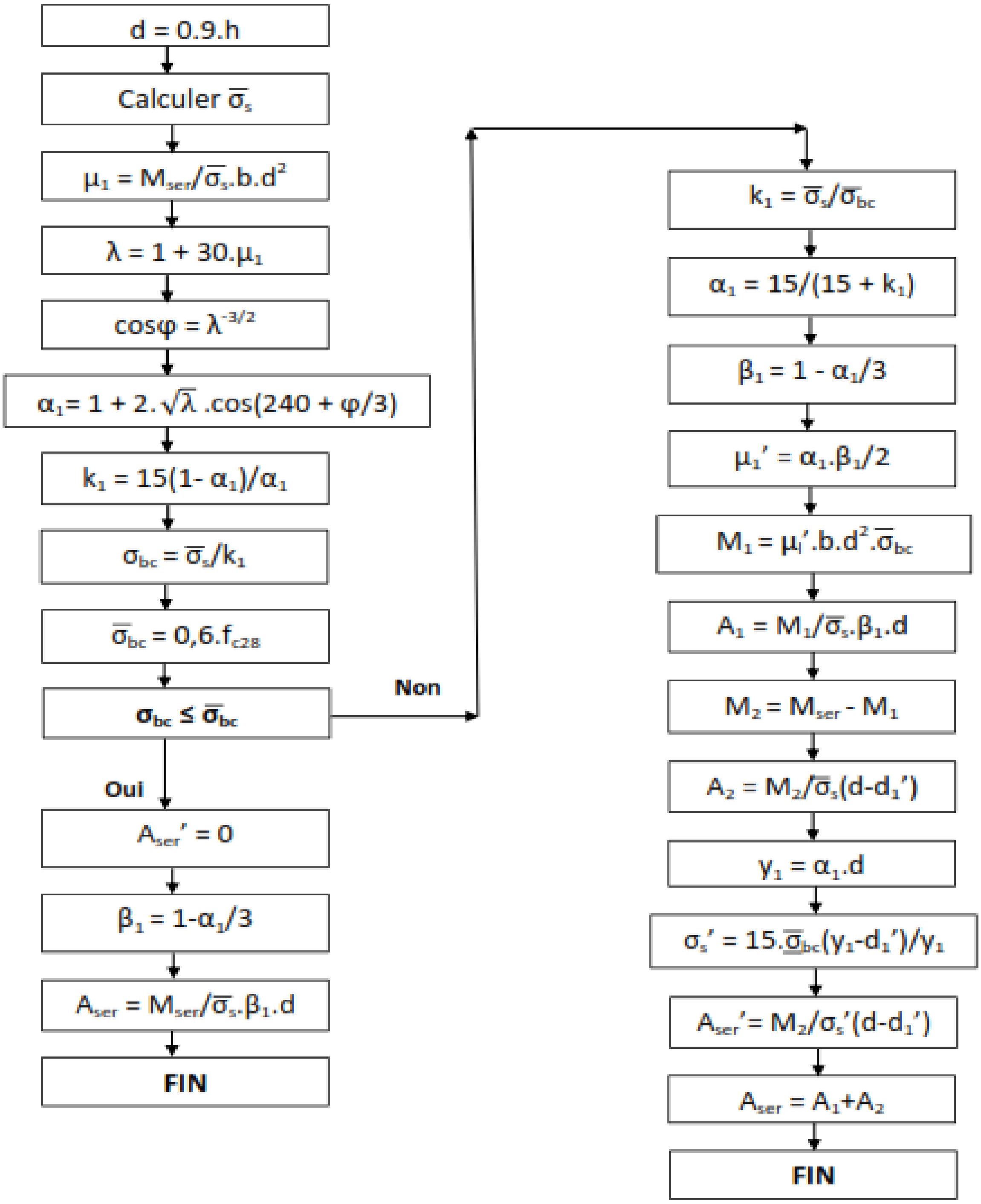
Annexe

Organigramme A :

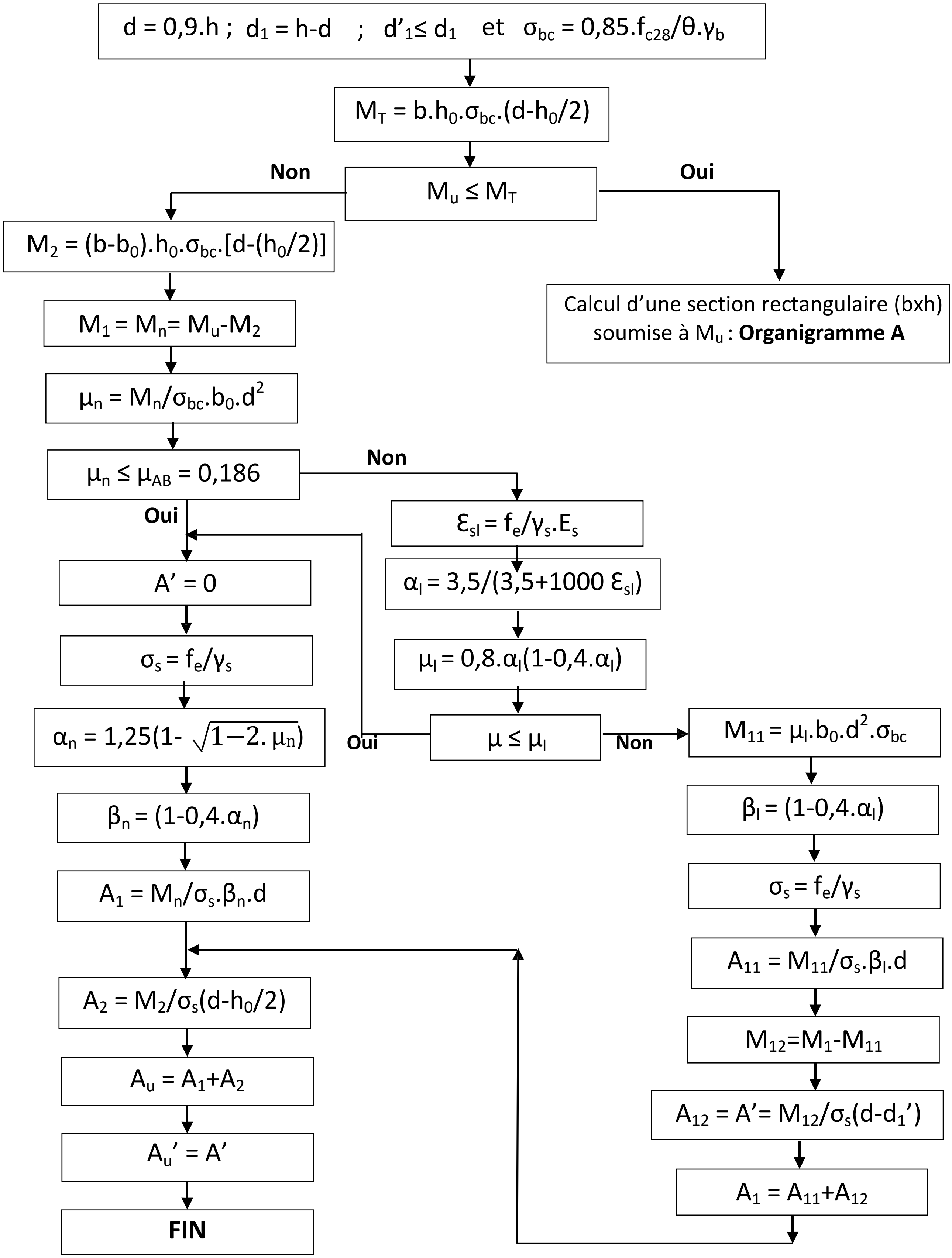
ELUR, Flexion simple, section rectangulaire



Organigramme B :
ELS, Flexion simple, section rectangulaire



Organigramme C :
ELUR, Flexion simple, section en "T"



Section en cm² de 1 à 20 armatures de diamètre Ø en mm

Ø	5	6	8	10	12	14	16	20	25	32	40
1	0,20	0,28	0,50	0,79	1,13	1,54	2,01	3,14	4,91	8,04	12,57
2	0,39	0,57	1,01	1,57	2,26	3,08	4,02	6,28	9,82	16,08	25,13
3	0,59	0,85	1,51	2,36	3,39	4,62	6,03	9,42	14,73	24,13	37,70
4	0,79	1,13	2,01	3,14	4,52	6,16	8,04	12,57	19,64	32,17	50,27
5	0,98	1,41	2,51	3,93	5,65	7,70	10,05	15,71	24,54	40,21	62,83
6	1,18	1,70	3,02	4,71	6,79	9,24	12,06	18,85	29,45	48,25	75,40
7	1,37	1,98	3,52	5,50	7,92	10,78	14,07	21,99	34,36	56,30	87,96
8	1,57	2,26	4,02	6,28	9,05	12,32	16,08	25,13	39,27	64,34	100,5
9	1,77	2,54	4,52	7,07	10,18	13,85	18,10	28,27	44,18	72,38	113,1
10	1,96	2,83	5,03	7,85	11,31	15,39	20,11	31,42	49,09	80,42	125,7
11	2,16	3,11	5,53	8,64	12,44	16,93	22,12	34,56	54,00	88,47	138,2
12	2,36	3,39	6,03	9,42	13,57	18,47	24,13	37,70	58,91	96,51	150,8
13	2,55	3,68	6,53	10,21	14,70	20,01	26,14	40,84	63,81	104,6	163,4
14	2,75	3,96	7,04	11,00	15,83	21,55	28,15	43,98	68,72	112,6	175,9
15	2,95	4,24	7,54	11,78	16,96	23,09	30,16	47,12	73,63	120,6	188,5
16	3,14	4,52	8,04	12,57	18,10	24,63	32,17	50,27	78,54	128,7	201,1
17	3,34	4,81	8,55	13,35	19,23	26,17	34,18	53,41	83,45	136,7	213,6
18	3,53	5,09	9,05	14,14	20,36	27,71	36,19	56,55	88,36	144,8	226,2
19	3,73	5,37	9,55	14,92	21,49	29,25	38,20	59,69	92,27	152,8	238,8
20	3,93	5,65	10,05	15,71	22,62	30,79	40,21	62,83	98,17	160,8	251,3