

PEOPLE'S DEMOCRATIC REPUBLIC OF ALGERIA

وزارة التعليم العالي و البحث العلمي

MINISTRY OF HIGHER EDUCATION AND SCIENTIFIC RESEARCH

جامعة أحمد زبانة غليزان

Ahmed Zabana University Relizane

FACULTY OF SCIENCE AND TECHNOLOGY

PHYSICS DEPARTMENT

LMD 3rd cycle DOCTORATE THESIS

Sector: **Energetic Physics**

Specialty : **physics**



THERMAL MODEL NUMERIC DEVELOPMENT BY USED NANOFLUIDE IN COMPLEX CAVITY

Presented by:

CHIKR DJAOUTSI ZINEB

President	Houari Mohammed	MCA	Ahmed Zabana University Relizane
Theses Director	Fares Redouane	MCA	Ahmed Zabana University Relizane
Theses Co-Director	Naim Houcine	MCA	Mohamed Boudiaf University Oran
Examiners	Lounis Mourad	Pr	Djilali Bounaama University Khemis Meliana
	Ould Melha Khellaf	Pr	Hassiba Benbouali university, Chlef, Algeria
	Mir Ali	Pr	Ahmed Zabana University Relizane

2025/2026

Acknowledgments

In the preamble to this thesis, we thank ALLAH the Almighty powerful and merciful who helped us and gave us patience and the courage during these long years of study.

Our heartfelt thanks also extend to our promoter. Mr. Fares Redouane for having agreed to supervise us, for his guidance and wise advice, which has been immensely helpful to us, and his unwavering availability. In particular to our Co-promoter Mr. Naim Houcine, who was always available listening, by our side at all times, as well as for inspiration, help, his advice.

Great thanks to Dr.Houari Mohammed , Pr.Lounis Mourad , Pr.Ould Melha Khellaf and Pr. Mir Ali . for agreeing to review our work.

Our sincere and heartfelt thanks also go to
all the people who helped us closely or from afar for the
completion of this work.

Finally, we express our warmest thanks to our families.
who have always been by our side and who have always encouraged us
during the completion of this thesis.

Dedications

To those who instilled in me the love of knowledge,

To those who taught me patience and perseverance,

To my dear parents, symbols of sacrifice and unconditional love,

To my esteemed professors, who generously shared their wisdom and guidance,

To my brothers, sisters, and friends who supported me throughout this journey,

To my classmates, with whom I shared moments of challenge, growth, and success,

*To everyone who believed in me and encouraged me with kind words or heartfelt
prayers,*

I dedicate this humble work,

oping always to live up to your trust and expectations.

Liste of contents

<i>Liste of publications</i>	v
<i>Abstract:</i>	vii
<i>Résumé:</i>	viii
<i>المخلص</i>	x
<i>Introduction:</i>	1
CHAPTER I GENERALITIES & BIBLIOGRAPHIC SYNTHESIS	5
<i>I.1. General principles of heat transfer by convection</i>	6
I.1.1. Introduction to heat transfer by convection:	6
Forced convection:	12
<i>I.2. Mixed Convection In Differentially Heated Cavite: State Of The Art:</i>	13
<i>I.3 Influence of radiation on heat and momentum transfers</i>	16
<i>I.4 The impact of radiation on natural convection flows:</i>	17
I.5 The impact of radiation on mixed convection flows:	18
<i>I.6. Magnetohydrodynamics</i>	19
I.6.1. Definition	19
I.6.2 Convection Magnetohydrodynamique	20
I.6.3 Aspect of Magnétohydrodynamics Fluid Mechanics	22
I.6.4 Porous media	23
<i>Models of Fluid Flow in Porous Media</i>	25
CHAPTER II MATHEMATICAL FORMULATION & NUMERICAL MODEL	26
<i>II.1 Introduction</i>	27
<i>II.2 Essential Fondement Mathmetics</i>	28
Key parameter definitions	28
Force of Buoyancy	28
II.2.1 Volumetric Coefficient of Expansion	29
II.2.2 The Reynolds number (Re):	29
II.2.3 The Prandtl number (Pr):	29
II.2.4 The Nusselt number (Nu):	30
II.2.5 The Grashof number (Gr):	30
II.2.6 The Rayleigh number (Ra):	31
II.2.7 The Richardson number (Ri):	31
II.2.8 The Bejan number (Be):	32
II.2.9 Hartmann number	32
II.2.10 Nusselt number	32
II.2.11 Reynolds number	33

II.2.12 Richardson number	33
II.2.13 The Prandtl (Pr):.....	34
<i>Continuity equation:.....</i>	<i>34</i>
<i>II.5 Governing equations for porous media.....</i>	<i>35</i>
<i>II.6 Conditions at Boundaries.....</i>	<i>36</i>
<i>II.8 Thermophysical properties of the working fluids In the mixet convection</i>	<i>37</i>
<i>II.9 Computational procedures.....</i>	<i>38</i>
II.9.1. Computational Fluid Dynamics	38
II.9.3. Finite Element Method	39
II.9.4. Mesh Generation	40
CHAPTER III Results & Discussions.....	44
<i>III.1 Introduction</i>	<i>44</i>
<i>Case 01: Investigating the Synergistic Effects of Magnetic Fields, Rotation, and Permeability on Heat Transport in Porous Systems</i>	<i>44</i>
<i>Abstract:</i>	<i>44</i>
2. Mathematical model	45
3. Validation and mesh evaluation	50
4. Results and discussion	51
<i>Case 02: Effect of MHD nanofluid natural convection on thermal radiation in a unique cavity</i>	<i>57</i>
<i>1.Abstract:</i>	<i>58</i>
<i>2. Problem Description.....</i>	<i>58</i>
<i>3. Numerical Solution and Validation.....</i>	<i>61</i>
<i>4. Results and Discussion.....</i>	<i>62</i>
<i>References.....</i>	<i>71</i>

List of publications

- Ghali, D., Redouane, F., Abdelhak, R., Belhadj Mohammed, A., Zineb, C. D., Jamshed, W., ... & Mohd Nasir, N. A. A. (2022). Mathematical entropy analysis of natural convection of MWCNT—Fe₃O₄/water hybrid nanofluid with parallel magnetic field via Galerkin finite element process. *Symmetry*, 14(11), 2312. <https://doi.org/10.3390/sym14112312>
- Lotfi, C., Redouane, F., Zineb, C. D., Jamshed, W., Eid, M. R., Ibrahim, R. W., ... & Hussain, S. M. (2023). Thermal energy analysis of multi-walled carbon nanotubes-Fe₃O₄/H₂O flow over non-uniformed surface with Darcy–Forchheimer model. *Energy & Environment*, 0958305X231183689. <https://doi.org/10.1177/0958305X231183689>
- Zineb, C. D., Redouane, F., Rachid, H., & Houcine, N. (2024). Exploring the complex interplay of factors in convective heat transfer: a comprehensive parametric analysis in three-dimensional permeable cavities. *Energy Sources, Part A: Recovery, Utilization, and Environmental Effects*, 46(1), 6064-6080. <https://doi.org/10.1080/15567036.2024.2346181>
- Redouane, F., Zineb, C. D., & Rachid, H. (2024). Vehicle Engine Cooling System: Review Research. *Journal of Nanofluids*, 13(3), 625-637. <https://doi.org/10.1166/jon.2024.2172>
- Zineb, C. D., Redouane, F., Abdelkader, A., Hidki, R., & Houcine, N. (2024). Thermal radiation impact of MHD nanofluid natural convection in a special cavity. *Journal of Thermal Analysis and Calorimetry*, 1-11. <https://doi.org/10.1007/s10973-024-13701-8>
- Aissani, A., Redouane, F., Hidki, R., Chikr Djaoutsi, Z., & Henini, O. (2024). Numerical investigation of MHD nanofluid natural convection in a cavity considering thermal radiation and nanoparticle shape factor effects using CVFEM. *Numerical Heat Transfer, Part B: Fundamentals*, 1-17. <https://doi.org/10.1080/10407790.2024.2391495>
- Lounis, A., Redouane, F., Djaoutsi, Z. C., Jamshed, W., Eid, M. R., Fadhl, B. M., ... & Hussain, S. M. (2025). Exploring Natural Convection Dynamics of TiO₂-Water Nanofluids in Complex Geometries Via Finite Volume Methodology. *Results in Engineering*, 107032. <https://doi.org/10.1016/j.rineng.2025.107032>

Nomenclature

C	Strength of magnetic field Specific heat ($J\ kg^{-1}K^{-1}$)	B_0	Magnetic field intensity
Ha	Hartman number	Fc	Forcheimer coefficient
K	Permeability m^2	k	Thermal conductivity ($W.m^{-1}.K^{-1}$)
Nu	Nusselt number	R	Radiation parameter
Pr	Prandtl number	Gr	Grashof number
T	Temperature (K)	P	the dimensionless fluid pressure
S	entropy	u, v	Velocity components (m/s)
Ra	Rayleigh number	x, y	Coordinates (m)
m	Shape factor of nanoparticles	F_c	Forchheimer coefficient
Greek symbols		k_{nf}^*	Mean absorption coefficient
β	volumetric thermal expansion coefficient of the fluid K^{-1}	σ	Electrical conductivity $\Omega.M$
λ	Thermal expansion coefficient K^{-1}	θ	non-dimensional temperature
ε	Porosity	ρ	Density ($kg\ m^{-3}$)
μ	Dynamic viscosity ($W.m^{-1}.K^{-1}$)	φ	Density $kg.m^{-3}$
α	effective thermal diffusivity of porous medium and fluid m^2s^{-1}		
Indices			
s			
F	Fluid	c	Cold
loc	Local	h	Hot
avg	average	nf	Nanofluid
σ^*	Stefan–Boltzmann constant	p	Porous medium

Abstract:

Heat and mass transport by natural convection in magnetohydrodynamics in traditional fluids has drawn substantial research interest due to its broad Science and engineering applications, especially in solar energy technologies. Incorporating factors such as heat sources/sinks, chemical reactions, radiation absorption into fluid flow and porous medium analyses introduces greater complexity but also increases the practical relevance of these studies for advanced engineering systems. These aspects have become key focus areas in contemporary research, given their critical importance across multiple industrial and scientific fields.

This work computationally simulates the convective motion of nanoparticles in permeable space in the presence of a magnetic field using a non-Darcy model and porous terms in momentum equations. A nano fluid serves as the working fluid. The radiation effect for various nanoparticle forms will be reported. The impact of physics parameters on nanofluid performance will also be illustrated, and the impacts of Rayleigh number, magnetic force, nanoparticle shape, and shape factor radiation parameter on nanofluid will be investigated.

The creation of entropy in nanofluid mixed convection flow has been the subject of further research. A mathematical model has been constructed to describe this phenomenon. The Galerkin weighted residual finite element method was used to discretize the resulting nondimensional equations.

This thesis uses computational fluid dynamics (CFD) software to investigate the steady-state incompressible free convection flow of nanofluid in various enclosures.

The study examines the impact of Rayleigh numbers (Ra) from 10^2 to 10^6 , Hartman numbers (Ha) from 0 to 100, Darcy numbers (Da) from 10^{-5} to 10^{-2} , geometrical parameters (undulation number and inner cylinder position), and solid volume fractions (0, 2%, 3%, 4%, 5%, and 8%) on isotherm, streamline, average Nusselt number (Nu_{avg}), and the total entropy generation.

According to the study, injecting nanofluid under particular circumstances increased heat transmission. The biggest increase in heat transmission occurs in the conduction-dominated flow regime, where the improved thermal characteristics of nanofluids are crucial. When convection is the main route of heat transmission, nanofluids do not considerably increase efficiency. The buoyant force grew as the temperature differential between hot and cold sources widened.

Résumé

Le transport de chaleur et de masse par convection naturelle en magnétohydrodynamique dans des fluides traditionnels a suscité un intérêt de recherche considérable en raison de ses larges applications en sciences et en ingénierie, notamment dans les technologies de l'énergie solaire. L'incorporation de facteurs tels que les sources/puits de chaleur, les réactions chimiques, l'absorption des radiations dans l'écoulement des fluides et les analyses des milieux poreux introduit une plus grande complexité mais augmente également la pertinence pratique de ces études pour les systèmes d'ingénierie avancés. Ces aspects sont devenus des axes de recherche clés dans les études contemporaines, compte tenu de leur importance cruciale dans de nombreux domaines industriels et scientifiques.

Cette étude utilise un modèle non-Darcy pour intégrer des termes poreux dans les équations de mouvement afin de simuler numériquement le mouvement convectif des nanoparticules dans un espace perméable en présence d'un champ magnétique. Le fluide de travail est un nanofluide. Un rapport sur l'effet de la radiation sera fourni pour différentes formes de nanoparticules. L'influence des facteurs physiques sur la performance des nanofluides sera également démontrée, et les effets du paramètre de radiation du facteur de forme, de la force magnétique, de la forme des nanoparticules et du nombre de Rayleigh sur les nanofluides seront examinés.

Une étude supplémentaire s'est concentrée sur la génération d'entropie dans l'écoulement de convection mixte des nanofluides. Un modèle mathématique expliquant cette occurrence a été développé. Les équations non dimensionnelles résultantes ont été discrétisées en utilisant l'approche des éléments finis par résidus pondérés de Galerkin.

Cette thèse examine l'écoulement de convection libre incompressible en régime permanent d'un nanofluide dans différentes enceintes en utilisant un logiciel de dynamique des fluides computationnelle (CFD). La forme de l'enceinte a été variée, avec différentes configurations de murs chauds et froids, des parois droites et ondulées. Avec ou sans barrières in situ.

L'étude examine l'impact des nombres de Rayleigh (Ra) de 10^2 à 10^6 , des nombres de Hartman (Ha) de 0 à 100, des nombres de Darcy (Da) de 10^{-5} à 10^{-2} , des paramètres géométriques (nombre d'ondulations et position du cylindre intérieur), et des fractions volumiques solides (0, 2%, 3%, 4%, 5% et 8%) sur l'isotherme, la ligne de courant, le nombre de Nusselt moyen (Nu_{avg}) et la génération totale d'entropie.

L'étude a révélé que l'ajout de nanofluide dans des conditions spécifiques augmentait la transmission de chaleur. Les propriétés thermiques améliorées des nanofluides jouent un rôle significatif dans le régime d'écoulement dominé par la conduction, entraînant la plus grande augmentation du transfert de chaleur. Les nanofluides n'améliorent pas de manière significative l'efficacité du transfert de chaleur lorsque la convection est le mode principal. Augmenter la différence de température entre les sources chaudes et froides a augmenté la force de flottabilité.

الملخص

نقل الحرارة والكتلة بالحمل الطبيعي في الديناميكا المائية المغناطيسية في السوائل التقليدية قد جذب اهتمامًا بحثيًا كبيرًا نظرًا لتطبيقاته الواسعة في العلوم والهندسة، خاصة في تقنيات الطاقة الشمسية. إدخال عوامل مثل مصادر/مصارف الحرارة، التفاعلات الكيميائية، امتصاص الإشعاع في تدفق السوائل وتحليل الوسائط المسامية يزيد من التعقيد ولكنه أيضًا يزيد من الأهمية العملية لهذه الدراسات للأنظمة الهندسية المتقدمة. أصبحت هذه الجوانب مجالات تركيز رئيسية في الأبحاث المعاصرة، نظرًا لأهميتها الحاسمة عبر مجالات صناعية وعلمية متعددة.

تستخدم هذه الدراسة نموذجًا غير دراسي لاستخدام المصطلحات المسامية في معادلات الزخم لمحاكاة الحركة الحملية للجسيمات النانوية في الفضاء المسامي في وجود المجال المغناطيسي رقميًا. السائل العامل هو سائل نانو. سيتم تقديم تقرير عن تأثير الإشعاع لأشكال الجسيمات النانوية المختلفة. سيتم أيضًا توضيح تأثير العوامل الفيزيائية على أداء السائل النانوي، وسيتم فحص تأثير معامل الشكل ومعامل الإشعاع، والقوة المغناطيسية، وشكل الجسيمات النانوية، وعدد رايلي على السائل النانوي.

تركزت دراسة إضافية على توليد الإنتروبيا في تدفق الحمل المختلط للسوائل النانوية. تم تطوير نموذج رياضي يشرح هذا الحدث. تم تفكيك المعادلات غير البعدية الناتجة باستخدام طريقة العناصر المنتهية بوزن غاليركين. تتناول هذه الأطروحة تدفق الحمل الحر غير القابل للانضغاط في حالة الاستقرار للسوائل النانوية في حاويات مختلفة باستخدام برنامج ديناميكا الموائع الحسابية (CFD). تم تغيير شكل الحاوية، مع تكوينات مختلفة للجدران الساخنة والباردة، وجدران جانبية مستقيمة ومتموجة. مع أو بدون حواجز في الموقع.

تدرس الدراسة تأثير أرقام رايلي (Ra) من 10^6 إلى 10^2 ، وأرقام هارتمان (Ha) من 0 إلى 100، وأرقام دراسي (Da) من 10^{-2} إلى 10^{-5} ، والمعاملات الهندسية (عدد التمدد وموقع الأسطوانة الداخلية)، ونسب الحجم الصلبة (0، 2، 3%)، 4، 5، 8%) على خطوط الحرارة، وخطوط التدفق، ومتوسط عدد نوسلت (Nuavg)، وتوليد الإنتروبيا الكلي. وجدت الدراسة أن إضافة السائل النانوي تحت ظروف محددة عزز نقل الحرارة. تلعب الخصائص الحرارية المحسنة للسوائل النانوية دورًا كبيرًا في نظام التدفق الذي تهيمن عليه التوصيل، مما يؤدي إلى أكبر زيادة في نقل الحرارة. لا تحسن النانوسوائل بشكل كبير كفاءة نقل الحرارة عندما يكون الحمل الحراري هو الوضع الأساسي. زيادة فرق درجة الحرارة بين المصادر الساخنة والباردة زادت من القوة الطافية.

Introduction:

Enhancing heat transfer in fluids is a critical pursuit in the realm of thermal engineering, driven by the need to optimize the efficiency of various industrial and technological processes. Achieving superior heat transfer capabilities involves a multifaceted approach, encompassing alterations in fluid properties, adjustments in flow dynamics, modifications to container geometries, and the application of external forces. Among the pivotal factors that significantly impact convective heat transfer is the fluid's thermal conductivity. employed.

The specific applications for the considered geometry of a three-layered permeable chamber that is an exposure to a magnetic field and has a revolving cylinder include central air (Warming, Ventilation, and Cooling) systems, solar energy use, electronic cooling applications, oil extraction processes, automotive technologies, and medical applications . These applications can benefit from the optimization of heat transfer in nano-fluids, which is the focus of the study.

Nano-fluids are utilized across an expansive range of designing and modern applications, remembering their use for central air (Warming, Ventilation, and Cooling) systems [1] solar energy use [2] electronic cooling applications [3] oil extraction processes [4] automotive technologies [5] and even medical applications. The aim of research on heat transfer in nano-fluids is to reduce energy losses and improve overall efficiency by optimizing heat transfer in nano-fluids. The two basic types of convection, which relate to the impact of fluid movement of heat transfer, are forced convection and free convection. Combination or mixed convection occurs when both types of convection occur simultaneously. Numerous studies [6], [7] show that much research has been done to examine the mixed convective heaviness of nano-fluids.

One of the most significant problems facing our society is energy management. In order to preserve our natural resources, it is imperative that we optimize our energy usage. Energy generation, storage, and cooling all utilize convective evaporation. As a result of this movement, fluid particles mix. natural convection is the mechanism of heat transmission.

The numerical simulation of Heat transmission and fluid flow phenomena holds a significant position both in research and industry. The mathematical simulation of the phenomenon is based on mechanical equations. Using one of the many numerical techniques, fluids coupled with the energy equation and their resolution become possible. One of the most popular approaches is the use of finite elements and volumes. The latter is the source of several other CFD codes, such as the COMSOL Multiphysics 6.2 software, which we will use in our work.

The ability to considerably boost heat transfers for specific applications Adding a tiny concentration of nanoparticles to a pure fluid has led to a recent substantial development in research on nanofluids. Remember The colloidal solutions known as nanofluids are created by scattering solid particles inside a base fluid that are nanometric in size ($10^{-9}m$). Certain solutions have demonstrated remarkable efficacy in enhancing heat transfer under specific circumstances, even at very low concentrations.

The requirement to maximize the efficiency of several industrial and technical processes has made improving heat transmission in fluids a crucial goal in the field of thermal engineering. It takes a comprehensive strategy that includes changes to fluid characteristics, flow dynamics, container geometries, and the application of external forces to achieve greater heat transfer capabilities. The fluid's thermal conductivity is one of the key elements that greatly affects convective heat transmission. The thermophysical characteristics of the liquid and solid phases both affect heat transport in a nanofluid.

Objective:

Energy efficiency has been one of the most sought-after development sectors in recent years. energy, and this mostly depends on enhancing industrial and technological systems' and processes' thermal performance. Convection-mixed heat transfer in devices with cavities, which enable the creation of recirculation flows, has grown in relevance as a research topic. These arrangements are seen in many engineering domains, including solar collectors, heat exchangers, combustion chambers, and systems for cooling electronic devices [8].

Convective heat transfer in enclosures includes a range of fluid flow and heat transfer scenarios within different kinds of cavities, despite the fact that it has been extensively studied in recent years. Enhancing heat transfer rates in convective situations has become more important in recent years. There are several scientific and industrial applications for convection heat transfer, including oil extraction, solar energy, and electronic cooling. The literature has discussed a number of topics, such as models, laminar and turbulent approaches, coolants, boundary conditions, and different enclosure shapes.

We assess A Comparative Analysis of Heat Transfer in Porous and Nanofluid-Filled Cavities and the Conjugation of Local Thermal Equilibrium and Radiation Effects The results, together with sensitivities pertaining to geometrical and porous media factors, will be displayed using streamlines, isotherm contours, and the Nusselt number output. This section will address two instances. The initial one illustrates the transfer of heat through conjugate local thermal equilibrium in a three-dimensional cavity containing a rotating cylinder composed of porous

material. The impact of natural convection in MHD nanofluid on heat radiation in a specific cavity is the second.

The research was directed by the following goals, which also influenced the content of each chapter. The following are the goals of the study:

- Enhancing the efficiency of heat transport across a range of applications.
- Examining the effects of nanofluid processes that facilitate heat transmission
- nanoparticles with different thermal properties and volume fractions.
- Evaluating the impact of different internal barriers and enclosure geometry.
- Being aware of the potential effects on energy transport of altering the central cylinder's rotational speed and direction within the chamber.
- investigating the possibility of adding interior baffles to the cavities.
- Determine the efficiency of heat transmission and the flow patterns inside the geometry by putting forth various heating source configuration scenarios.
- Compare the overall entropy and average Nusselt number values for each of the examined situations.

Establish a clear, optimistic, and forward-looking course for future work.

Layout of the thesis:

The following is how the current thesis has been put together:

First Chapter: This chapter examines the body of research on fluid models used in the doctoral dissertation. The main goal is to provide a summary of the main concepts that underpin this thesis.

Chapter II: In this section, we delve into the subtleties of dimensionless parameters and basic concepts that form the basis of our theory. We start by reviewing the mathematical formulas that govern the flow of nanofluids in the presence of a magnetic field and a porous medium.

Additionally, we look into the connections between the thermo-physical properties of nanofluids and the non-dimensional numbers relevant to our study.

We also examine numerical solution techniques and emphasize their application in computational fluid dynamics (CFD). Lastly, we provide an in-depth explanation of the computational domain's mesh, form, and validation procedures.

Chapter III: The primary focus of this chapter is the thesis research on the analysis of Heat Transfer Mechanisms in Porous and Nanofluid-Filled Cavities: Conjugate Local Thermal

Equilibrium and Radiation Effects and their outcomes. We will look at two different cases in this chapter. The first example is a three-dimensional, triangular chamber that is partially porous and filled with a hybrid nanofluid. The second example is a novel kind of cavity that is filled with Cu-H₂O nanofluid while being continuously magnetized.

CHAPTER I

GENERALITIES

&

BIBLIOGRAPHIC

SYNTHESIS

I.1. General principles of heat transfer by convection

I.1.1. Introduction to heat transfer by convection:

The convection mode is encountered in situations where energy is transferred in the form of heat to a fluid in flow at the surface where the flow occurs. We distinguish three types of convection: natural convection caused by Archimedes' buoyancy, forced convection caused by an external force, and mixed convection caused by the effects forced convection, which is due to an external force, and mixed convection, which is due to the combined effects coupled with Archimedes' buoyancy and the external force. In this chapter, we will present a reminder on the mode of thermal transfer by convection, particularly mixed convection.

This mode basically involves conduction in a very thin fluid layer at the surface followed by flow-induced mixing. Energy is transferred by a mixture of bulk flow and molecular diffusion. The heat flux solely depends on the fluid's characteristics and is unaffected by the surface material's characteristics.

Conduction refers to The conveyance of energy from particles of greater energy of a material to lower energetic particles as a consequence of particle interactions. Greater temperatures are associated with more intensive molecular energy because of heat transmission by conduction phenomenon.

Radiation is the energy that nonzero-temperature materials emit. Despite the fact that we shall concentrate on radiation from solid objects,, it is also possible to emit radiation between gasses and liquids. Whatever the nature of the substance, the emission can be explained by changes in the electron configurations of the constituent molecules or atoms. Through electromagnetic waves, the radiation field's energy is conveyed . Radiation, unlike conduction and convection, is not dependent on the existence of a tangible medium. to carry energy. Radiation transfer is indeed most effective in a vacuum.

The study of heat movement is known as convective heat transfer. mechanisms caused by fluid flow. The human desire In order to understand and forecast how a fluid flow acts as a "conveyor belt" or "carrier" for matter and energy, the development of convective heat transfer into modern science. Heat transfer and fluid mechanics are two more established sciences, and convective heat transfer is unquestionably a field at their intersection. Because of this, the analysis of every convective heat transfer case must be established on a strong understanding of fundamental concepts in fluid mechanics and heat transfer. The goal of this chapter is to review these concepts.

Convective heat transfer has two branches: forced convection and natural convection. Forced convection refers to heat transmission via fluid motion that is produced that is compelled to occur. Consistent mechanical power is required for this induced flow. But natural convection is not the same as forced convection. caused by fluid flow driving force. Because of the presence of a density gradient and a gravitational field, the flows are driven by the buoyancy forces. The density differential causes buoyancy effects, which lead flow to occur. The combination of a fluid density differential and body force causes buoyancy. The temperature potential drives the fluid currents' intensity, which in turn determines the temperature distribution in natural convection. These two mechanisms can interact in a variety of ways. The mix of a buoyancy-driven flow (natural convection) and an imposed flow (forced convection) simultaneously refers to a mixed convection mechanism.

Furthermore, convection heat transfer is categorized as either exterior or internal. In internal convection, the fluid is surrounded by a surface, like a cooling tube filled with water in an internal combustion engine or a pipe carrying steam. In external convection, the fluid surrounds a surface, such as flow across a flat or curved surface. There are three types of fluid flows: transitory (changing from laminar to turbulent), turbulent, and laminar.

The field of thermodynamics focuses on quantifying the transfer of energy during the transition of a system between equilibrium states, regardless of the duration of the process. However, in engineering, there is often a keen interest in the heat transfer rate, which is the main objective of research on thermal exchanges [9].

Heat transfer plays a crucial role in various industrial applications, encompassing both macroscopic and microscopic scales. Scientists are currently striving to predict and optimize heat transfer rates across a wide range of practical scenarios. The three fundamental mechanisms of heat transfer recognized are conduction, convection, and radiation [10].

Numerous [11] studies have focused on improving heat transfer through convection, and in order to do so, many researchers have carried out a great deal of experimental and numerical testing on the properties of the fluids involved (physicochemical properties), the nature of the systems in which convection occurs (particularly geometry), and the description of the phenomena governing convection.

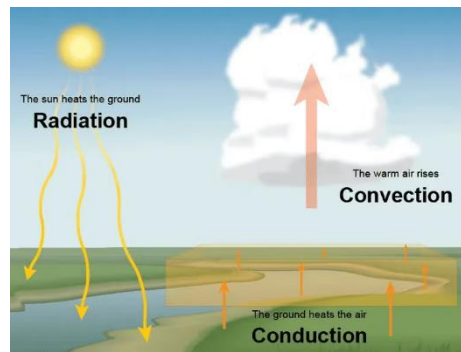


Figure I-1 Modes of heat transfer.

Convection

Convection is the process by which heat moves through a moving fluid, such as a liquid or gas. in a gravitational field, any temperature difference in a liquid or gas alters its density, causing movement within the fluid due to the buoyancy force. This mixing movement, in which the hottest parts of the fluid tend to rise and the cold and dense parts tend to descend, is called convection. The movement of the fluid can be natural, forced, or mixed.

Natural convection:

In this type of convection, the movement results from the variation of the fluid's density with temperature, this variation creates a gravitational force field that conditions the movement of the fluid particles.

The process of heat transfer that occurs when fluid particles move between areas at different temperatures is known as natural convection. This movement leads to a mixing of fluid particles, which exchange energy and momentum with each other. Unlike forced convection, where the fluid moves due to an external energy input, natural convection is caused by variations in the fluid's density related to temperature changes. The nature of the fluid, the geometry of the area where the process takes place, and the external thermal requirements that cause it are directly related to the structure and intensity of natural convection.

The temperature gradient, heat flux, geometry, fluid density, fluid viscosity, fluid heat capacity, fluid thermal conductivity, fluid direction, and the pivot of the enclosure are all known to affect natural convection. In porous materials, porosity and permeability play a significant role in the efficiency of natural convection. Natural convection has a more complicated reaction, especially in conjugate domains. Temperature has an impact on the physical properties of a fluid. This influence is expressed in the form of streamlines and isotherm lines rather than the

expression of the thermal boundary layer used in forced convection. Whether the surfaces are heated isothermally or with a continuous heat flux, the process of free convection can be modified. Natural convection in cavities has been analyzed experimentally and numerically, and research has produced increasingly reliable results over time, up to the present day.

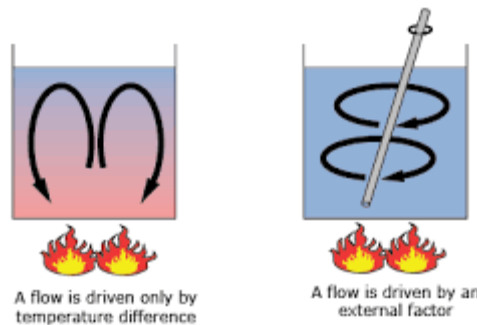


Figure I-2 Representative diagram of natural convection

The study [12] presents an analytical calculation procedure for predicting heat transmission at vertical flat plates in air through unstable natural convection. It approximates mean deviations and the temporal evolution of momentary convective heat transfer through experimental studies and observations. The computation process makes the assumption that fluid molecules have varying amounts of potential and kinetic energy. The validation results are in good accord with the measurement data that was cited.

The study examines [13] the Entropy creation and improvement of natural convection heat transfer in nano liquid-metal fluids, specifically copper-Ga, diamond-Ga, and carbon nanotubes-Ga (CNT single bond Ga), in a differentially heated cavity. The findings indicate that as the Grashof number increases, so do convective intensity, heat transfer, and local entropy generation and nanoparticle volume fraction.

The study [14] investigates natural convection and the impact of a magnetic field Al_2O_3 -water in a heated annular enclosure, highlighting the complex interactions between nanoparticles and magnetic fields, crucial for optimizing thermal management systems.

The article [15] presents a digital study on natural convection and thermal transfer using nano-fluid (Cu-Water (H_2O)). It uses FLUENT 6.3 code and geometry model, showing increased Nusselt number due to volume fraction and Rayleigh number growth.

This article examines heat and mass transfer in a trapezoidal cavity heated from the bottom and cooled from the inclined upper wall. Results show flow configurations depend on the θ angle

inclination, with weak angles forming Rayleigh-Benard cells and large angles forming differentially heated rectangular enclosures. The study [16] also examines the influence of thermo-solutal solicitations, geometry, and species nature.

The study [17] explores the coupling of MHD natural convection in Al_2O_3 -water nanofluid cavities, revealing that heat transfer enhancement increases with Rayleigh number and nanoparticle volume fraction, but decreases with Hartmann number.

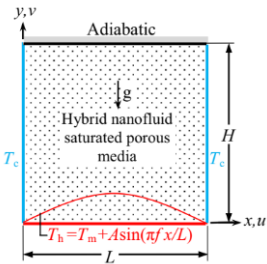
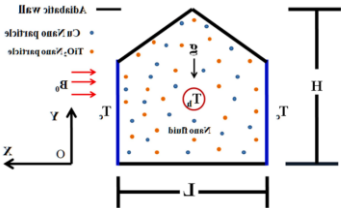
Numerous new experimental studies have been published in the last few years, discussing various scenarios related to the optimization of natural convection of hybrid-Nano fluids.

Table I.1 Examples of current natural convection experiment research.

Authors / Year	Subject	Nano fluid	Results
Dey & Dash, 2023 [18]	Examining the nanofluids in a hollow under natural convection both with and without a rotating magnetic field through experimentation	ferromagnetic and non-magnetic,	The study explores the convection behavior of nanofluids in a square cavity, examining the impact of ferro-magnetic and non-magnetic concentrations on heat transfer performance, using a rotating magnetic field for data analysis..
[19]	Experimental Investigation into Natural Convection of Zinc Oxide/Water Nanofluids in a Square Cavity	Zinc Oxide/Water	ZnO nanoparticles in DIW were used to create nanofluids, demonstrating stability, enhanced heat transfer coefficient, Nusselt number, and heat transfer rate at varying concentrations.
Torki & Etesami, 2020 [20]	Experimental study of SiO_2 /water nanofluid heat transfer by natural convection in an inclined enclosure	SiO_2 /water	The study examined the natural convection heat transfer of SiO_2 /water nanofluids in a rectangular enclosure, finding that

			concentration decreased the heat transfer coefficient and tilt angle significantly affected Nusselt number.
Dey & Sahu, 2021[21]	Investigation of nanofluids in a natural convection cavity	ZnO (zinc oxide) and Al_2O_3 (alumina) nanoparticles/water	The importance of the stability of nanofluids and their potential to improve heat transfer in industrial applications.

Table I.2 Examples of current natural convection numerical research.

Authors/year	Geometry	Nanofluid	Outcomes
Biswas et al., 2021 [22]		Cu– Al_2O_3 /water	The study reveals that half-sinusoidal heating enhances heat transfer in a porous cavity filled with a Cu– Al_2O_3 /water hybrid nanofluid, with multi-frequency heating being more effective for thermal system design.
Islam et al., 2023 [23]		Cu– TiO_2 – H_2O	The study investigates convective heat transfers using Cu TiO_2 – H_2O nanofluid in a closed cavity, revealing its superior thermal properties, increased heat transfer rate, and improved Nu_{avg} development.

Forced convection:

Convection is referred to as forced when an external force field causes the fluid to move. Under natural settings, forced convection results from the vent, which is itself the product of the local gradient of atmospheric pressure. The fluid exchanges heat with parois, but temperature variations practically have no effect on its movement, movement is a gradient of external pressure that causes the displacement of fluid particles. The study of heat transfer by convection is therefore closely related to that of fluid flow.

The air is blown into the cabin by a fan, but the buoyancy effect is not negligible, especially when positioned far from the air intakes. The dimensionless group determines which of these convection types is dominant.

$$Ri = \frac{Gr}{Re^2}$$

if: $Ri < 0.1$ forced convection that dominates.

if: $Ri > 10$ forced convection that dominates.

if : $0.1 < Ri < 10$ forced convection that dominates.

The study [25] on forced convection heat transfer of nanofluids reveals that particle migration mechanisms like Brownian motion and thermophoresis enhance heat transfer coefficient, indicating potential for performance optimization. [24] The study explores the thermal behavior of nanofluids in a cubic cavity, revealing nanoparticles enhance heat transfer, decrease entropy generation, and minimize temperature variation, offering potential engineering applications.

The study [26] on forced convection of Al_2O_3 -nanofluids revealed that adding Al_2O_3 nanoparticles to water enhances heat transfer rates, with Reynolds number influencing velocity and temperature distribution. Nanofluids [27] enhance heat transfer efficiency by 2% to 50%, with optimal concentration increasing heat transfer but increasing pressure losses, and performance varies between laminar and turbulent flow regimes.

The study [28] explores the impact of uncertainties in nanofluids' physical properties on their thermohydraulic performance during forced convection heat transfer, highlighting the need for accurate predictions and experimental data

The study [29] reveals that nanofluids' thermal conductivity increases with concentration and temperature, varying based on type, flow conditions, and particle shape, highlighting the complexity of interactions.

The study [30] investigates the hydrodynamic and thermal properties of nanofluids, specifically Ethylene Glycol– Al_2O_3 and water– Al_2O_3 , using numerical simulations, revealing increased heat transfer and wall shear stress.

The study [31] explores nanofluid's laminar forced convection heat transfer through a bent channel, revealing increased Nusselt number with nanoparticle volume fraction, particularly at higher Reynolds numbers.

I.2. Mixed Convection In Differentially Heated Cavite: State Of The Art:

Mixed convection is a combination of forced and free convections, determined by both outer and inner volumetric forces. It is most commonly observed in geophysics and is found in smaller scale systems like channel flows. Mixed convection is often realized when heating or cooling channel walls and at small velocities of laminar flow. Studies of turbulent channel flows with gravity field effects have become important in engineering practice since the 1960 [32].

The issue of laminar mixed convection with lid-driven flows presents a wide array of practical applications within thermal engineering. These include electronic device cooling, high-performance building insulation, multi-layered shielding for nuclear reactors, food processing, glass production, solar power collection, furnaces, drying technologies, and more. Extensive research on lid-driven cavity flow and heat transfer, exploring diverse cavity setups, fluid types, and temperature gradients, remains a recurrent topic in published literature.

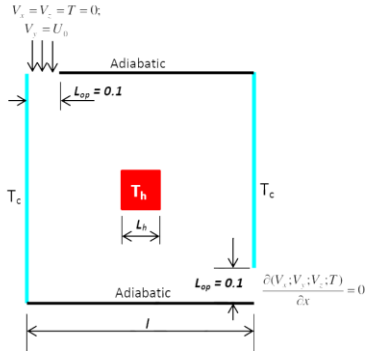
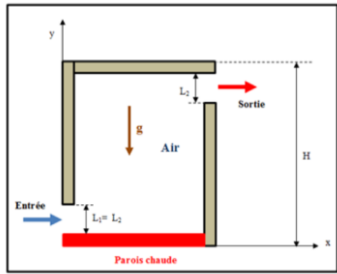
The authors [33] conducted a numerical study on mixed convection in a double lid-driven cubic cavity, analyzing flow structures and heat transfer rates under varying Reynolds and Richardson numbers, and comparing results with published data.

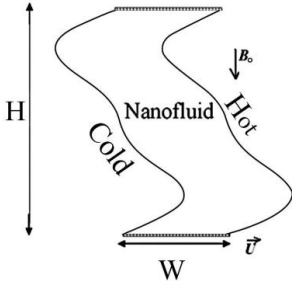
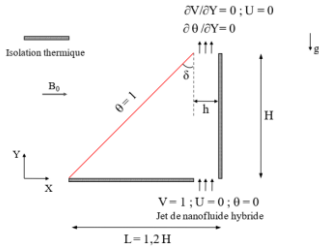
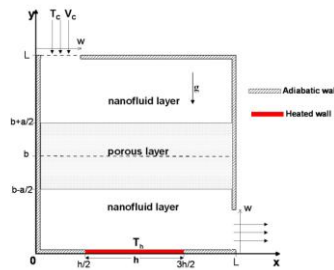
The article [34] explores mixed convection and surface radiation heat transfer in an air-filled ventilated cavity, analyzing parameters like Reynolds number, Richardson number, and aspect ratio, emissivity, and Grashof number. Its comprehensive approach, comparative analysis, detailed experimental setup, and rigorous numerical simulations contribute to knowledge. The article [35] explores the use of Al_2O_3 -water nanofluids in natural convection within wavy cavities, analyzing their heat transfer enhancement and comparing them to base fluids, highlighting their potential benefits in thermal engineering applications. The study predicts [36] convective behavior of nanofluids in shallow cavities using parallel flow approximation and integral energy equation, predicting Rayleigh number. It enhances credibility and reliability of findings.

The study [37] explores the effect of an external magnetic field on heat transfer and entropy generation in Ag-MgO/water hybrid nanofluid flow, finding that intensifying the magnetic field reduces heat transfer convection.

Mohammad Lakhi [38] explores heat transfer in a 2D channel using the Lattice Boltzmann method, analyzing parameters like optical thickness, Planck number, and Richardson number, and their impact on temperature and fluid flow distribution.

Table I.3 Examples of current mixet convection numerical research.

Authors/year	Geometry	Nanofluid	Outcomes
[39]		Al ₂ O ₃ the Aluminium Oxide/Water nanofluid	The study reveals that the Nusselt number increases with Rayleigh number and nanoparticle volume fraction, indicating enhanced heat transfer performance and a significant increase in entropy generation.
[40]		air	The study validated the thermal lattice Boltzmann method's reliability in simulating mixed convection phenomena, revealing thicker boundary layers and intensified upward flow in a ventilated cavity.

[41]		Al_2O_3 /Water nanofluid	The study explores mixed convection flow in a wavy lid-driven cavity filled with alumina-water nanofluid, revealing that wavy wall geometry, Rayleigh and Hartmann numbers, and solid volume fraction significantly impact flow efficiency.
[42]		Ag-MgO/Water	The study on mixed convection of a hybrid nanofluid under an external magnetic field improved hydrodynamic effects and thermal performance, with nanoparticle addition enhancing heat transfer.
[43]		Fe_3O_4 , Cu , Ag and Al_2O_3	Messaoud Hamdi's study on mixed convection heat transfer in a square cavity revealed Reynolds number, porous layer height, and superior thermal

			performance of silver nanoparticles.
--	--	--	--------------------------------------

The study reveals that the Nusselt number increases with Rayleigh number and nanoparticle volume fraction, indicating enhanced heat transfer performance and a significant increase in entropy generation

I.3 Influence of radiation on heat and momentum transfers

In the context of natural convection flows in cavities representative of the habitat where the temperature is on the order of 300 K, the low temperature amplitudes have often led to considering radiative exchanges as negligible compared to convective and conductive transfers. Given the difficulty of numerically solving radiative transfers, this hypothesis has long resonated in the literature.

However, some authors have shown that radiation has a significant impact on natural convection flows [Velusamy et al., 2001; Sergent et al., 2013a; Ibrahim et al., 2013] compared to forced convection, even at low to moderate Rayleigh numbers. In order to study the impact of radiation on natural convection flows, several experimental [Bratis and Novotny, 1974; Fusegi and Farouk, 1989; Clergent, 2000; Rouger, 2009; Saury et al., 2011] and numerical studies have been conducted, in which we can highlight two main categories:

- **the surface convection-radiation coupling, considering only the exchanges** radiative between the different surfaces of the domain, associating the fluid with a non-participating transparent medium. Also, the coupling between convection and radiation does not occurs only at the walls through the convecto-radiative equilibrium.
- **the coupling of surface convection-radiation and gas, considering the gas as** being a participative medium that can absorb and emit heat through radiation, thereby altering the local temperature of the fluid, in addition to the exchanges between walls.

1.3.1 Surface convection-radiation interactions

Two different approaches can be considered for accounting for the coupling: explicit coupling and implicit coupling. The first approach involves numerically solving the set of equations governing convection and radiation problems and will be described in the following paragraph.

I.4 The impact of radiation on natural convection flows:

The study [44] investigates the influence of internal heat generation and thermal radiation on mixed convection flow from a heated vertical plate, using mathematical modeling, numerical solutions, and parameter analysis.

The study [45] examined radiation heat transmission and natural convection heat transfer in vertical sinusoidal wavy fins that are positioned horizontally. According to the results, wavy fins improved heat transmission more than rectangular fins. However, because of the obstruction of fluid mobility, natural convection was adversely affected by larger wave amplitudes. According to the study, radiation plays a significant role in the overall heat transfer in finned surface heat sinks.

The study [46] investigates combined radiation and natural convection in annular spaces and sectors using gray, emitting, absorbing, and isotropically scattering gas. It uses finite volume radiation solution method and control volume approach to analyze coupled equations, obtaining numerical solutions for $Pr = 0.7$.

The study [47] investigates combined radiation and natural convection in a medium between concentric or vertically eccentric horizontal cylinders. It uses a two-cylindrical coordinate system, stream function, and vorticity to solve coupled equations. Numerical solutions are obtained for Rayleigh numbers $10^3 - 10^5$ and radiation-conduction parameters 0 to ∞ .

The study [48] reveals that radiation absorption significantly improves the temperature distribution and heat transfer characteristics of magnetohydrodynamic hybrid nanofluids, leading to increased thermal energy, improved Nusselt number, and potential boundary layer behavior.

The study [49] explores radiative transfer's impact on turbulent convection flow in turbulent flow turbulence, using a Rayleigh-Bénard cubic cavity and numerical direct simulations. The ADF model captures gas properties, and the Spectral Vanishing Viscosity model is used for large-scale simulations.

The study [50] explores the unsteady magnetohydrodynamic convection of a hybrid nanofluid in a U-shaped porous cavity, crucial for optimizing thermal management systems in electronic devices and industrial applications. Results show that increasing the Hartmann number reduces the average Nusselt number, while high wave amplitude increases the average Nu .

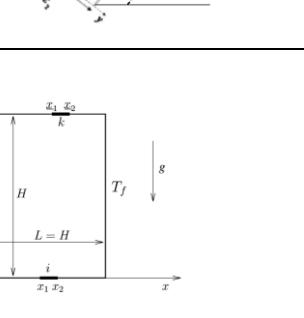
Thermal radiation improves cooling performance in electronic components by enhancing heat dissipation and affecting temperature distribution within nanofluids. Mathematical models validate the influence of radiation on nanofluids' flow and temperature, highlighting its role in electronic device cooling systems[51].

I.5 The impact of radiation on mixed convection flows:

Mixed convection with surface radiation heat transfer is a thermal phenomenon in an enclosed space influenced by natural and forced convection, as well as radiative heat exchange between surfaces. The term "mixed convection with surface radiation heat transfer with nanofluid hybrid" refers to a system where heat transfer is influenced by a combination of natural and forced convection, surface radiation heat transfer, nanofluid, and a hybrid system. The presence of nanofluid as a hybrid medium significantly affects heat transfer characteristics and overall thermal behavior. This understanding and optimization are crucial for applications such as thermal management in electronics, renewable energy systems, and heat exchangers.

In this type of situation, various methods of thermal transfer (convection, radiation) can be combined, especially through the walls (surface radiation), and when the fluid itself behaves like a semi-transparent medium, that is, it absorbs and emits radiation directly. It is therefore necessary to consider an internal heat source in the medium, which results from the disparity between the radiative energy absorbed and that emitted by each element of the volume. Convection is influenced by radiative transfer in one direction, with a direct influence on the thermal field and an indirect effect on the dynamic field (changing the temperature and thus Archimedes' push forces).and in my work I will compared the effect of radiation coupled by convection and the heat transfert without radiation

The study [52] on mixed convection boundary layer flow in a porous medium containing thermal radiation and nanofluid surrounding a permeable vertical cylinder revealed bifurcation in flow behavior, with positive eigenvalues indicating stability and negative eigenvalues indicating instability.

	Numerical study of volumetric convection-radiation coupling in an inclined channel using the lattice Boltzmann method [53]
	Wang et al.[54] developed a numerical code to analyze the interaction between natural convection and surface radiation in a square cavity, focusing on radiative heat exchange's impact on flow, temperature, and stability thresholds.

I.6.1. Definition.

It is described as a field of physics that integrates concepts from fluid dynamics and magnetism to investigate the behavior of electrically conducting fluids, including plasmas, liquid metals, and some ionized gases. The fundamental focus of MHD is the interaction between fluid motion and the magnetic fields it produces or is exposed to. However, a comprehensive theory known as magnetohydrodynamics (MHD) did not arise until the 20th century. The term Magnetohydrodynamics (MHD) was originally used in 1942 by the scientist Hannes Olof Gösta Alfvén (born 30 May 1908 in Norrköping, Sweden – died 2 April 1995 in Djursholm, Sweden). For his work on this topic, he was granted the 1970 Nobel Prize in Physics. The main goal of MHD is to understand how an electrically conducting fluid— a liquid or an ionized gas known as plasma—moves when exposed to a magnetic field. Electromagnetic effects and the velocity of the conductive fluid are coupled ".

Mass, momentum, energy, and magnetic induction in a conducting fluid are all explained by the fundamental equations of MHD. These frequently intricate computations take electromagnetic force effects into account. In short, the equations that define MHD are derived from the combination of the Navier-Stokes equations for fluid dynamics and the electromagnetic Maxwell's equations. These partial differential equations can be solved both analytically and numerically at the same time. MHD is produced when a conducting fluid moves across a magnetic field and is influenced by it. A fluid running over a transverse magnetic field experiences an electromagnetic force as a result, and the current and magnetic field that are produced work against the fluid's movement. Additionally, the original magnetic field is altered by the magnetic field produced by the current.

I.6.2 Convection Magnetohydrodynamique

Magnetohydrodynamics (MHD) is of practical importance in various industrial sectors such as propulsion reactor cooling, electronic devices, heat exchangers, and energy production, among other applications. It can be used as a design and manufacturing tool in the industrial field. Natural MHD convection involving heated obstacles has been the subject of numerous studies over the decades. Moreover, it can be beneficial in areas such as phase change materials (PCM), passive ship insulation, as well as other applications in the field of renewable energy. The experiments and figures are displayed in the references [55]. It has been proven that the height of the heat source has a significant impact on the velocity fields and, consequently, on the efficiency of heat transfer. Paroncini et al. [56] investigated the convective heat transfer produced by a hot source situated in the center of a square cage that had three distinct heights. The 2D particle image velocimetry (PIV) technology and holographic interferometry were used for the experimental study. The distributions of dynamic fields are shown using the Particle Image Velocimetry (PIV) technology, while temperature fields are analyzed using holographic interferometry. The study demonstrates how the height of the source reduces heat transfer by natural convection.

Another study examined natural convection experimentally in a square air-filled chamber by Corvaro et al. [55] and Esmail et al. [57]. For three distinct layouts, they attached two heat sources to the cavity's vertical walls. Measurements were made using varying hot source temperatures for each arrangement. They investigated the interaction between the dynamic and thermal fields and found that the temperature of the hot source and the location of the cold source both affect heat transport within the cavity. To statistically examine the effect of

thermophysical characteristics on laminar natural convection in a cavity containing nanofluids, he chose to use the finite difference approach.

The physical processes behind the movement of nanoparticles have been thought to involve thermophoresis and Brownian motion. According to a parametric analysis, the viscosity of a nanofluid has a direct correlation with its heat transfer efficiency, although its thermal conductivity has a secondary effect. Hoseinpour et al. [58] investigated the entropy production in a porous cavity saturated with nanofluid (Cu-water) using the Lattice Boltzmann method (LBM).

According to their findings, the average Nusselt number rises as the volume percentage of nanofluids grows, while the total entropy production falls. In a similar vein, the total entropy output rises as the porosity of the porous medium increases. Gorla and associates [59] They introduced heat sources for heating and fins for cooling after performing a computational research on MHD natural convection in a porous cavity filled with a hybrid nanofluid (Cu-Al₂O₃-water). Their findings suggest that changing the heat source's location reduces heat transfer for hybrid nanoparticles. Furthermore, they came to the conclusion that, in comparison to other suspensions, the addition of Cu nanoparticles results in a faster rate of heat transmission.

Another conclusion of their study is that for higher values of the Hartmann number, the current function intensity of hybrid nanofluids decreases.

Esfahani and Alinejad. [59] Selimefendigil and his [60] colleagues examined the generation of two-dimensional entropy caused by natural double-diffusion convection. They used the lattice Boltzmann method as a numerical tool. Their conclusions indicate that by increasing the Rayleigh number, the Bejan number decreases, which improves heat transfer and increases the total entropy generation rate. Moreover, they demonstrated that an increase in the thermal conductivity ratio leads to an increase in the Nusselt number and the total entropy generation rate, while the Bejan number decreases. A numerical study. [61] on MHD mixed convection of nanofluid in various geometric configurations (a cavity with a downward step, a triangular cavity with an adiabatic rotating cylinder, and a driven cavity) was conducted. The finite element method was used to solve the system of equations.

The results indicate that heat transfer decreases with the increase in the Richardson number and the Hartmann number. On the other hand, the heat transfer increases with the increase in the volume fraction of the nanoparticles. The presence of a magnetic field exerts a retarding effect

on convection. Moreover, studies have revealed that the maximum generation of total entropy is influenced by the orientation of the magnetic field and the geometry studied. Al Amiri et al. [61] analyzed natural convection in a square enclosure containing a heated block at the bottom. For the numerical solution of the transfer equations, a finite element formulation based on the Galerkin method was employed. The effects of the height, width, and position of the heated block were examined, as well as the impact of the Rayleigh number. The results demonstrate that increasing the height, width, and placement of the heated block leads to an improvement in heat transfer, attributable to the increase in the heating surface area. Sarris et al. [60] conducted a numerical analysis of natural convection in rectangular reservoirs locally heated from the bottom, using a fluid with a high Prandtl number. The researchers observed that, for shallow reservoirs with symmetric heating, an increase in the Rayleigh number leads to an increase in the thermal penetration length from the symmetry plane towards the side walls. However, this increase in Ra also induces a local rise in temperature above the heat source, without a significant increase in temperature in the colder regions of the reservoir. Moreover, they found that increasing the aspect ratio of the cavity and the width of the heat source intensifies the flow and raises the fluid temperature. The position of the heat source also plays a crucial role in the flow model, temperature distribution, and thermal penetration length. A recent review of natural convection in enclosures with localized heating is referenced in the article.

I.6.3 Aspect of Magnétohydrodynamics Fluid Mechanics

The study of electrically conducting fluid dynamics is known as magnetohydrodynamics (MHD), often known as magnetofluid dynamics or hydromagnetics. Salt water or electrolytes, liquid metals, and plasmas are a few examples of these fluids.

such as freely moving ions or electrons. A charged particle, such an electron, experiences three different types of forces in accordance with non-relativistic electromagnetic principles. The motion of charged particles and magnetic materials produces magnetism, which is symbolized by the magnetic field (B). The Lorentz force, so named in honor of the Dutch physicist Hendrik A. Lorentz, is the force acting on a charged particle (q) moving with a velocity (v) within an electric field (E) and a magnetic field (B). This force can be expressed mathematically as

$$F = qE + qV \times B \quad (I.1)$$

For simplified cases, the Lorentz force may be described as:

$$F = qV \times B \quad (I.2)$$

I.6.4 Porous media

Definition.

Solid materials with voids or pores integrated into their structure are known as porous materials, or porous media. Both permeable and nonpermeable porous media are categorized according to these materials' capacity to promote fluid flow in the presence of external forces. Porous media can have a variety of properties, such as homogeneity or heterogeneity in their internal structure, dispersion or nondispersion of pores, and the potential to integrate numerous structural components. One important characteristic that distinguishes porous media is porosity. It is a number between 0 and 1 that represents the percentage of empty space in a substance in relation to its overall volume. The porosity of naturally occurring porous materials is normally around 0.6. On the other hand, highly high porosity levels, close to 0.99, are possible with manufactured materials like metal foams. A material's porosity is determined by both its intrinsic properties and the manufacturing process it goes through.

Heat exchangers, heat storage systems, geothermal applications, and drying techniques are just a few of the industries and engineering domains where researching the heat transfer properties of porous media is crucial. One passive way to increase the efficiency of heat transfer in mechanical systems is to incorporate porous materials. This is because fluid flow patterns are altered and the system's total thermal conductivity is improved when porous materials are present [62].

In the mid-19th century, Darcy experimentally demonstrated that the flow rate of water through a column of sand was proportional to the pressure gradient applied between two sections of this column. The first mathematical formulation of fluid movement in a saturated porous medium was proposed by Darcy.

For an isotropic medium, the generalized Darcy equation can be written as:

$$\vec{\nabla}p = -\frac{\mu}{K}\vec{V} + \rho\vec{g} \quad (I.3)$$

where $\vec{\nabla}p$ is the pressure gradient.

μ and ρ are respectively the viscosity and the density of the fluid. K is the permeability that depends only on the geometry of the solid matrix, v is the filtration velocity or the superficial velocity of the fluid, defined as the volumetric flow rate of the fluid per unit area normal to the flow (average velocity over a representative elementary volume).

The filtration velocity v is related to the average interstitial pore velocity $\vec{V}_p$ by the following relation:

$$\vec{V} = \varepsilon \vec{V}_p \quad (I.4)$$

where ε is the porosity defined by the relation:

$$\varepsilon = \frac{V_f}{V_f + V_s} \quad (I.5)$$

V_f is the volume occupied by the fluid and V_s is the volume occupied by the solid. From the beginning of the 20th century, Forchheimer experimentally demonstrated that recirculations appear at the pore scale when the flow rate is high. These recirculations produce an additional pressure drop expressed as a quadratic function of the filtration velocity. According to Almalki et al. [63], the appropriate modification to Darcy's equation is to replace the equation. (1.1) by:

$$\vec{\nabla} p = -\frac{\mu}{K} \vec{V} + \rho \vec{g} - \frac{\rho_f C_F}{\sqrt{K}} |\vec{V}| \vec{V} \quad (I.6)$$

According to Ward [64], C_F could be a universal constant, with a value of approximately 0.55, but later it was found that C_F varies with the nature of the porous medium. Beavers et al. [65] showed that the boundary walls could have a substantial effect on the value of C_F . The numerical calculations by Coulaud et al. [66] showed that C_F is an empirical coefficient that depends on the porosity and microstructure of the porous medium.

The transition from Darcy flow to Darcy-Forchheimer flow occurs when the flow velocity is sufficiently high. Vafai and Tien [67], Lauriat and Prasad [68], and Lage [69] show in their works that the simple Darcy equation was insufficient to describe flow in porous media, in this case they opt for the term Forchheimer to account for inertial effects.

In the context of calculating the viscous force exerted by a fluid on spherical particles composing a porous medium, Brinkman [70] extended Darcy's law by introducing a term equivalent to the viscous diffusion term in Stokes' law.

This term also accounts for the frictional forces that occur at the surfaces delimiting the porous medium (in the presence of this term, a boundary condition of adherence to these surfaces is considered, whereas in the absence of this term, it is considered that there is sliding without friction).

The modified Darcy-Brinkman-Forchheimer (DBF) law is then represented by the following equation:

$$\vec{\nabla} p = -\frac{\mu}{K} \vec{V} + \rho \vec{g} - \frac{\rho_f C_F}{\sqrt{K}} |\vec{V}| \vec{V} + \tilde{\mu} \nabla^2 \vec{V} \quad (I.7)$$

Where μ is an effective viscosity that depends on the fluid viscosity (μ) and the porosity (ε)

Models of Fluid Flow in Porous Media

The particulars of the porous media, the flow regime (such as laminar or turbulent), and the engineering or research project all influence the fluid flow model selection. To effectively depict the flow and heat transfer characteristics in each porous medium, researchers and engineers choose the best model. Convective heat transmission in porous medium is modeled using a variety of fluid flow models. For convective heat transfer in porous media, some of the most significant and often applied fluid flow models are as follows [71].

The study explores the behavior of MWCNT+Fe₃O₄ nanopowder in a permeable cavity, revealing that permeability positively influences convective flow, resulting in faster [72] nanomaterial movement.

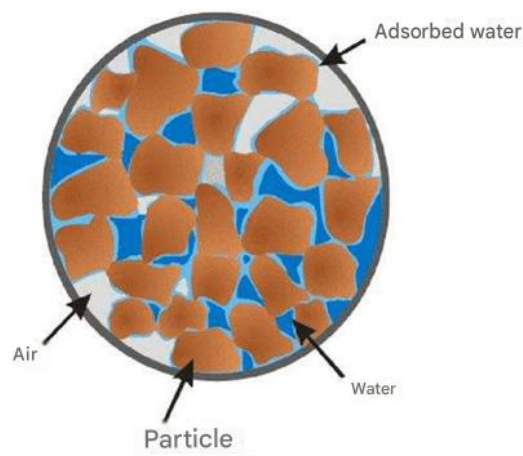


Figure I-3 Representation of a porous medium[73].

CHAPTER II

***MATHEMATICAL
FORMULATION
&
NUMERICAL MODEL***

II.1 Introduction

Given the increasing importance of studying nanofluids for energy conservation, there is a significant demand for nanofluid-based heat transfer systems, which requires the acquisition of experimental data. However, experimental methods are not only costly but also restrict the ability to gather sufficient information. A key limitation lies in the inefficiency of these procedures to accurately determine various influencing factors and the extent of their impact on the system. These challenges in experimental approaches have led to the growing adoption of computational simulations in nanofluid research [74].

In many scientific and technical fields, such as the physical sciences, chemistry, engineering, and environmental sciences, convection is a significant and intricate process. Its significance lies in its ability to help us understand the mechanisms of heat and matter transport in liquids and gases, such as air and water. This phenomenon is observed at several scales, ranging from the micro to the macro, and can appear naturally or artificially.

By examining the principles of convection, it becomes possible to understand how various hydrodynamic and thermal variables interact to produce heat and material fluxes. The derived partial equations, which are based on the conservation of mass, motion quantity, energy, and entropie, are used to represent these convection processes.

The preservation of a fluid's mass is expressed by the continuity equation, but the quantity of motion is conserved by the motion equations. By taking into consideration heat transfers through conduction, convection, and radiation, the energy equation reflects energy conservation. Finally, the increasing entropie of a system as a result of thermal energy loss is described by the entropie generation equation.

It is possible to create more effective and long-lasting systems in many fields, including climate change, energy production, and material processing, by understanding the laws of convection and using mathematical models to predict the flow of mass and heat.

In this chapter, we present the physical model and the simplifying assumptions. We let's formulate the physical problem governing the phenomenon of mixed convection in a cavity wavy cavity with a sliding wall. We express the conservation equations and boundary conditions limits in a dimensionless form. Finally, we introduce the main quantities dynamic and thermal.

II.2 Essential Fondement Mathmetics

In this study, we examined the physical properties of heat transfer and buoyancy-induced flows in cavities when a magnetic field is present. Furthermore, the cavity geometry has been researched in heat transfer because to its essential significance in numerous applications. The behavior of convection heat transfer of nanofluids and hybrid-nanofluids in a number of enclosures with various heat source configurations and geometrical shapes is investigated. The finite element method used to numerically solve the resulting non-linear system of partial differential equations after the governing equations for the current problems and the relevant boundary conditions have been converted into a non-dimensional form. In this section, numerical solutions for non-dimensional parameters such as the Rayleigh number (Ra), Hartmann number (Ha), and Prandtl number (Pr) are obtained. The results are discussed as streamlines and isothermal lines. Finally, the average Nusselt numbers and the entropy generation are computed

Key parameter definitions

Some important parameters and dimensionless variables related to the mathematical modeling of convective heat transfer are as follows:

Force of Buoyancy

The lighter fluid is pushed upward from the heavier fluid by a net force in the gravitational field.

$$F_{buoyancy} = \rho_{fluid} g V_{body} \quad (II.1)$$

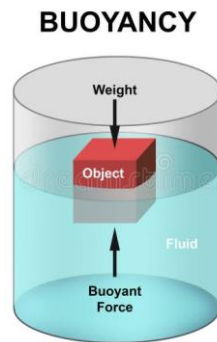


Figure II.1 Buoyancy force.

where ρ_{fluid} is the average density of the fluid, g is gravity's acceleration, and V_{body} is the volume of the body immersed in the fluid. In the absence of other effects, the net vertical force acting on the body is equal to the difference between the body's weight and buoyancy force.

II.2.1 Volumetric Coefficient of Expansion

The density of a fluid varies with its temperature. Understanding how density varies with temperature while pressure remains constant is so essential. These characteristics are connected via the coefficient of thermal (or volumetric) expansion. Denoted by β , it is defined as the fractional change in volume to temperature change at constant pressure and can be expressed as follows:

The upward force that a fluid applies to a body submerged entirely or partially in it is known as the buoyancy force. The weight of the fluid that the body displaces determines the buoyancy force.

$$\beta = -\frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_p \quad (\text{II.2})$$

Use $v = \frac{1}{\rho}$

Experimentation is often conducted on a scale different from industrial reality. The necessity to define correlations that are applicable to devices of different sizes operating in different situations leads to the use of dimensionless parameters that allow for the definition of similarities.

II.2.2 The Reynolds number (Re):

The Reynolds number allows for the quantification of laminar and turbulent flows. High Reynolds number flows are generally turbulent, whereas flows where inertial forces are weak compared to viscous forces are more often laminar.

$$Re = \frac{\rho LV}{\mu} \quad (\text{II.3})$$

Where V: The average speed [m/s].

L: The characteristic length [m].

II.2.3 The Prandtl number (Pr):

Ludwig Prandtl, who made major contributions to boundary layer theory and created the boundary layer idea in 1904, is associated with the Prandtl number, represented by (Pr). For

liquid metals, the Prandtl value is less than 0.01; for heavy oils, it is over 100,000. It is noteworthy that water has a Prandtl number of around 7. Heat and momentum dissipate through fluids at about the same rate when the Prandtl numbers of gases are near to one. Consequently, the thermal boundary layer is significantly thicker for liquid metals and thinner for oils when compared to the velocity boundary layer.

The Prandtl number describes the ratio between the diffusion of momentum and the diffusion of heat in the fluid, it is written as:

$$Pr = \frac{\nu}{\alpha} \quad (\text{II.4})$$

With ν and α being the kinematic viscosity and thermal diffusivity, respectively.

II.2.4 The Nusselt number (Nu):

The relationship between convectional and conductive exchange in a fluid couch is described by Nusselt's name, which is as follows:

$$Pr = \frac{\text{kinematic viscosity}}{\text{Thermal diffusivity}}$$

$$Pr = \frac{\frac{\nu}{k}}{\frac{\rho \nu C_p}{k}} = \frac{\rho \nu C_p}{k} = \frac{\mu C_p}{k} \quad \text{as} \quad \rho \nu = \mu$$

$$Nu = \frac{qL}{K\Delta T} = \frac{hL}{K} \quad (\text{II.5})$$

q and l stand for flux density, characteristic length, and heat flux density, respectively, whereas h is the heat transfer coefficient and k is the thermal conductivity.

II.2.5 The Grashof number (Gr):

Represents the comparison between the thrust force resulting from heating and the viscosity force. It compares the effects of gravity with the effects of thermal dissipation due to viscosity, it is written as:

$$Gr = \frac{\rho^2 g \beta Q V L^4}{\mu^2 K} \quad (\text{II.6})$$

With g ; β ; μ are respectively the gravity, the coefficient of thermal expansion, the density, and the dynamic viscosity, ΔT being the temperature difference between the walls and L a characteristic length.

II.2.6 The Rayleigh number (Ra):

Rayleigh number is the most influential independent parameter. It describes the balance of thermal force to viscous and thermal dissipation and gives an idea about the strength of the thermal driving force in natural convection. The Rayleigh number can also be considered as a ratio of buoyancy forces and the product of thermal and momentum diffusivities. It is affected by the problem's geometrical conditions. When Ra exceeds a critical Rayleigh number, the nature of heat transfer in the medium shifts from conduction to convection. It is the product of Grashof number and Prandtl number. It expresses the ratio between the buoyant forces due to viscous effects and thermal diffusion. It characterizes the strength of natural convection. For a fixed temperature condition (Dirichlet), the Rayleigh number (Ra) is the combination of the Grashof number and the Prandtl number, it is written as:

$$Ra = Pr \cdot Gr = \frac{g\beta}{\nu\alpha} (T_s - T_\infty) x^3 \quad (II.7)$$

For a prescribed heat flux density (Neumann boundary condition), the Rayleigh number Ra_q is generally the combination of the Grashof number and the Prandtl number, it is written as:

$$Ra_q = Pr \cdot Gr \cdot Nu = \frac{g\beta QL^4}{\alpha\nu K} \quad (II.8)$$

In general, the Rayleigh numbers strongly depend on the intended application. For the cooling of electronic components, studies are conducted at low modified $Ra \leq 10^5$ Rayleigh numbers, whereas for large-scale configurations, authors aim to achieve modified Rayleigh numbers that are $Ra \geq 10^7$ often limited by the size of the experimental device.

II.2.7 The Richardson number (Ri):

The Richardson number (Ri) is a dimensionless number used particularly in thermodynamics, developed by Lewis Fry Richardson, an English physicist and mathematician. It is the ratio between the gravitational potential energy of a parcel of fluid and its kinetic energy:

$$Ri = \text{Potential energy} / \text{Kinetic energy}.$$

The Richardson number can be explained in several ways:

$$Ri = \frac{\rho\beta\Delta TL}{\nu^2} \quad \text{Ou} \quad Ri = \frac{Gr}{Re^2} \quad (II.9)$$

II.2.8 The Bejan number (Be):

It is a dimensionless number used in thermodynamics and fluid mechanics. Two versions of this number are used.

This number is named after Adrian Bejan, an American professor of Romanian origin.

$$Be = \frac{S_{gen,\Delta T}}{S_{gen,\Delta T} + S_{gen,\Delta p}} \quad (II.10)$$

With: $S_{gen,\Delta T}$: entropy generated by heat transfer

$S_{gen,\Delta p}$: entropy generated by viscous friction (pressure drop)

II.2.9 Hartmann number

The Hartmann number is the ratio of electromagnetic force to viscous force named after the German engineer who handled important experiments in the 1930s. It is defined as follows:

$$Ha = B_0 L \sqrt{\frac{\sigma}{\mu}} \quad (II.11)$$

II.2.10 Nusselt number

The number of Nusselt It is defined as the ratio of the rate of heat transfer by convection to the rate of heat transfer by conduction along a characteristic length 'L' under a unit temperature gradient. It calculates the difference between convection and conduction in terms of how much better heat is transferred through a fluid layer.

The local Nusselt number at the cavity's heated surface, as defined by the following expression:

$$Nu = \frac{\text{convection heat transfert}}{\text{conduction heat transfert}} = \frac{hL}{K} \quad (II.12)$$

With:

$$Nu_{loc} = \begin{cases} \frac{k_{nf}}{k_f} \frac{\partial T}{\partial x} \\ \frac{k_{nf}}{k_f} \frac{\partial T}{\partial y} \end{cases} \quad (II.13)$$

The expression of the average Nusselt number is the following:

$$Nu = \int_0^L Nu dx \quad (II.14)$$

The Nusselt number, also called the dimensionless convection heat transfer coefficient, was created by Wilhelm Nusselt, who made important discoveries about convective heat transfer in the early 20th century. A higher Nusselt number indicates high convective heat transport and a big temperature gradient at the surface. When a fluid layer has a Nusselt number of $Nu = 1$, it means that conduction is the only way for heat to move across it. Convective heat transfer takes over when the Nusselt number exceeds unity ($Nu > 1$).

II.2.11 Reynolds number

The ratio of inertial forces to viscous forces is given by the dimensionless number (Re). The flow is laminar when the fluid has a high viscosity or a low fluid motion; however, the flow is turbulent when the fluid has a high Reynolds number or a low viscosity.

$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\rho V L}{\mu} \quad (\text{II.15})$$

In a variety of fluid flow scenarios, the Reynolds number helps forecast the flow patterns. Laminar flow is the passage of fluid particles in layers along smooth channels. tends to dominate flows at low Reynolds numbers. While at high Reynolds numbers, flows tend to be turbulent.

II.2.12 Richardson number

Lewis Fry Richardson is the name of the Richardson number (**Ri**). It is. an expression of mixed convection It represents the proportion of the buoyancy term to the flow gradient term as a dimensionless number. Ri reflects the significance of natural convection in relation to forced convection in the thermal convection problems. When the Richardson number is much greater than unity, buoyancy is the dominant force. When $Ri < 0.1$, the natural convection is often insignificant. However, the forced convection is considered as negligible when **Ri** > 10 .

$$Ri = \frac{\text{buoyancy term}}{\text{flow gradient term}} \quad (\text{II.16})$$

$$Ri = \frac{g\beta(T_{hot}-T_{ref})L}{v^2} = \frac{Gr}{Re^2} \quad (\text{II.17})$$

$$Ri = \frac{Gr}{Re^2}$$

II.2.13 The Prandtl (Pr):

The ratio (kinematic viscosity ν and thermal diffusivity α) is known as the Prandtl number, which is used to evaluate the relative significance of thermal and viscous effects. This nomenclature is derived from Ludwig Prandtl, a physicist of German origin.

II.3 Base mathematical elements (governing equations):

The Navier-Stokes equation and the equation of heat are the equations that govern the problem. Showing fluid flow and heat within the rectangular cavity de conservation. The problems of The basic principles of conservation of mass (continuity equations), conservation of momentum (momentum equations), and conservation of energy (energy equations) are used to solve fluid flow and heat transfer. These laws together create a system of coupled, nonlinear partial differential equations. Additionally, a laminar flow of fluid with minimal viscous heating dissipation is considered.

Every physical characteristic of the working fluid is taken to be constant in accordance with the Boussinesq approximation. A constant magnetic field (B_0) used in each of the scenarios under study. Using a homogeneous applied magnetic field, the model used to formulate laminar incompressible stable equations in two or three dimensions. The gravitational force (g) acts downhill and vertically.

II.4 The three-dimensional mixed convection flow governing equations are as follows:

Continuity equation:

The continuity equation for the working fluid that assumed as incompressible retains its original form: $\nabla \cdot \vec{q} = 0$

where x , y , and z components of velocity are represented by u , v , and w in the vectorial expression.

$$\vec{u} = u.\vec{i} + v.\vec{j} + w.\vec{k} \quad (\text{II.18})$$

The paths of individual particles show up as steady streamlines if the flow is steady. The streamlines can be described using the expression of a stream function $\psi(x, y) = \text{constant}$, where each value of the constant designates a different streamline. The velocity vector ($\vec{u}$) is always perpendicular to the streamlines because they represent the pathways that working fluid particles take.

So that, the continuity equation for 3D models can be expressed as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (\text{II.19})$$

$$\nabla \cdot \vec{u} \equiv \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (\text{II.20})$$

Momentum equation:

The momentum equations are defined by the Navier-Stokes equation. The mechanical forces generated by the magnetic field alter the fluid flow. When a uniform magnetic field B_0 is applied horizontally in a three-dimensional laminar flow, the momentum equation can be expressed as follows incompressible steady fluid flow:

In x-direction:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho_{hnf}} \frac{\partial P}{\partial x} + Pr \frac{\rho_f}{\rho_{hnf}} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (\text{II.21})$$

In y-direction:

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{1}{\rho_{hnf}} \frac{\partial P}{\partial y} + Pr \frac{\rho_f}{\rho_{hnf}} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \quad (\text{II.22})$$

In z-direction:

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho_{hnf}} \frac{\partial P}{\partial z} + Pr \frac{\rho_f}{\rho_{hnf}} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (\text{II.23})$$

Energy equation:

The first law of thermodynamics is the source of this equation. This is the outcome of applying this law: heat flow in = heat flow out in steady-state conditions.

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \frac{k}{\rho c_p} \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] \quad (\text{II.24})$$

II.5 Governing equations for porous media

As mentioned before, studies on convective heat transfer do consider the existence of porous medium to be a significant problem. Numerous geometries under analysis were assumed to be completely or partially porous media.

We made the assumption that the issues with the presence of porous media were steady and incompressible in order to make the numerical computations easier. Additionally, an isotropic homogeneous porous matrix with constant porosity (ε) and permeability (K) is traversed by the fluid.

The following list of Darcy-Brinkman-Forchheimer generalized formulae provides the governing equations for the porous medium area in the case of mixet convection:

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0. \quad (\text{II.25})$$

$$\frac{1}{\varepsilon^2} \left(v \frac{\partial u}{\partial y} + u \frac{\partial v}{\partial x} \right) = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2} \right). \quad (\text{II.26})$$

$$\frac{1}{\varepsilon^2} \left(v \frac{\partial v}{\partial y} + u \frac{\partial v}{\partial x} \right) = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} \right) - \frac{F_c}{\sqrt{K}} v |u| + (\rho\beta)_{nf} g(T - T_c) - \frac{\sigma_f B_0^2}{\rho_{nf}} v \quad (\text{II.27})$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \frac{k_{nf}}{(\rho C_p)_{nf}} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right). \quad (\text{II.28})$$

Where $\alpha = \frac{k_{nf}}{(\rho C_p)_{nf}}$ (The thermal diffusivity)

II.6 Conditions at Boundaries

The proper version of the heat equation needs to be solved in order to ascertain the temperature distribution in a medium. But such a solution depends on the physical circumstances at the limits of the medium.

Any of the examined examples' numerical models must be solved with suitable boundary conditions.

The computational domain's boundary conditions are defined for each scenario. Generally speaking, the geometry under study includes walls that are retained adiabatically, walls that are heated (TH), and others that are cooled (Tc).

The boundary conditions for the natural heat convection problem with uniform temperatures are as follows:

For hot walls: $u = v = 0$ and $T = T_H$.

For cold walls: $u = v = 0$ and $T = T_C$.

For adiabatic walls: $u = v = 0, \frac{\partial T}{\partial n} = 0$

II.7 Formulations without dimensions

Dimensionless versions of The boundary conditions and governing equations are created. format using additional non-dimensional variables or numbers in addition to the dimensionless parameters already mentioned.

$$X, Y, Z = \frac{x, y, z}{L}, U, V, W = \frac{(u, v, w)L}{\alpha_{nf}}, \theta = \frac{T - T_c}{T_h - T_c}, P = \frac{(p)}{\rho_{nf} \frac{\alpha_f^2}{L^2}}, Ra = \frac{g \beta_f (T_h - T_c) L^3}{\alpha_f \nu_f}, Da = \frac{k}{L^2}, Pr = \frac{\nu_f}{\alpha_f}, Ha = BL \sqrt{\frac{\sigma_{hnf}}{\mu_{hnf}}}, Ri = \frac{g \beta_T (T_H - T_L) L}{u_0^2} = \frac{Gr}{Re^2}, Br = \frac{\beta_c (C_H - C_L)}{\beta_T (T_H - T_L)}, \alpha = \frac{k}{\rho C_p}, \nu = \frac{\mu}{\rho}, Gr = \frac{g \beta \Delta T L^3}{\nu^2}$$

II.8 Thermophysical properties of the working fluids In the mixet convection

μ_{nf} , k_{nf}^* , $(\rho C_p)_{nf}$ are defined as follows

$$\rho_{nf} = \phi \rho_s + (1 - \phi) \rho_f, \mu_{nf} = (1 - \phi)^{-2.5} \mu_f, (\rho C_p)_{nf} = \phi (\rho C_p)_s + (1 - \phi) (\rho C_p)_f \quad (II.29)$$

When considering the influence of nanoparticle shape, the thermal conductivity of the nanofluid, k_{nf} , is described as follows:

$$\frac{k_{nf}}{k_f} = \frac{(m-1)\phi(k_s - k_f) + k_s - (1-m)k_f}{k_s - (1-m)k_f - \phi(k_s - k_f)}. \quad (II.30)$$

In this case, m represents the form factor. The form factor for different particle forms is shown in Table 1. The thermophysical properties of the nanofluid are also listed in Table 2.

Properties

Hybrid nanofluid

Density

$$\rho_{np} = \frac{\varphi_{Fe_3O_4} \rho_{Fe_3O_4} + \varphi_{MWCNT} \rho_{MWCNT}}{\varphi}$$

$$\textbf{Heat capacity} \quad (C_p)_{np} = \frac{\varphi_{Fe_3O_4}(C_p)_{Fe_3O_4} + \varphi_{MWCNT}(C_p)_{MWCNT}}{\varphi}$$

$$\textbf{Coefficient of thermal expansion} \quad \beta_{np} = \frac{\varphi_{Fe_3O_4}\beta_{Fe_3O_4} + \varphi_{MWCNT}\beta_{MWCNT}}{\varphi}$$

$$\textbf{Electrical conductivity} \quad \sigma_{np} = \frac{\varphi_{Fe_3O_4}\sigma_{Fe_3O_4} + \varphi_{MWCNT}\sigma_{MWCNT}}{\varphi}$$

$$\textbf{Thermal conductivity} \quad k_{np} = \frac{\varphi_{Fe_3O_4}k_{Fe_3O_4} + \varphi_{MWCNT}k_{MWCNT}}{\varphi}$$

$$\textbf{Viscosity} \quad \mu_{np} = \frac{\varphi_{Fe_3O_4}\mu_{Fe_3O_4} + \varphi_{MWCNT}\mu_{MWCNT}}{\varphi}$$

$$\textbf{Volume fraction} \quad \varphi = \varphi_{MWCNT} + \varphi_{Fe_3O_4}$$

The mixture-nanofluid's density is represented by ρ_{hnf} , while its viscosity and thermal conductivity are represented by μ_{hnf} and k_{hnf} , respectively. The hybrid-nanofluids' thermal expansion coefficient is $c_{p,hnf}$, and the thermal expansion factor of the nanofluid

II.9 Computational procedures

II.9.1. Computational Fluid Dynamics

The method of computational fluid dynamics, or CFD, is becoming more and more popular for scientific and technical applications that don't need testing because it solves fluid flow problems quickly and affordably. It was challenging to perform suitable experimental study on temperature transmission and fluid movement due to the limited flexibility of fluid dynamics. Analytical solutions are inadequate due to the existence of numerous variables, complicated

geometric entities, diverse boundary forms, and conditions. The most effective method for resolving realistic partial differential equations is through numerical approaches [75].

Engineering problems involving intricate geometry or significant elements that cannot be resolved using traditional techniques are addressed by CFD, a computer-based simulation tool. Algebraic equations are used in CFD to approximate a natural living system. The results of computations provide insight into the performance of a system.

To produce realistic physical solutions with acceptable accuracy, researchers use CFD simulation tools using finite grids. High dependability and economic challenges are satisfied by the exact and accurate projections of complex geometry. In CFD, these tendencies are frequently observed. Additionally, CFD has been used extensively for numerical fluid dynamics calculations. Large-scale industrial issues like turbulent flow are currently successfully handled by it [75]

The initial step in solving Numerical discretization is a mathematical representation of a physical phenomenon. This suggests that every element of a differential equation is converted to a numerical analog, which is subsequently handled by computer programs that use algorithms. Several discretization approaches are available for high-performance numerical computing in CFD, including the boundary element method (BEM), boundary volume method (BVM) [75], finite element method (FEM), finite difference method (FDM), and finite volume method (FVM).

II.9.3. Finite Element Method

Finite element modeling (FEM) is capable of handling intricate geometric bodies and boundaries when other approaches are ineffective. One advantage of this strategy is that it takes into account the body's disintegration.

In the current numerical computation, Finite element approach with Galerkin weighted residuals was employed. It is the best method for calculating approximate solutions to a system of PDEs using numerical computing. FEM is gaining popularity as a way to solve problems in fluid dynamics. This method is global enough to manage nonlinear and time-dependent flow issues in irregular domains. An essential part of FEM is the mathematical model, which is created by combining the local approximations of the processes being studied. One major benefit of FEM is its capacity to handle any complex shape. Additionally, each component

may be readily disconnected, and the grid is easily customizable. FEM is used to obtain the equations for each component separately from the others. Only after the equations have been created and integrated The relationships between the elements are included into a global matrix. investigated.

To obtain these ideal features, FEM is used in the majority of computer procedures. There are no limitations on how elements can be linked in FEM as long as their sides are correctly aligned and share nodes with their neighboring components. Its adaptability allows us to express extraordinarily intricate geometries.

A persistent A physical problem is transformed into a discretized restricted element problem with unknown nodal values. To solve a linear problem, one must solve a system of linear algebraic equations. Finite element values can be recovered using nodal values. Important steps in the FEM of a typical problem are as follows:

Mesh construction involves discretizing a domain into a finite number of components.

- Analyze a differential equation with weak formulation or weighted integral.
- The problem's FEM is created with the weak or weighted integral form.
- Developed a form function for elements.
- To create a global system of algebraic equations, combine the finite elements.
- Setting boundaries and constraints.
- An equation's solution and interest amount after computation.

II.9.4. Mesh Generation

A domain is divided into finite components using grid generation or FEM mesh. To separate a local approximation obtained from a wide area, mesh cells are utilized. The goal is to produce a mesh with high-quality (well-shaped) cells that precisely represent the geometry of the input domain and aren't so dense as to prevent further computations. The mesh should have tiny, delicate components in key places to allow for precise calculations. Meshes are employed in physical simulation, including FEM and CFD, as well as computer screen rendering.

For finite element analysis, three-dimensional meshes made of tetrahedra, pyramids, prisms, or hexahedra are necessary. They can be any kind of polyhedral and are employed in the finite volume approach. Multi-block structured meshes, sometimes referred to as piecewise structured arrays of hexahedra, are used in finite difference methods. The Delaunay triangular

method will be used to discretize the computational domain into unstructured triangles using the current numerical techniques. A geometric structure that gained popularity when mesh generation research was just getting started is the Delaunay triangulation. Maximizing the shortest angle between a vertex set's potential triangulations is the aim of a two-dimensional Delaunay triangulation.

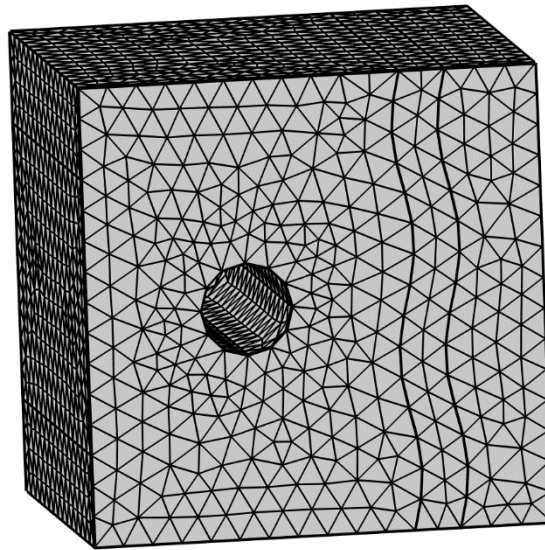


Figure II.2 Example of mesh discretization

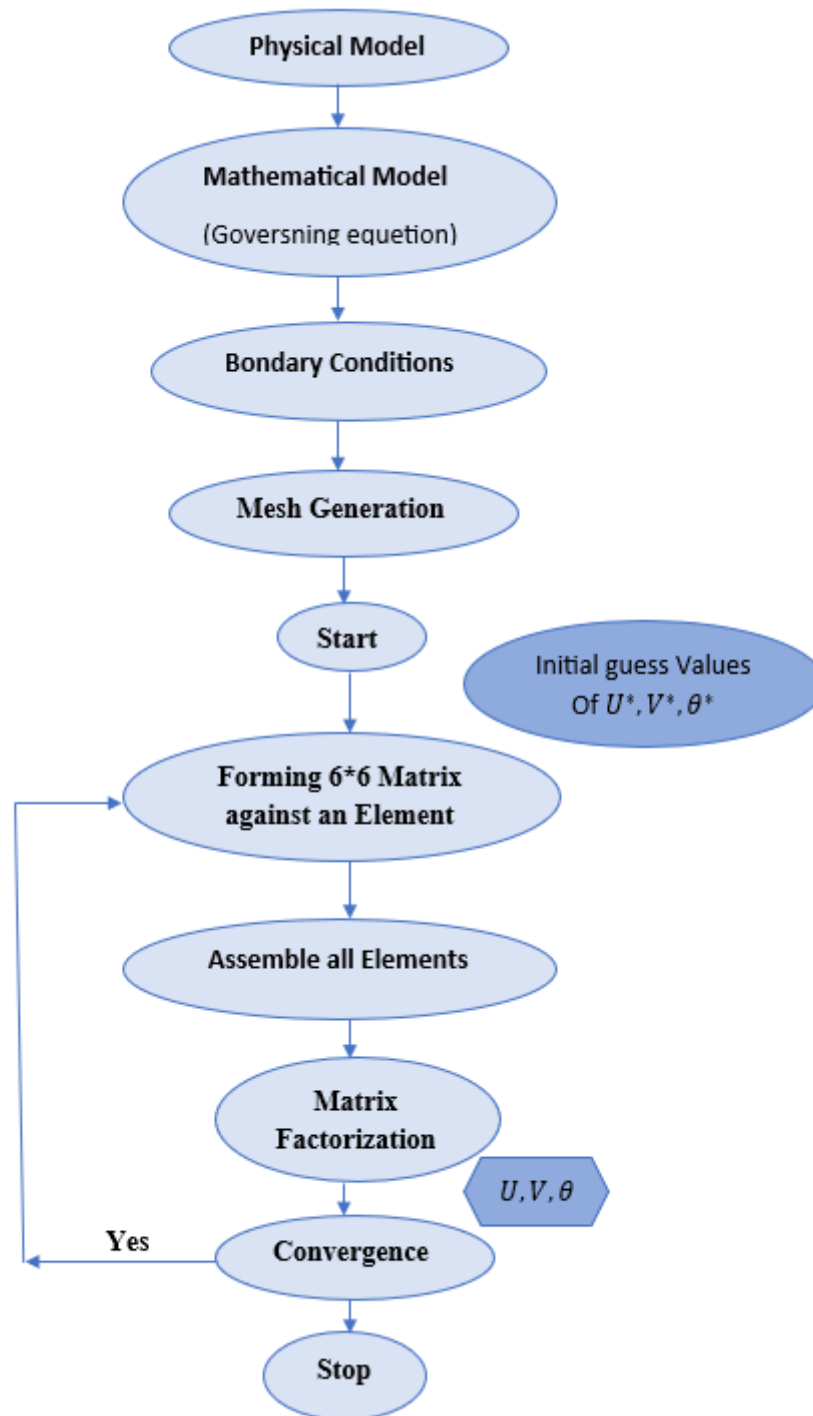


Figure II.3 Flow diagram illustrating FEM computational process [76] β_{hnf} .

CHAPTER III
Results
&
Discussions

"Comparative Analysis of Heat Transfer Mechanisms in Porous and Nanofluid-Filled Cavities: Conjugate Local Thermal Equilibrium and Radiation Effects"

III.1 Introduction

Enhancing heat transfer in fluids is a critical pursuit in the realm of thermal engineering, driven by the need to optimize the efficiency of various industrial and technological processes. Achieving superior heat transfer capabilities involves a multifaceted approach, encompassing alterations in fluid properties, adjustments in flow dynamics, modifications to container geometries, and the application of external forces. Among the pivotal factors that significantly impact convective Thermal conductivity is the measure of heat transport fluid employed.

Nanofluids offer advantages such as increased stability, direct impact on viscosity based on nanoparticle characteristics, and improved wetting, spreading, and dispersion capabilities on solid surfaces, even at low concentrations.

In this chapter, we examine a Comparative Analysis of Heat Transfer Mechanisms in Porous and Nanofluid-Filled Cavities and Conjugate Local Thermal Equilibrium and Radiation Effects. The results will be presented through streamlines, isotherms contours and the output of Nusselt number and including sensitivities related to geometrical and porous medium parameters. Two cases will be discussed in this section. First one represents conjugate local thermal equilibrium heat transmission in a cavity filled by a spinning cylinder made of porous material in three dimensions. and the second is how heat radiation in a particular cavity is affected by the natural convection of MHD nanofluid.

Case 01: Investigating the Synergistic Effects of Magnetic Fields, Rotation, and Permeability on Heat Transport in Porous Systems [77]

Abstract:

This study's objective was to use numerical methods to examine In a three-dimensional enclosure, mixed convective heat transfer with an undulating porous cavity and a rotating circular cylinder. Numerical simulations were performed using the FEM for a variety of parameters, such as the Hartmann number, the Darcy number (which ranges from 10^{-5} to 10^{-2}), and Ha (varying from 0 to 100), angular speed of rotation (between -4000 and 4000), and the

number of undulations in the porous cavity. The results, depicted through stream function, isotherms, and isentropic contours, reveal the influence of these parameters on motion, heat transfer, and entropy generation. The findings suggest that, to enhance heat transfer rates in a three-dimensional undulating porous cavity with a rotating cylinder and subjected to a magnetic field, specific conditions are recommended: a Darcy number greater than or equal to 10^{-2} , a Hartmann number of 0, undulation of the porous cavity ($N=3$), and a rotation speed exceeding 2000 in the flow direction.

This was done using a single-phase nanoliquid model and a Forchheimer-Darcy model for porous cavity enclosures, this study aims to address precisely convective heat transfer involving nanoparticles. The focus is on examining the characteristics of mixed convection heat transfer in a corrugated enclosure housing a rotating cylinder. To fill the cavity, a porous layer was incorporated and the flow inside was rigorously simulated using the Galerkin finite element scheme. Several dimensionless parameters were taken into account, including Hartman number ($0 \leq Ha \leq 100$), Darcy number ($10^{-6} \leq Da \leq 10^{-2}$), cylinder angular velocity (-4000 to 4000) and nanoparticle concentration ($0 \leq \phi \leq 0.08$). This comprehensive analysis strives to shed light on the complex interaction of variables influencing fluid dynamics and heat transfer in this unique enclosure.

2. Mathematical model

This study presents a laminar, steady, three-dimensional convective flow. Figure III.1 displays the computational domain, which is made up of porous wavy medium at $(x=0.7, y=0, z=0; x=0.8, y=0, z=0)$ sidewalls are heat on the left and cold on right go through the points $((0;0.2;0);(0;0.8;0);(0;0.2;1);(0;0.8;1))$ and $((1;0.2;0);(1;0.8;0);(1;0.2;1);(1;0.8;1))$ respectively and a cylindric in the cube and the related boundary conditions. Every insulated wall is visible hatched,. In order to encounter the influence of enclosure geometry, various numbers of wavy wall undulations are taken into consideration. The influence is taken into account by applying a constant magnetic field along the positive x-axis. Fe_3O_4 and MWCNT nanoparticles, as well as water as a base liquid, are components of the worked suspensions. Both the base fluid and the nanoparticles' thermophysical properties are shown in Table III.1 .

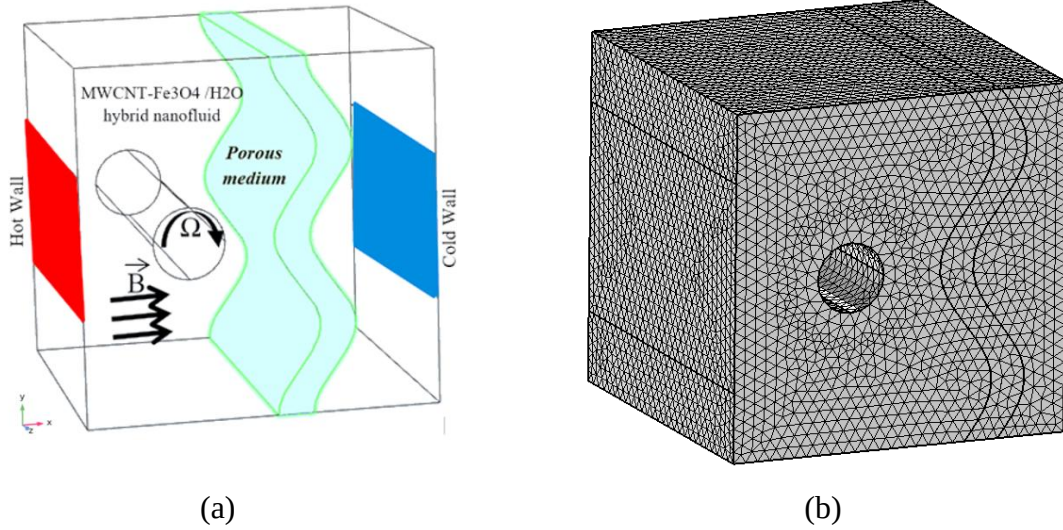


Figure III.1 The computational domain with the boundary conditions (a) and Grid mesh (b).

Table III.2 Properties of a simple liquid's thermophysics and MWCNT-Fe₃O₄ /H₂O nanoparticles (50%/50%)[78].

Material	$C_p(J/kg \cdot K)$	$k(W/m \cdot k)$	$\rho(kg/m^3)$	$\beta \times 10^{-5}(K^{-1})$	$\sigma(s/m)$
Water	4179	0.613	997.1	21	5.5×10^{-6}
Fe ₃ O ₄	670	6	5810	1.3	2.5×10^{-4}
MWCNT	711	3000	2100	4.2	1.9×10^{-4}

The following are the dimensional formulas for the conservation equations in a three-dimensional Cartesian coordinate system for mass, momentum, and energy :

$$\frac{\partial U}{\partial X} + \frac{\partial U}{\partial Y} + \frac{\partial U}{\partial Z} = 0 \quad (III.1)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} + W \frac{\partial U}{\partial Z} = -\frac{1}{\rho_{hnf}} \frac{\partial P}{\partial X} + Pr \frac{\rho_f}{\rho_{hnf}} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial Z^2} \right) \quad (III.2)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} + W \frac{\partial V}{\partial Z} = -\frac{1}{\rho_{hnf}} \frac{\partial P}{\partial Y} + Pr \frac{\rho_f}{\rho_{hnf}} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + \frac{\partial^2 V}{\partial Z^2} \right) \quad (III.3)$$

$$U \frac{\partial W}{\partial X} + V \frac{\partial W}{\partial Y} + W \frac{\partial W}{\partial Z} = -\frac{1}{\rho_{hnf}} \frac{\partial P}{\partial Z} + Pr \frac{\rho_f}{\rho_{hnf}} \left(\frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2} + \frac{\partial^2 W}{\partial Z^2} \right) + RaPr \frac{\rho_f}{\rho_{hnf}} \left[1 - \phi + \phi \frac{(\rho\beta)_{hnf}}{(\rho\beta)_f} \right] \theta \quad (III.4)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} + W \frac{\partial \theta}{\partial Z} = \frac{\alpha_{hnf}}{\alpha_f} \left[\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} + \frac{\partial^2 \theta}{\partial Z^2} \right] \quad (III.5)$$

$$\frac{\partial U}{\partial X} + \frac{\partial U}{\partial Y} + \frac{\partial U}{\partial Z} = 0 \quad (III.6)$$

$$\begin{aligned} \frac{1}{\varepsilon^2} \frac{\rho_{hnf}}{\rho_f} \left[U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} + W \frac{\partial U}{\partial Z} \right] \\ = -\frac{1}{\rho_{hnf}} \frac{\partial P}{\partial X} + \frac{1}{\varepsilon} \frac{\nu_{hnf}}{\nu_f} Pr \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial Z^2} \right) - \frac{\nu_{hnf}}{\nu_f} \frac{Pr}{Da} U \\ - \frac{\rho_{nf}}{\rho_f} \frac{F_c}{\sqrt{Da}} \sqrt{U^2 + V^2 + W^2} U \end{aligned} \quad (III.7)$$

$$\begin{aligned} \frac{1}{\varepsilon^2} \frac{\rho_{hnf}}{\rho_f} \left[U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} + W \frac{\partial V}{\partial Z} \right] \\ = -\frac{1}{\rho_{hnf}} \frac{\partial P}{\partial Y} + \frac{1}{\varepsilon} \frac{\nu_{hnf}}{\nu_f} Pr \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + \frac{\partial^2 V}{\partial Z^2} \right) - \frac{\nu_{hnf}}{\nu_f} \frac{Pr}{Da} V \\ - \frac{\rho_{hnf}}{\rho_f} \frac{F_c}{\sqrt{Da}} \sqrt{U^2 + V^2 + W^2} V \end{aligned} \quad (III.8)$$

$$\begin{aligned} \frac{1}{\varepsilon^2} \frac{\rho_{hnf}}{\rho_f} \left[U \frac{\partial W}{\partial X} + V \frac{\partial W}{\partial Y} + W \frac{\partial W}{\partial Z} \right] \\ = -\frac{\partial P}{\partial Z} + \frac{1}{\varepsilon} \frac{\nu_{hnf}}{\nu_f} Pr \left(\frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2} + \frac{\partial^2 W}{\partial Z^2} \right) - \frac{\nu_{hnf}}{\nu_f} \frac{Pr}{Da} W \\ - \frac{\rho_{hnf}}{\rho_f} \frac{F_c}{\sqrt{Da}} \sqrt{U^2 + V^2 + W^2} W + \frac{\beta_{hnf}}{\beta_f} Pr \cdot Ra\theta - \frac{\sigma_f}{\sigma_{hnf}} \frac{Pr}{\varepsilon} Ha^2 W \end{aligned} \quad (III.9)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} + W \frac{\partial \theta}{\partial Z} = \frac{\alpha_{hnf}}{\alpha_f} \left[\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} + \frac{\partial^2 \theta}{\partial Z^2} \right] \quad (III.10)$$

The thermophysical characteristics of the mixture are calculated using the relationships described in Table III.3.

Table III.4 Correlations between classical and hybrid nanofluid properties [79].

Properties

Hybrid nanofluid

Density

$$\rho_{np} = \frac{\varphi_{Fe_3O_4} \rho_{Fe_3O_4} + \varphi_{MWCNT} \rho_{MWCNT}}{\varphi}$$

Heat capacity	$(C_p)_{np} = \frac{\varphi_{Fe_3O_4}(C_p)_{Fe_3O_4} + \varphi_{MWCNT}(C_p)_{MWCNT}}{\varphi}$
Coefficient of thermal expansion	$\beta_{np} = \frac{\varphi_{Fe_3O_4}\beta_{Fe_3O_4} + \varphi_{MWCNT}\beta_{MWCNT}}{\varphi}$
Electrical conductivity	$\sigma_{np} = \frac{\varphi_{Fe_3O_4}\sigma_{Fe_3O_4} + \varphi_{MWCNT}\sigma_{MWCNT}}{\varphi}$
Thermal conductivity	$k_{np} = \frac{\varphi_{Fe_3O_4}k_{Fe_3O_4} + \varphi_{MWCNT}k_{MWCNT}}{\varphi}$
Viscosity	$\mu_{np} = \frac{\varphi_{Fe_3O_4}\mu_{Fe_3O_4} + \varphi_{MWCNT}\mu_{MWCNT}}{\varphi}$
Volume fraction	$\varphi = \varphi_{MWCNT} + \varphi_{Fe_3O_4}$

The following are the non-dimensional parameters:

$$X, Y, Z = \frac{x, y, z}{L}, U, V, W = \frac{(u, v, w)L}{\alpha_{nf}}, \theta = \frac{T - T_c}{T_h - T_c}, P = \frac{(p)}{\rho_{nf} \frac{\alpha_f^2}{L^2}}, Ra = \frac{g\beta_f(T_h - T_c)L^3}{\alpha_f \nu_f}, Da = \frac{k}{L^2}, Pr = \frac{\nu_f}{\alpha_f}, Ha = LB \sqrt{\frac{\sigma_{hnf}}{\mu_{hnf}}} \quad (III.11)$$

The following are the governing equations' boundary conditions:

$$U = V = W = 0, \frac{\partial \theta}{\partial x} = 0 \quad (III.12)$$

Both sidewalls:

$$U = V = W = 0, \theta = 0 \quad (III.13)$$

For the front and back walls:

$$U = V = W = 0, \frac{\partial \theta}{\partial n} = 0 \quad (III.14)$$

For the bottom walls:

$$U = V = W = 0, \theta = 1 \quad (III.15)$$

And for the cylinders:

$$U = -\Omega(Y - Y_0), V = \Omega(X - X_0), \frac{\partial \theta}{\partial Y} = 0 \quad (\text{III.16})$$

The following correlations were used to assess the hybrid nanoliquid's thermos-physical properties:

It is anticipated that the amount of entropy in the nanoliquid's mobility will change. The total entropy can be explained as follows due to specific variations in thermal transport:

$$S_{tot} = S_{ht} + S_{ff} + S_{mf} \quad (\text{III.17})$$

$$S_{ht} = \frac{k_{hnf}}{T_0^2} \left[\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 + \left(\frac{\partial T}{\partial z} \right)^2 \right] \quad (\text{III.18})$$

$$S_{FF} = \frac{\mu_{hnf}}{T_0} \left\{ 2 \left(\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right) + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 \right\} + \frac{\mu_{hnf}}{T_0 K} (u^2 + v^2 + w^2) \quad (\text{III.19})$$

$$S_{mf} = \frac{\sigma_{hnf}}{T_0} (v \times B_0)^2 \quad (\text{III.20})$$

Entropy generation S_{TOT} can be expressed in the following non-dimensionless form:

$$S_{TOT} = S_{HT} + S_{FF} + S_{MF} \quad (\text{III.21})$$

$$S_{HT} = \frac{k_{hnf}}{T_0^2} \left[\left(\frac{\partial \theta}{\partial X} \right)^2 + \left(\frac{\partial \theta}{\partial Y} \right)^2 + \left(\frac{\partial \theta}{\partial Z} \right)^2 \right] \quad (\text{III.22})$$

$$S_{FF} = \frac{\mu_{hnf}}{\mu_f} N_\mu \left\{ 2 \left(\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right) + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 \right\} + \frac{\mu_{hnf}}{\mu_f} N_\mu \left(\frac{u^2 + v^2 + w^2}{Da} \right) \quad (\text{III.23})$$

$$S_{MF} = N_\mu \frac{\sigma_{hnf}}{\sigma_f} (Ha)^2 (U^2 + V^2) \quad (\text{III.24})$$

$$\text{Where } N_\mu = \frac{\varepsilon \mu_{hnf} T_0}{k_{hnf}} \left(\frac{\alpha_{nf}}{L \Delta T} \right)^2$$

The Nu_{loc} and Nu_{avg} are estimated in the heated wall as:

$$Nu_{loc} = -\frac{k_{eff}}{k_{fl}} \left(\frac{\partial \theta_{po}}{\partial n} \right)_{wall}, Nu_{avg} = \frac{1}{S^2} \int_0^S \int_0^S Nu_{loc} dy dz \quad (\text{III.25})$$

3. Validation and mesh evaluation

We develop incorporate the required boundary conditions into the governing equations. by using the Finite Element Method (FEM) to handle crucial parameters. In this transformation, a system of integral equations is created from nonlinear differential equations. When the field solution is decertified, non-uniform triangular Using grids. To guarantee the grid's independence and the current method's resilience, a mesh evaluation process was conducted. Non-uniform triangles are employed in the field solution's decertification. To ensure that the current approach is grid independent, a mesh evaluation procedure was carried out. In Table 1, seven different grids were examined. Because of the noted differences in the findings, the sixth grid, which has 230486 elements, was chosen as the final mesh. The number of elements needed to guarantee that altering the mesh size is feasible is determined by mesh convergence won't substantially affect the results of an investigation. The response of the system with relation to deformation and stress progressively converges toward a repeatable and stable solution as the element sizes decrease.

Depending on the particular situation, the precise number of elements needed for this convergence is determined by repeatedly improving the mesh until additional improvement no longer substantially alters the outcomes.

As shown in Table 2, our model was validated using Ghasemi et al.'s previous research as a primary criterion for arriving at conclusions.

This method looks for a compromise between the required level of precision for the given problem and the computational resources. The solution is said to be convergent when the relative error for each variable meets the provided convergence conditions, which are as follows: $\left| \frac{\Gamma^{i+1} - \Gamma^i}{\Gamma^i} \right| \leq \eta$, where 'i' is the number of iterations and η is the convergence threshold.

A convergence threshold of $\eta = 10^{-6}$ was determined in this study.

Table III.1 Variation of Nu_{avg} and S_{avg} with mesh elements number (EN) at $Ra = 10^6$, $Da = 0.01$ and $\varepsilon = 0.4$.

EN	10887	13865	29257	53447	95536	230486	298512
Nu_{avg}	1.9612	1.9837	2.0247	2.0550	2.0701	2.1466	2.1464
S_{avg}	12.473	12.863	13.690	14.384	14.769	17.219	17.207

Table III.2 Comparisons between the current model and earlier research

Ha		0	15	30	45	60
Nu_{avg}	Ghasemi[80]	9.672	9.148	7.978	6.856	5.564
	Our Work	9.752	9.251	7.856	6.847	5.839

4. Results and discussion

In a 3D porous cavity filled with Fe_3O_4 /MWCNT-water nanofluid, we investigated the effects of several important parameters on the Nusselt number (Nu), the generation of entropy and rationalization/isothermal contours, and the Hartmann number (Ha), the undulation (N) of the porous medium, the angular velocity (Ω), and the Darcy number (Da). A revolving cylinder inside the porous cavity is exposed to a magnetic field.

The current line intensity dropped as the Hartmann number (Ha) increased, as seen in Figure III.3 . The impact of the magnetic field on the motion of the nanoparticles is responsible for this decrease in intensity. Furthermore, as Ha increased, the effect of Ha on the isothermal contours showed higher temperatures surrounding the cylinders. A closer look at the total entropy contours revealed that, as Ha increases, entropy generation decreases across the domain, mostly as a result of the streamlines' declining velocities.

Figure III.4 The ripple velocity of the porous medium is investigated in four cases, with ripple values of 0, 3, 7, and 10. In all cases, the isothermal contours and the streamlines intensity remains almost constant, showing no significant difference. However, the total entropy in the 3D domain are notably influenced by the wave velocity. In particular, in cases 3 and 4, it is obvious that the maximum temperature

Figure III.5 presents the influence of angular velocity (Ω) on the simulation results. As the angular velocity increases from 0 to 4000, the streamlines intensify, with The greatest velocity magnitude fluctuated between 12 and 1000. Further, the angular velocity influences the temperature distribution, with lower temperatures above the cylinders being the result of higher velocities. This effect is due to flow blockage, which hinders the transfer of hot flows to the upper regions.

Rotation is distinguished between clockwise and counterclockwise in a noticeable way. When the cylinders rotate counterclockwise, the fluid flows naturally in that direction, increasing the speed at the upper regions. This encourages hot streams to travel upward, which improves the

temperature distribution. Contrarily, counterclockwise rotation generates a flow in the opposite direction (buoyancy effect), which has a different impact on the temperature distribution.

Additionally, as angular velocities increase, entropy generation which is directly related to streamline intensity—increases. The importance of taking into account the angular velocity in assessing the thermal and hydraulic performance of the system is shown by the fact that the greater angular velocity has a more noticeable impact on entropy formation.

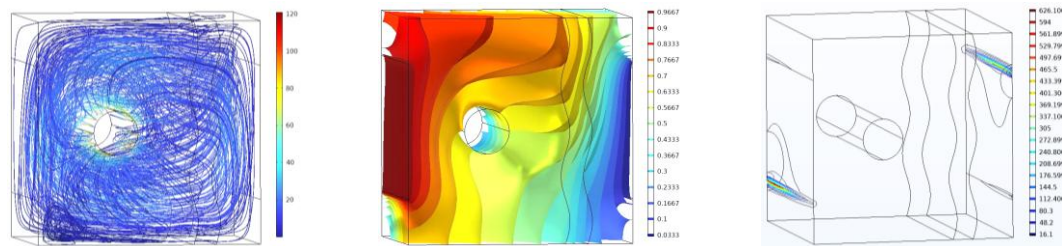
Lastly, Figure III.6 shows how the parameters that were discussed affected the average Nusselt numbers. We find that higher Ω and Da produce higher Nu , while higher Hartmann number (Ha) produces lower Nu values.

Nu is improved by 150 % when Ω is raised from 0 to 2000, even at the lowest Rayleigh number (Ra). However, at high Ra , the difference in Nu for different Da values is more noticeable. More specifically, Nu is increased by 180.72.% at the maximum Ra when Da is increased from 10^{-6} to 10^{-2} .

Regarding the effect of Ha , increasing Ha from 0 to 100 causes a reduction of 39.37% in Nu . It is worth noting that in configuration w and the porosity, has no influence on Nu numbers, whereas it does affect the differences in the values of Rayleigh numbers.

Figure III.7 illustrates how The thermal field, entropy generation, and flow pattern are all impacted by the Darcy number. A higher Darcy number (Da) indicates that the medium allows fluid flow more easily, leading to improved flow efficiency. Moreover, an increased Da enhances fluidity, enabling better heat dispersion and more effective heat spreading. However, this also causes the entropy distribution to widen. across the cavity, resulting in higher levels of entropy, as represented by the red lines. The Darcy number has a positive effect on heat transfer performance by accelerating fluid motion and increasing fluid friction in the system. Consequently, raising Da leads to a general rise in entropy generation arising from fluid friction and heat transfer.

$Ha = 0$



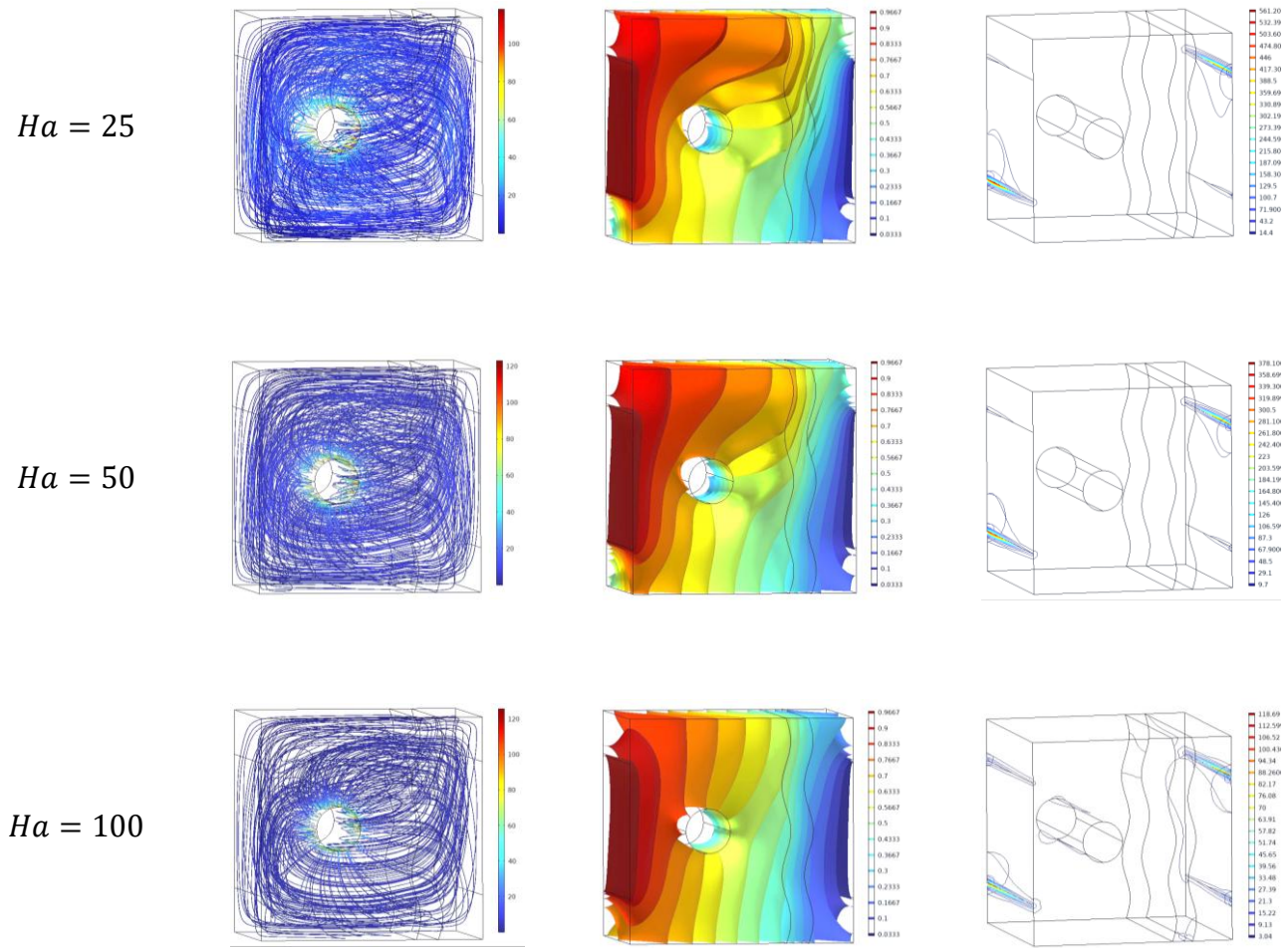
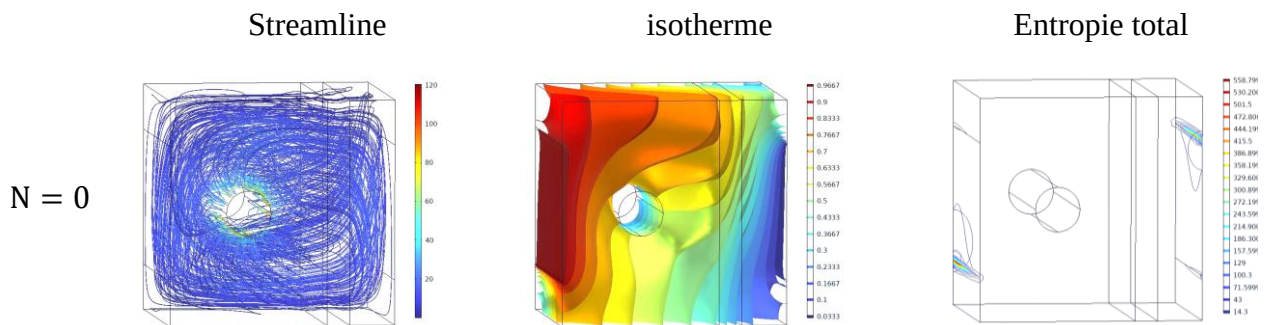


Figure III.4 Effect of Ha on isotherms, entropy contours, and streamlines with $\varphi = 0.02$; $\varepsilon = 0.1$; $\Omega = 100$; $Da = 0.01$, $Ha = 25$



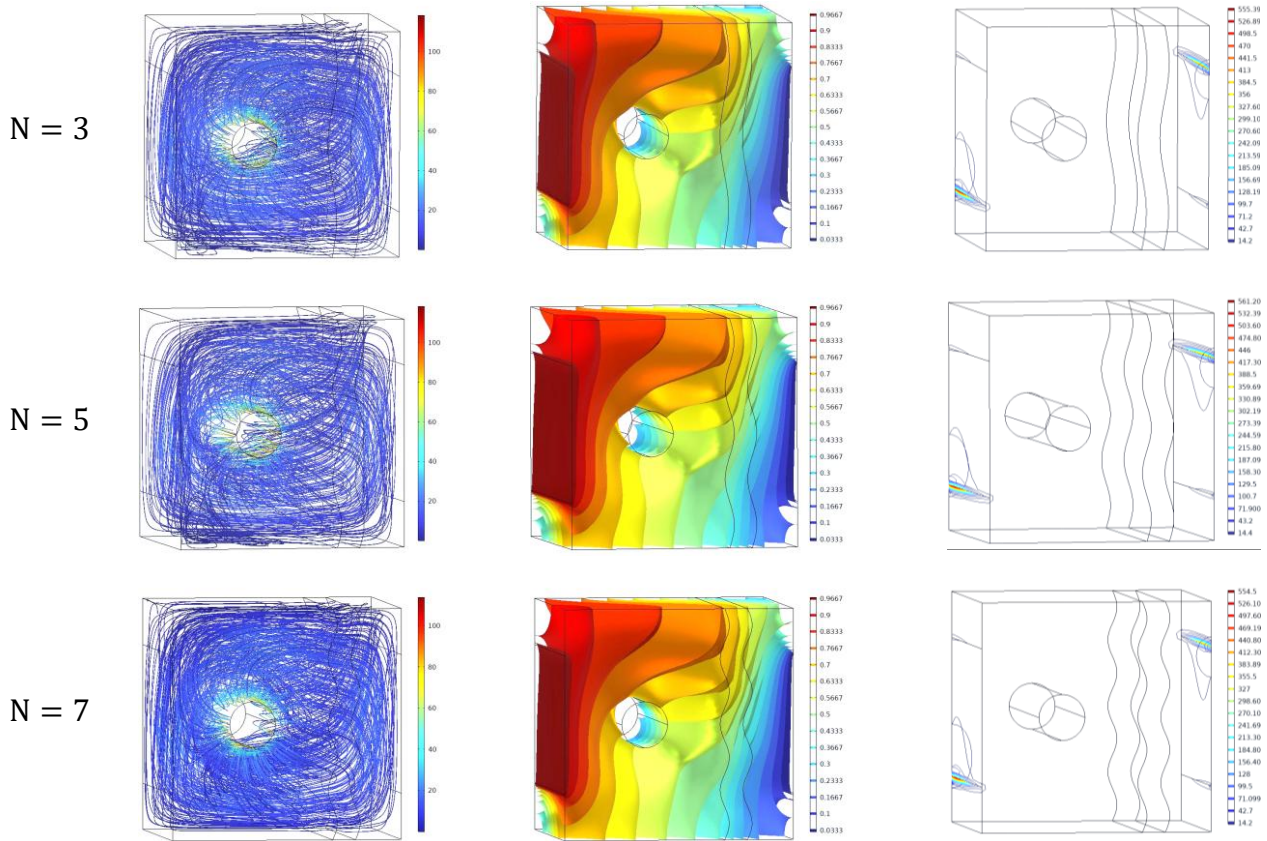
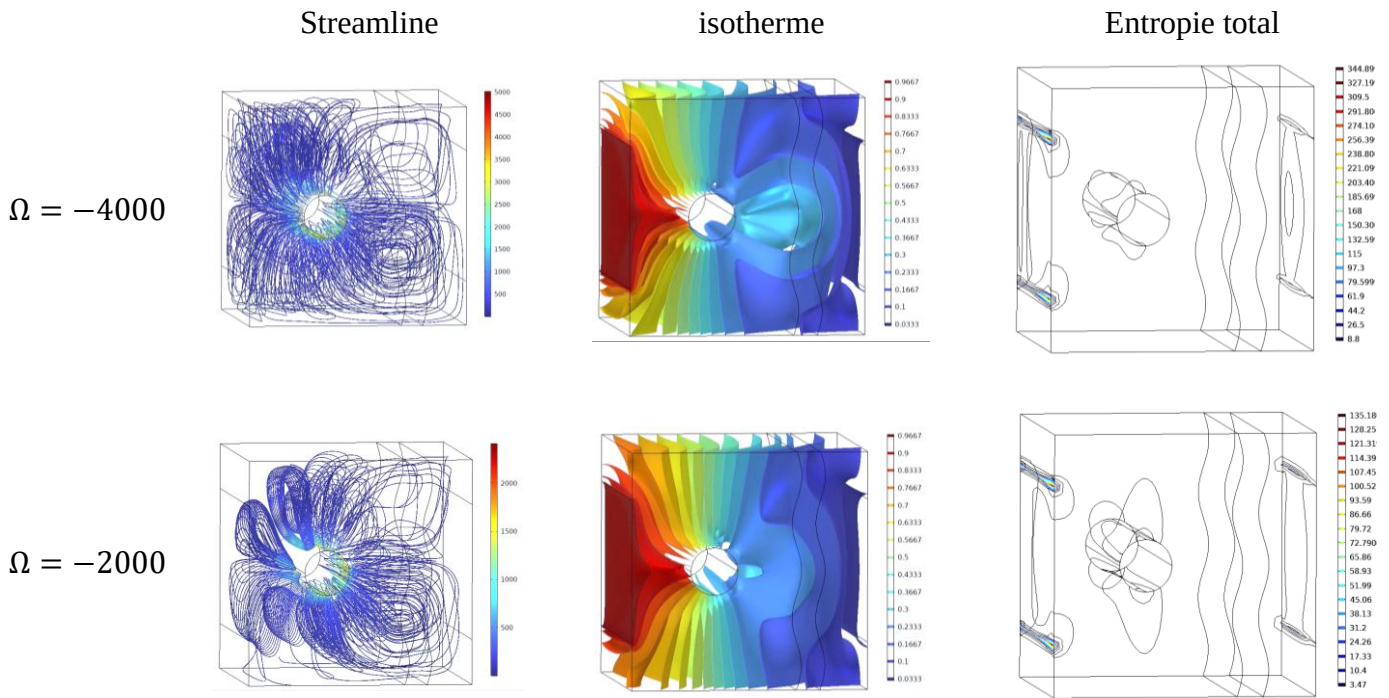


Figure III.5 The effect of the porous medium's configuration on entropy contours, streamlines, and isotherms with $\varphi = 0.02$; $\varepsilon = 0.1$; $\Omega = 100$; $Da = 0.01$, $Ha = 25$.



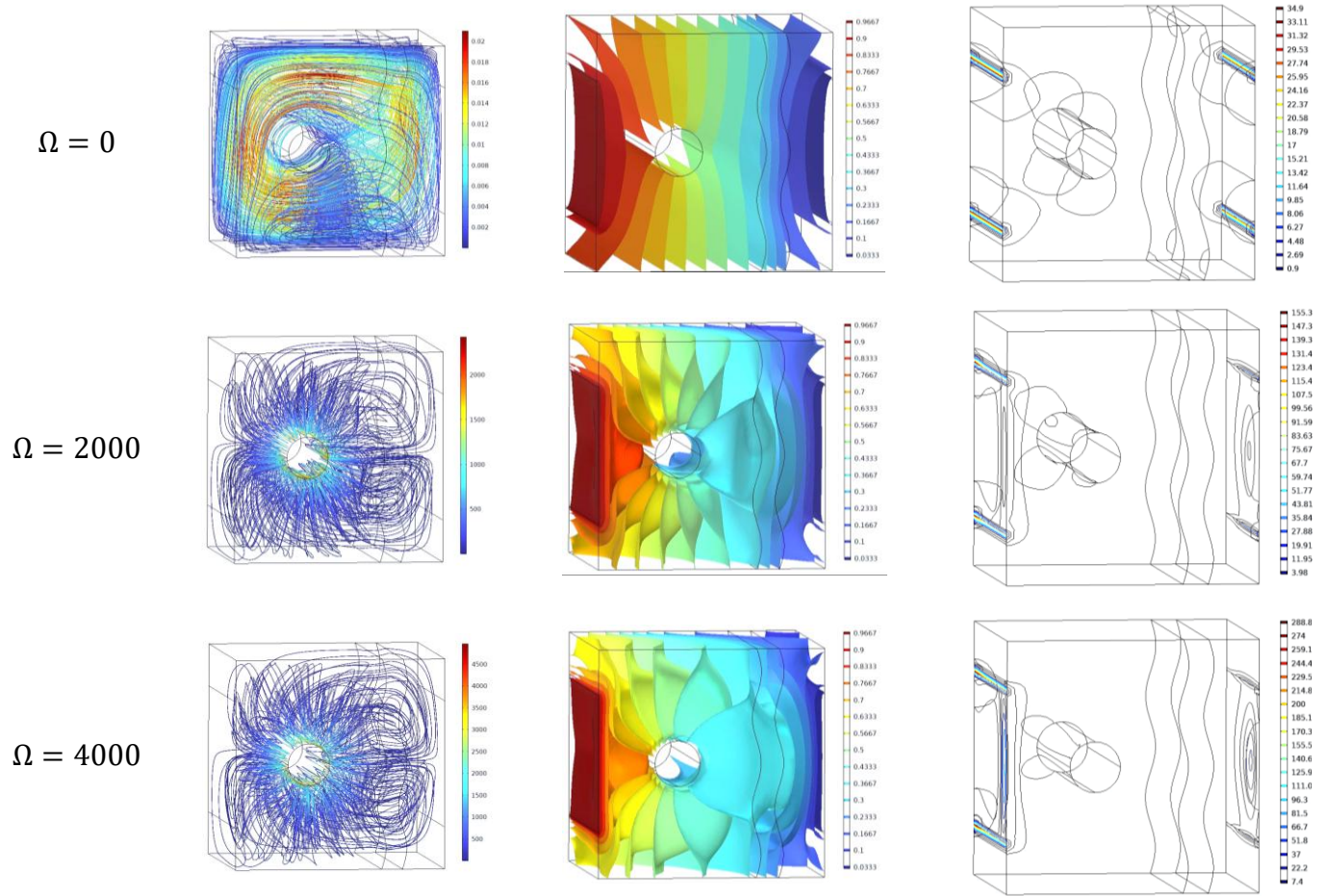


Figure III.6 The effect of Ω on entropy contours, streamlines, and isotherms with $\varphi = 0.02$; $\varepsilon = 0.1$; $Da = 0.01$, $Ha = 25$, $Ra = 10^5$.

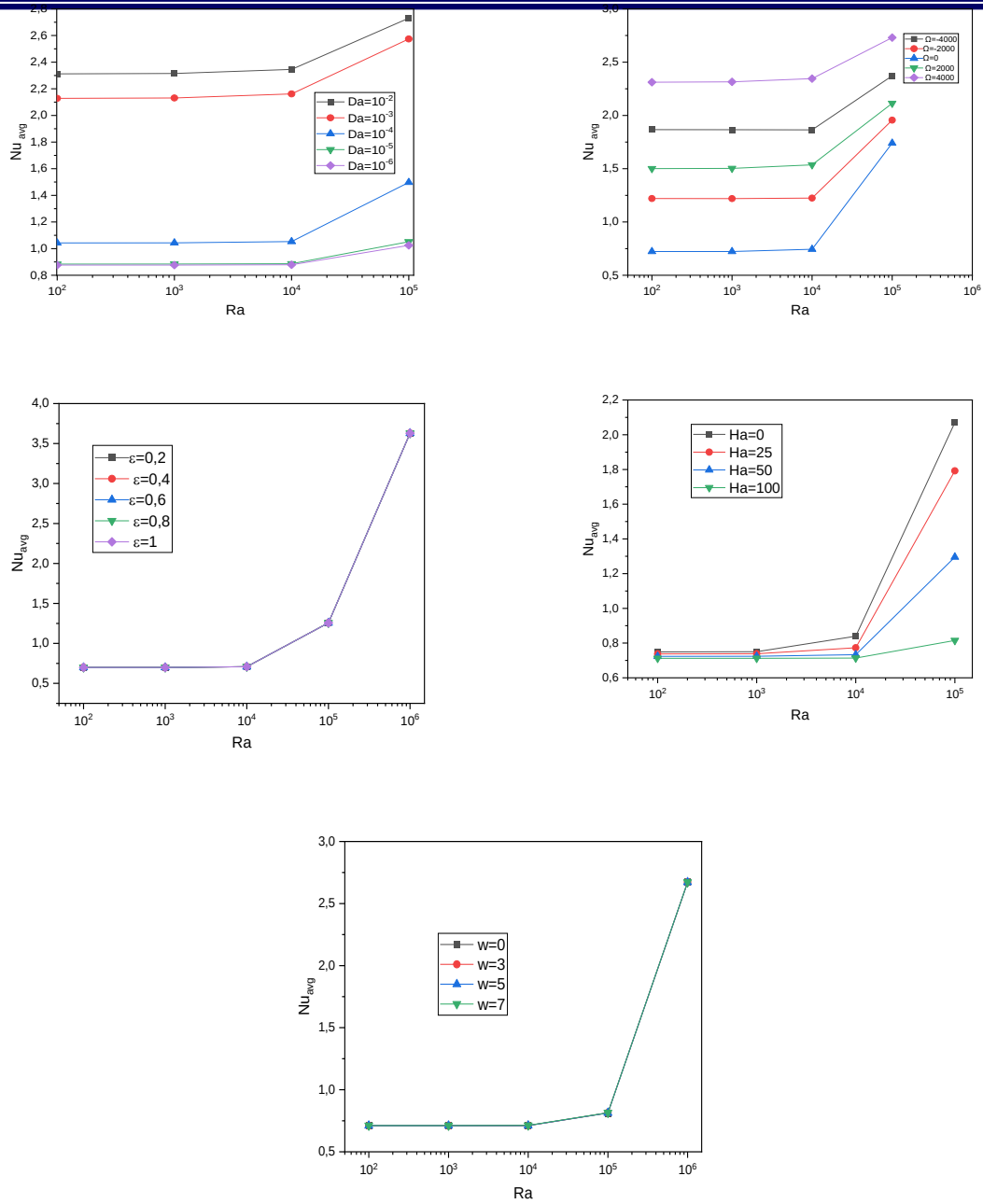


Figure III.7 The effect on the average Nusselt number of Ω , Da , Ha , and w with different Ra , at $Ha = 50$, $\varepsilon = 0.1$; $\varphi = 0.02$; $Da = 1e - 2$; $\Omega = 100$

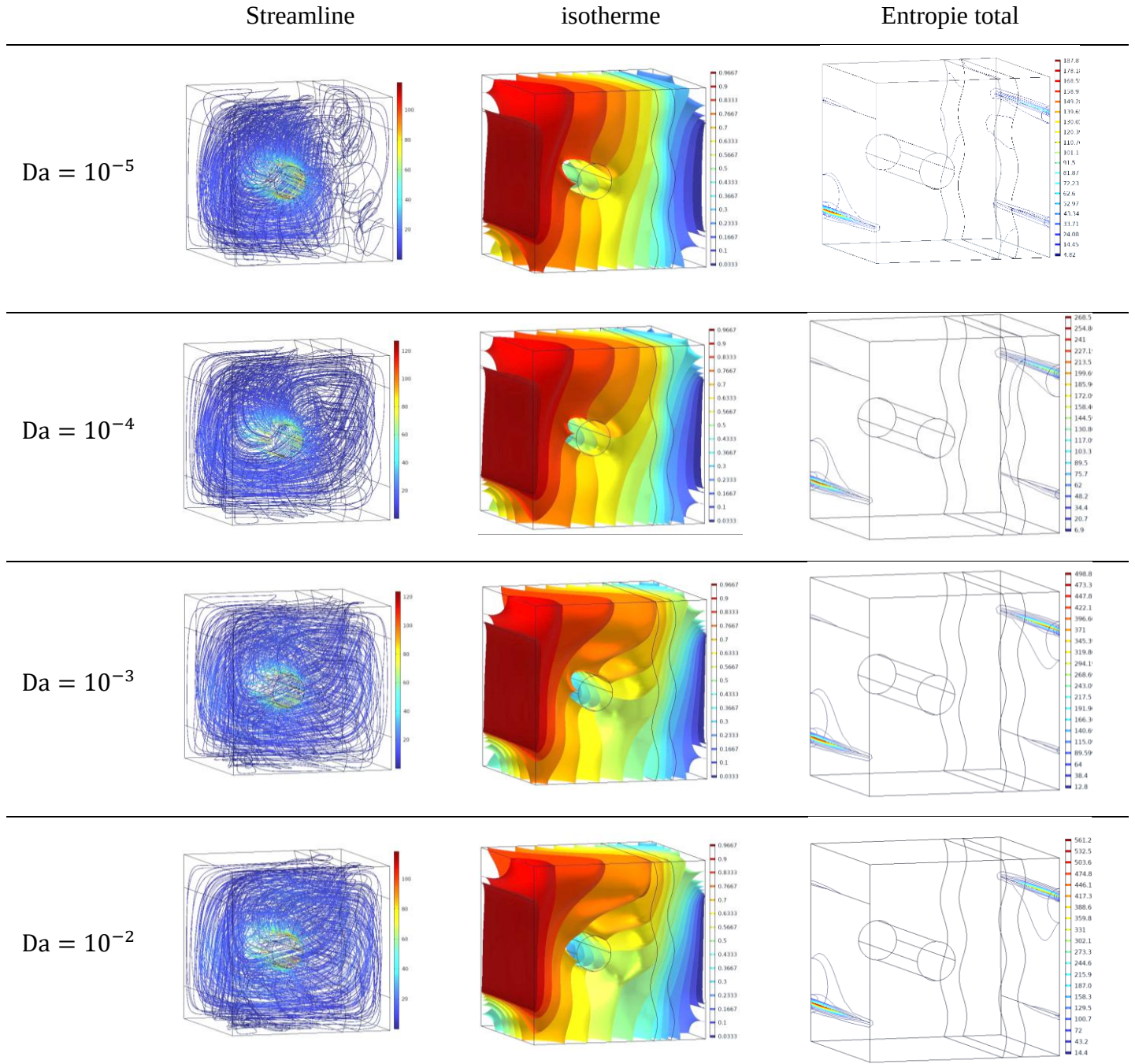


Figure III.8 Effect of Darcy on the results at $Ha = 50, \varepsilon = 0.1$; $\varphi = 0.02$; $Ra = 1e^5$; $\Omega = 100$.

Case 02: Effect of MHD nanofluid natural convection on thermal radiation in a unique cavity

1. Abstract:

In this particular study, we conducted an investigation into the intrinsic magneto-hydrodynamic (MHD) nanofluids convection phenomena. The influence of thermal radiation and the nanoparticle form factor on this convection process was numerically analyzed using the highly advanced comsol multiphysics software. To comprehensively understand The properties of nanofluids in terms of flow and heat transfer, we meticulously explored a wide range of parameters, including R, ϕ, m, Ha and Ra , Furthermore, the results obtained from our investigation unequivocally highlight the fact that the presence of laminar nanoparticles exerts a significantly more pronounced influence on both the Nusselt numbers average and local when compared to other shapes of nanoparticles that were considered in this study.

2. Problem Description

The investigation conducted revealed a novel type of cavity filled by Cu-H₂O nano fluid under a constant magnetic field. the natural convection behavior of a magnetohydrodynamic (MHD) nanofluid within an enclosure. This particular study considers both the nanoparticles' shape factor and the impact of thermal radiation. Figure III.1 displays a physical model and a grid distribution example. to give a visual depiction of the research. The problem is simplified by assuming certain things, such as the Newtonian and incompressible nature of the nanofluid under study. Additionally, laminar flow is thought to exist within the container. By taking into account the previously given assumptions and applying It is possible to define The equations for energy, momentum, and continuity controlling the present state using the Boussinesq approximation. Consequently, these equations serve as the foundation for further analysis and investigation in this study.

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0 \quad (III.1)$$

$$v \frac{\partial v}{\partial y} + u \frac{\partial u}{\partial x} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2} \right) \quad (III.2)$$

$$v \frac{\partial v}{\partial y} + u \frac{\partial u}{\partial x} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} \right) + \beta_{nf} g (T - T_c) - \frac{\sigma_f B_0^2}{\rho_{nf}} v \quad (III.3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho C_p)_{nf}} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - \frac{1}{(\rho C_p)_{nf}} \left(\frac{\partial q_{x,rad}}{\partial y} + \frac{\partial q_{y,rad}}{\partial x} \right) \quad (III.4)$$

In the equations listed above, the variables indicate the strength of the magnetic field, pressure, temperature, electrical conductivity, velocity component in both the x and y directions, and so forth. The final variable in these equations is the radiative heat flux.

The variables v, u, B_0, P, sf, T , and q_{rad} in the previously given equations match the magnetic field strength, the pressure, the temperature, the electrical conductivity, the velocity component in the x-direction and the y-direction, and so on the radiative heat flux is represented by the last variable.

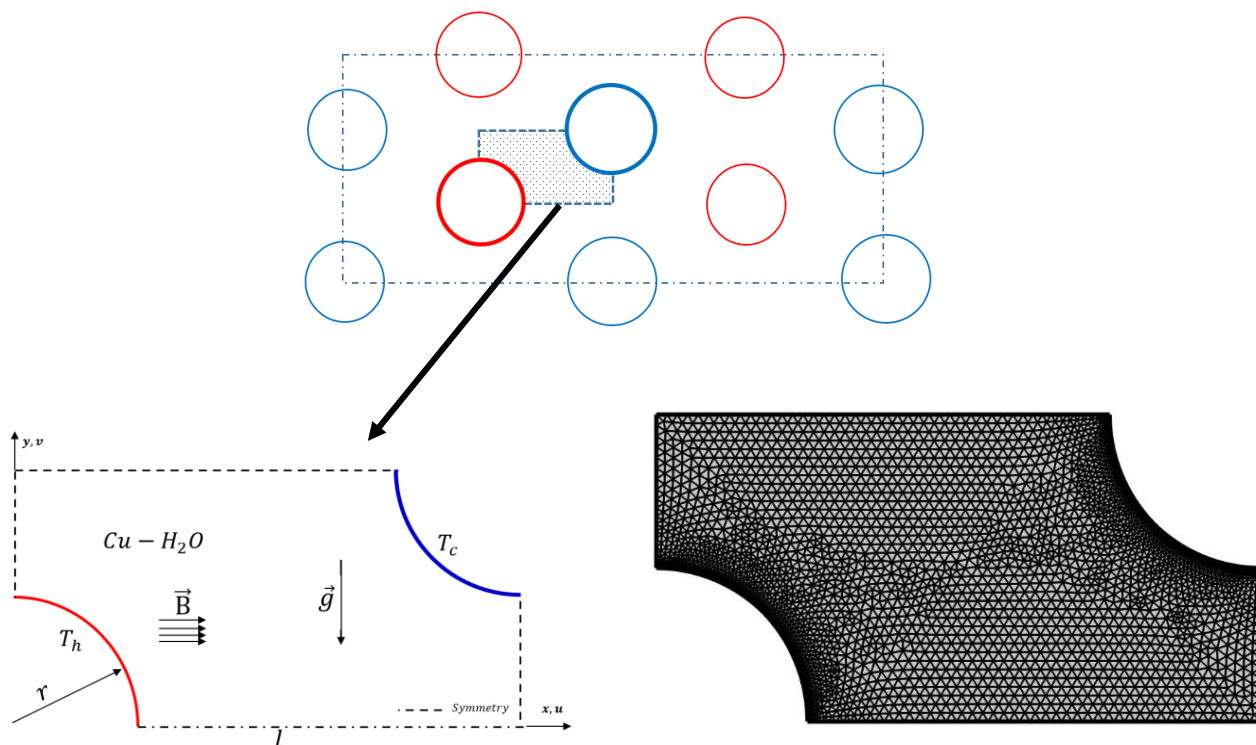


Figure III.9 (a) Actual control system and model; (b) mesh

$$q_{x,rad.} = -(4\sigma^*/3k_{nf}^*) \frac{\partial T^4}{\partial x}, q_{y,rad.} = -(4\sigma^*/3k_{nf}^*) \frac{\partial T^4}{\partial y}, T^4 \cong -3T_\infty^4 + 4T_\infty^3 T \quad (\text{III.5})$$

So, Equation (4) has been altered to

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho C_p)_{nf}} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{16\sigma^* T_\infty^3}{3k_{nf}(\rho C_p)_{nf}} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (\text{III.6})$$

μ_{nf} , k_{nf}^* , $(\rho C_p)_{nf}$ are defined as follows

$$\rho_{nf} = \phi \rho_s + (1 - \phi) \rho_f, \mu_{nf} = (1 - \phi)^{-2.5} \mu_f, (\rho C_p)_{nf} = \phi (\rho C_p)_s + (1 - \phi) (\rho C_p)_f \quad (\text{III.7})$$

The following describes the thermal conductivity of the nanofluid, k_{nf} , when the influences of nanoparticle shape are taken into when the effects of nanoparticle shape are considered, the heat conductivity of the nanofluid, k_{nf} consideration:

$$\frac{k_{nf}}{k_f} = \frac{(m-1)\phi(k_s - k_f) + k_s - (1-m)k_f}{k_s - (1-m)k_f - \phi(k_s - k_f)} \quad (\text{III.8})$$

In this case, m represents the form factor. The form factor for different particle forms is shown in Table III.5. The thermophysical properties of the nanofluid are also listed in Table III.6.

Table III.7 The form factor values for various nanoparticle forms.

Particle Shapes	Sphere	Column	Lamina
m	3	10	15

Table III.8 Thermal properties of water and nanoparticles [81]

	$\rho(kg/m^3)$	$C_p(J/kgK)$	$k(W/mK)$
Copper (Cu)	8933	385	40
Pure water	997.1	4179	0.613

Expression for the stream function and vorticity:

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}, \psi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad (III.9)$$

We define the dimensionless variables listed below:

$$Y = \frac{y}{L}, X = \frac{x}{L}, V = \frac{uL}{\alpha_f}, U = \frac{vL}{\alpha_f}, \theta = \frac{T-T_c}{T_h-T_c} \quad (III.10)$$

The governing equations produce the following non-dimensional forms.

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (III.11)$$

$$\frac{\rho_{hnf}}{\rho_f} \left[U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right] = -\frac{\partial P}{\partial X} + Pr \frac{\mu_{hnf}}{\mu_f} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (III.12)$$

$$\frac{\rho_{hnf}}{\rho_f} \left[U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right] = -\frac{\partial P}{\partial Y} + Pr \frac{\mu_{hnf}}{\mu_f} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - PrHa^2V + RaPr \frac{(\rho\beta)_{nf}}{(\rho\beta)_f} \frac{\partial \theta}{\partial X} \quad (III.13)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f} \frac{k_f}{k_{nf}} \left(1 + \frac{4}{3} Rd \right) \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (III.14)$$

Depending on the boundary conditions:

$$\begin{aligned} \theta &= 1 & \text{left quarter cercle} \\ \theta &= 0 & \text{right quarter cercle} \end{aligned}$$

The following definitions apply to the words Pr , Ra , R , and Ha in these equations:

$$Pr = \nu_f / \alpha_f, Ra = \frac{g\beta_f(T_h - T_c)L^3}{\alpha_f\nu_f}, R = \frac{4\sigma^*T_\infty^*}{k_{nf}k_{nf}^*}, Ha = B_0L\sqrt{\frac{\sigma_f}{\mu_f}}$$

One can determine the average and local Nusselt numbers along the hot wall by:

$$Nu_{loc} = \frac{k_{nf}}{k_f} \left(1 + \frac{4}{3}R\right) \frac{\partial \theta}{\partial X'}, Nu_{loc} = \int_0^1 Nu_{loc} \cdot dY \quad (III.15)$$

3. Numerical Solution and Validation

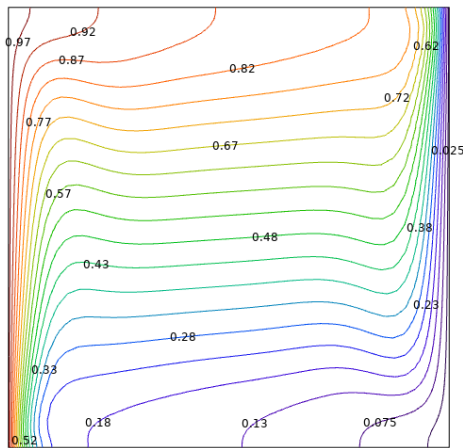
The finite element method based on control volume was applied to solve Equations [11–14]. Nu_{avg} for a various element count.

According to the investigation in Table III.9 , the average Nusselt number is influenced by the quantity of mesh elements. Nu_{avg} converges to stable values as the number of mesh elements rises. Still, there is a lot of diversity in the results from various mesh arrangements. With 2492 elements, this is the fifth mesh arrangement.

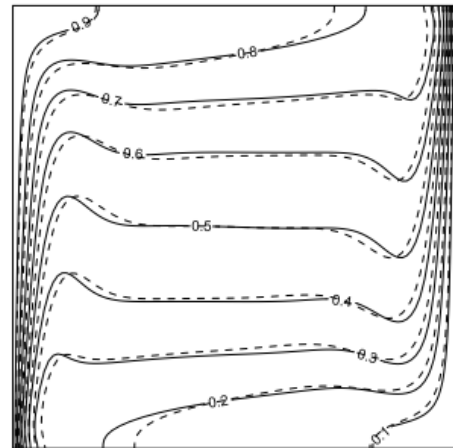
A key criterion for achieving outcomes is the numerical method's confidence level. Our model, as displayed in Figure III.10, was validated using earlier research by OUYAHIA's work [82].

Table III.10 Variation of Nu_{avg} with mesh elements number when $Ra = 10^6, Ha = 0, \phi = 0.02, R = 3$ and $m = 3$.

elements Number	214	422	860	1338	2492	5470
Nu_{avg}	0.484	0.506	0.519	0.522	0.523	0.523



Our work



OUYAHIA's work [82].

Figure III.11 a comparison between our results for parameter $Ra = 10^6$ and the work of OUYAHIA. [82] Our findings closely resemble those presented in the aforementioned literature.

Table III.11 An analysis at $\varphi = 0.06$; $Ha = 30$ comparing the current study and Ref [83], [84].

Ra	Present study	Alireza et al[83]	Ghasemi et al [84]
10^3	1.190	1.182	1.184
10^4	1.300	1.290	1.291
10^5	3.101	3.107	3.108

The finite element method is used to solve the governing equations and boundary conditions. mentioned above. in Comsol Multiphysics, it should be noted [85]. The realm of computing is separated into triangle components. Finite elements of triangular Lagrange various orders are employed for each flow variable in the computing area. To obtain The Estimates are incorporated into the governing equations. together with the residuals for each conservation equation. The momentum equations' nonlinear parts were simplified through the use of a Newton-Raphson iteration technique. Since i is the number of iterations and η is the convergence criteria, the convergence of When the relative error for every variable is known, the answer is presumed. satisfies the following convergence conditions $\left| \frac{r^{i+1} - r^i}{r^i} \right| \leq \eta$. the convergence condition was established at $\eta = 10^{-6}$.

4. Results and Discussion

in this research ,The Hartmann number ($Ha = 0 - 100$) , Rayleigh number($Ra = 10^2, 10^3, 10^4, 10^5$ and 10^6), The radiating parameter ($N=1-5$), The effects of the nanofluid volume percentage ($\varphi = 0.02 - 0.08$) and the form factor ($m = 3, 5, 10, 15$). on magnetohydrodynamics were investigated numerically. Natural convection of nanofluids in a cavity. The effects of heat radiation and the nanoparticles' form factor were identified.

The effects of the Ra and Ha on streamlines and isotherms are depicted in Figure III.12. and Figure III.13. , respectively. Ra growth suggests that the stream function's maximum absolute value has increased. The flow of the cavity accelerates as a result. This phenomenon is caused

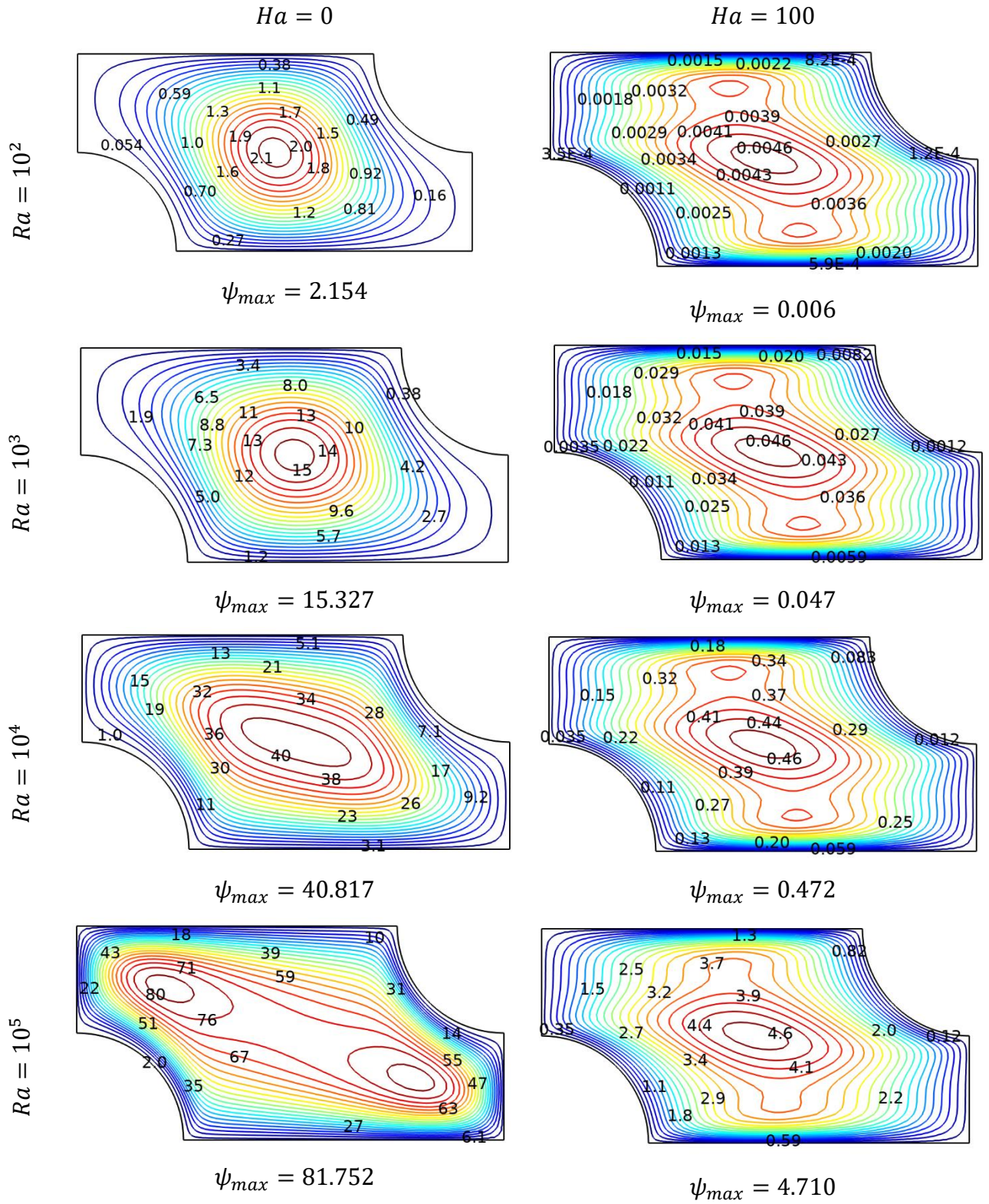
by the degree of natural convection, which is closely associated with the largest absolute value of the stream function. On the other hand, higher Ra distorts the isotherms and, consequently, the isothermal vertical stratification, especially at lower Ha and higher Ra . In other words, the thermal boundary layer is formed by the two side walls.

Convection, which has greater Ra and lower Ha values, is really the most common heat transport mechanism. Moreover, the flow function's absolute maximum value and the strength of natural convection both decrease with increasing Ha . Additionally, when Ha grows, the nanofluid's thermal stratification reduces and the central line widens.

Figure III.14 illustrates the connection along with the local Nusselt values, between R and ϕ . Three terms are multiplied to create the non-dimensional local Nusselt number: (k_{nf}/k_f) , $((4/3)R + 1)$, and $\partial\theta/\partial x$. Because the ratio's increase (k_{nf}/k_f) will triumph over a lesser $\partial\theta/\partial x$ for a given R , Nu_{loc} increases ϕ increases. However, for a constant ϕ , increasing ϕ will offset the lower value of $\partial\theta/\partial x$, resulting in a bigger $((4/3)R + 1)$ term. Nu_{loc} On the other hand, the average and local Nu numbers behave similarly.

Figure III.15 demonstrates the impact of the local Nusselt number on Ha and m . 5 accordingly. According to Table 1 and Equation (8), for a specific nanofluid, the development of (k_{nf}/k_f) with regard to the form factor is seen. Among the many nanoparticle forms investigated in this work, the spherical nanoparticle ($m = 3$) has the lowest thermal conductivity when a Cu–water nanofluid with $\phi = 0.02$ is used, whereas the laminar nanoparticle ($m = 15$) has the lowest thermal conductivity. If the shape factor and the local Nusselt number increases since the greater ratio (k_{nf}/k_f) will remove the drop statuses.

Rising Ra and dropping Ha can be used to explain these fluctuations in Nu_{loc} in the following ways: Lower Ra values reduce the buoyancy force of the system, favoring the conduction heat transfer mechanism. However, because of the increased buoyancy force, convection prevails over conduction at higher Ra values. Furthermore, because the buoyancy force inside the hollow is weakened by the insertion of a magnetic field, the conduction mode dominates at greater Ha values.



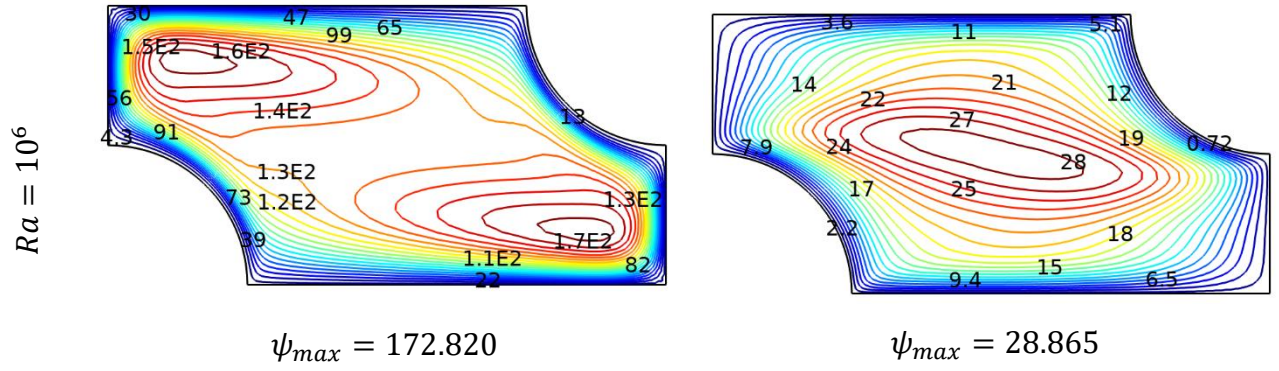
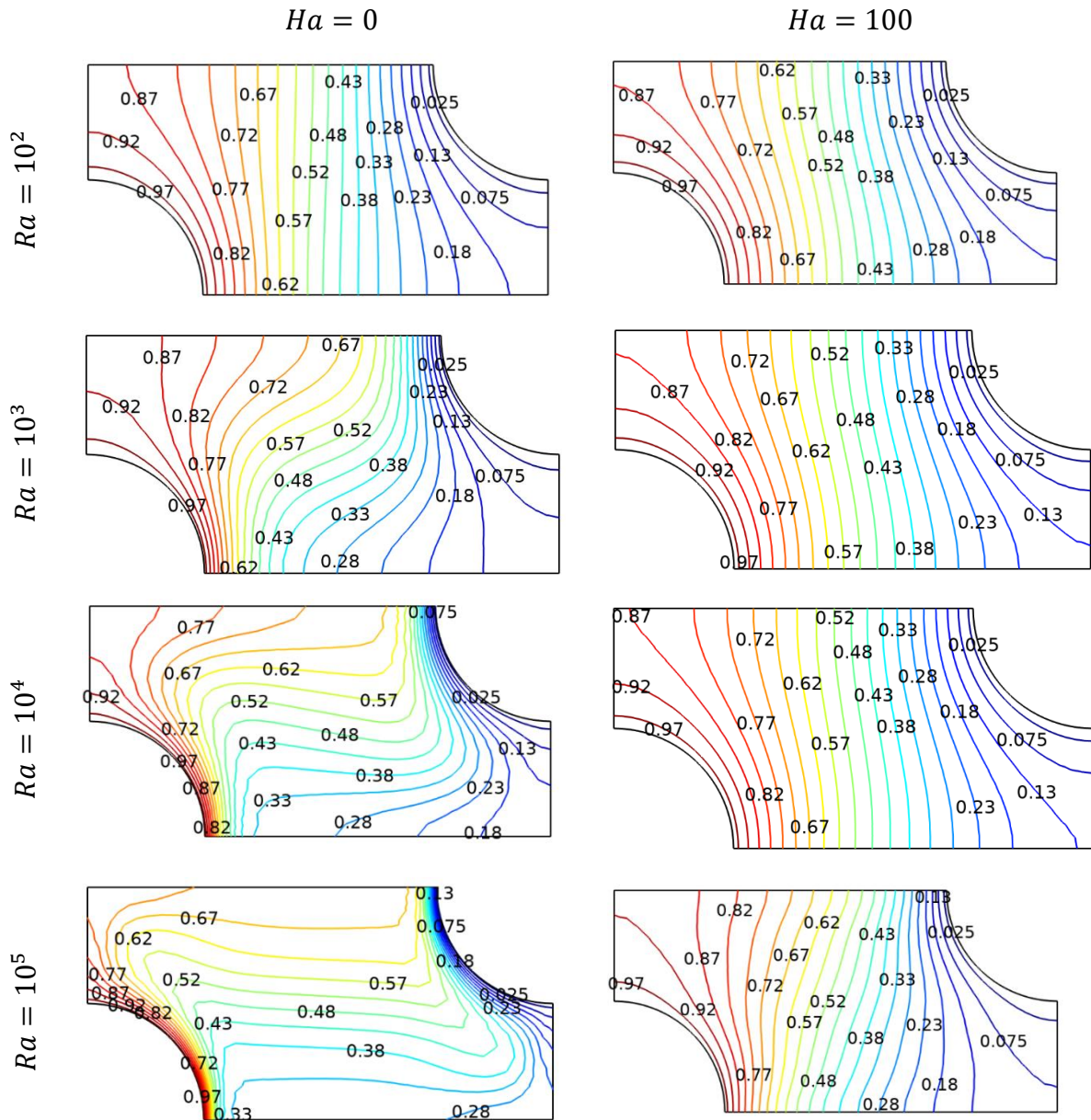


Figure III.16. smooth curves for different Ra and Ha values when $R = 3, m = 3$, and $\varphi = 2\%$.



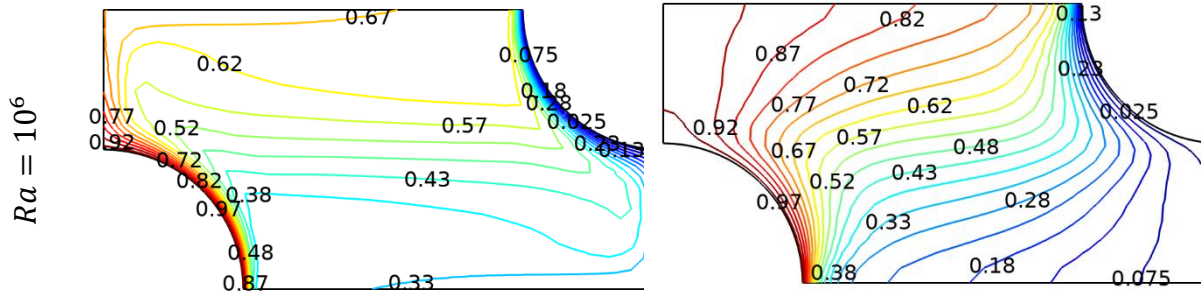


Figure III.17 Shapes of the isotherms for different Ra and Ha when $Ha = 0$, $R = 3$, $m = 3$, and $\phi = 0.02$.

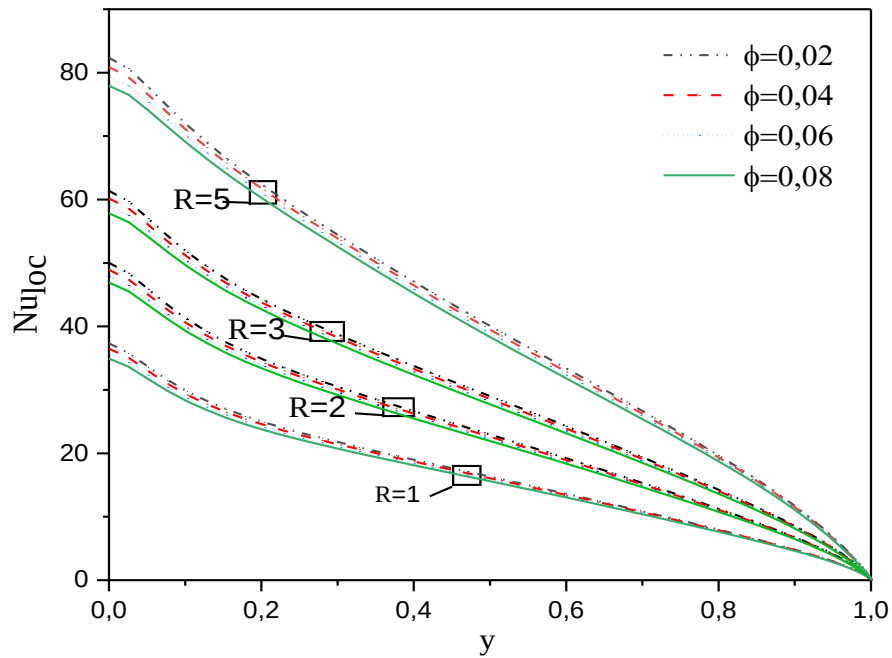


Figure III.18 variation values of the nanofluid volume percentage and radiation parameters, local Nusselt (Nu) numbers $Ra = 10^6$, $Ha = 0$, and $m = 3$.

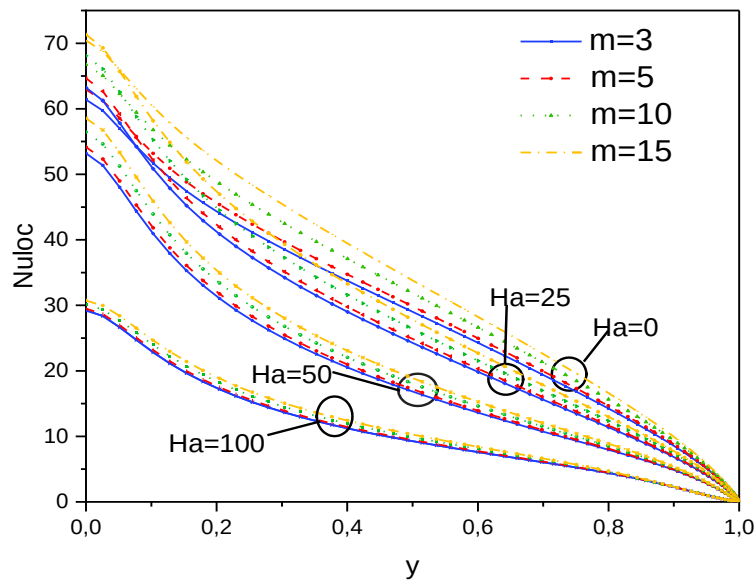


Figure III.19 Local Nusselt Nu_{loc} for different radiation and nanofluid parameters values of Ha when $Ra = 10^6$, $Ha = 0$, and $m = 3; 5; 10; 15$.

According to the two comparative analysis of heat transfer mechanisms in porous and nanofluid-filled cavities and conjugate local thermal equilibrium and radiation effects. the key results gained are summarized as follows:

- examines the magnetohydrodynamic nanofluid's natural convection in an enclosure while taking heat radiation and nanoparticle formation factors into account.

The Emphasizes the significance of taking convection, radiation, and magnetic fields into account while improving heat transfer procedures.

- Draws attention to how Brownian motion significantly affects the process of convective heat transfer.
- Created a 3D model of a permeable hole containing a turning chamber in an attractive field and loaded with $Fe_3O_4/MWCNT$ -water nanofluid.
- Findings reveal that increasing Hartmann number significantly reduces entropy generation.
- Higher rakish speed and Darcy number positively affect Nusselt number (Nu).
- Findings suggest that increasing Hartmann's number from 0 to 100 causes a significant reduction in Nusselt number.

- Provides insights for optimizing heat transfer systems, exploring nanofluid dynamics, highlighting external forces' impact on convective heat transfer, and enhancing efficiency in thermal engineering processes.

Conclusion

The primary goal of This study made a 3D model of a permeable hole containing a turning chamber in an attractive field and loaded up with $\text{Fe}_3\text{O}_4/\text{MWCNT}$ -water nanofluid. The objective of the review was to research the impacts of significant boundaries, for example, the (Ha) , (Ω) , and (Da) on the Nusselt number, entropy age, conveyance smoothest out, and isotherms.

The study found implies a large reduction in the (Ha) from 0 to 100 entropy generation in a system. It also revealed that increasing (Ω) and (Da) positively impacted the (Nu) , with higher values resulting in improved Nu values. The study also revealed that increasing Hartmann's number (Ha) from 0 to 100 caused a significant reduction in Nu , specifically a 39.37% decrease. The findings can be used for optimizing heat transfer systems, exploring nanofluid dynamics, highlighting external forces' impact on convective heat transfer, and enhancing efficiency in thermal engineering processes.

In the second research, heat radiation and nanoparticle shape factors are taken into account as the magnetohydrodynamic nanofluid naturally convects in an enclosure. The findings indicate that while local The Nusselt populations rise in tandem with rising Ra , R , and φ , they decrease with increasing Hartmann numbers.

Investigations on Heat Transfer and Convection in Magnetohydrodynamic Nanofluids Study on Convective Heat Transfer and Fluid Dynamics

- Optimization of Heat Transfer Systems: Insights can be used to develop efficient systems for various applications.
- Exploration of Nanofluid Dynamics: The mathematical model integrates nanofluids, offering potential for further research.
- Impact of External Forces: Study highlights external forces' impact on convective heat transfer.
- Application in Thermal Engineering: Insights can enhance efficiency in processes like central air systems, electronic cooling, and solar energy utilization.
- Exploration of Porous Media Dynamics: Study's focus on porous media within cavity configuration.
- Study explores effects of nanoparticle form factor and magnetohydrodynamic nanofluid natural convection under heat radiation.
- offers possible uses in energy efficiency and temperature management systems.

Conclusion

- Stresses the value of taking radiation, convection, and magnetic fields into account while maximizing heat transfer processes.
- Draws attention to the important influence that Brownian motion has on the process of convective heat transfer.

References

- [1] M. Biglarian, M. Rahimi Gorji, O. Pourmehran, et G. Domairry, « H₂O based different nanofluids with unsteady condition and an external magnetic field on permeable channel heat transfer », *Int. J. Hydrog. Energy*, vol. 42, n° 34, p. 22005-22014, août 2017, doi: 10.1016/j.ijhydene.2017.07.085.
- [2] A. K. Hussein, « Applications of nanotechnology to improve the performance of solar collectors – Recent advances and overview », *Renew. Sustain. Energy Rev.*, vol. 62, p. 767-792, sept. 2016, doi: 10.1016/j.rser.2016.04.050.
- [3] K. R. Aglawe, R. K. Yadav, et S. B. Thool, « Preparation, applications and challenges of nanofluids in electronic cooling: A systematic review », *Mater. Today Proc.*, vol. 43, p. 366-372, 2021, doi: 10.1016/j.matpr.2020.11.679.
- [4] T. Liang, J.-R. Hou, M. Qu, J.-X. Xi, et I. Raj, « Application of nanomaterial for enhanced oil recovery », *Pet. Sci.*, vol. 19, n° 2, p. 882-899, avr. 2022, doi: 10.1016/j.petsci.2021.11.011.
- [5] F. Abbas *et al.*, « Nanofluid: Potential evaluation in automotive radiator », *J. Mol. Liq.*, vol. 297, p. 112014, janv. 2020, doi: 10.1016/j.molliq.2019.112014.
- [6] S. Izadi, T. Armaghani, R. Ghasemiasl, A. J. Chamkha, et M. Molana, « A comprehensive review on mixed convection of nanofluids in various shapes of enclosures », *Powder Technol.*, vol. 343, p. 880-907, févr. 2019, doi: 10.1016/j.powtec.2018.11.006.
- [7] S. Aberoumand et A. Jafarimoghaddam, « Mixed convection heat transfer of nanofluids inside curved tubes: An experimental study », *Appl. Therm. Eng.*, vol. 108, p. 967-979, sept. 2016, doi: 10.1016/j.applthermaleng.2016.06.032.
- [8] A. Atek et F. Nezlioui, « Etude numérique des échanges convectifs dans une cavité munie d'ouvertures », 2015.
- [9] L. Theodore, F. Ricci, et T. Vanvliet, *Thermodynamics for the practicing engineer*. John Wiley & Sons, 2011.
- [10] A. Mammeri, « Amélioration des performances énergétiques des systèmes de refroidissement industriels: Application aux serveurs informatiques », 2014.
- [11] E. K. Fatima, « Modélisation numérique et théorique de la convection naturelle des nanofluides en cavité rectangulaire verticale sous l'effet d'un champ magnétique », 2017.

- [12] M. Schaub, M. Kriegel, et S. Brandt, « Analytical prediction of heat transfer by unsteady natural convection at vertical flat plates in air », *Int. J. Heat Mass Transf.*, vol. 144, p. 118665, déc. 2019, doi: 10.1016/j.ijheatmasstransfer.2019.118665.
- [13] X. Zhou *et al.*, « Numerical investigation of heat transfer enhancement and entropy generation of natural convection in a cavity containing nano liquid-metal fluid », *Int. Commun. Heat Mass Transf.*, vol. 106, p. 46-54, août 2019, doi: 10.1016/j.icheatmasstransfer.2019.05.003.
- [14] F. Berrahil, A. Bouttout, S. Benissaad, et C. Abid, « Effet du champ magnétique sur la convection naturelle du nanofluide (Al_2O_3 - eau) à propriétés variables dans une enceinte annulaire », 2020, doi: 10.25855/SFT2020-013.
- [15] S. Bekhadra, D. Bellakhal, M. Abdi, et M. Bekhadra, « Convection naturelle dans une cavité carrée avec nano-fluide », *Commun. Sci. Technol.*, vol. 21, n° 1, p. 34-45, 2023.
- [16] M. Boussaid, A. Mezenner, et M. Bouhadeb, « Convection naturelle de chaleur et de masse dans une cavité trapézoïdale », *Int. J. Therm. Sci.*, vol. 38, n° 4, p. 363-371, avr. 1999, doi: 10.1016/S1290-0729(99)80103-3.
- [17] B. P. Geridonmez et H. F. Oztop, « MHD natural convection in a cavity in the presence of cross partial magnetic fields and Al_2O_3 -water nanofluid », *Comput. Math. Appl.*, vol. 80, n° 12, p. 2796-2810, déc. 2020, doi: 10.1016/j.camwa.2020.10.003.
- [18] D. Dey et S. K. Dash, « An experimental investigation on the nanofluids in a cavity under natural convection with and without the rotary magnetic field », *Heliyon*, vol. 9, n° 11, p. e22416, nov. 2023, doi: 10.1016/j.heliyon.2023.e22416.
- [19] M. Sharifpur, S. O. Giwa, K.-Y. Lee, H. Ghodsinezhad, et J. P. Meyer, « Experimental Investigation into Natural Convection of Zinc Oxide/Water Nanofluids in a Square Cavity », *Heat Transf. Eng.*, vol. 42, n° 19-20, p. 1675-1687, nov. 2021, doi: 10.1080/01457632.2020.1818384.
- [20] M. Torki et N. Etesami, « Experimental investigation of natural convection heat transfer of SiO_2 /water nanofluid inside inclined enclosure », *J. Therm. Anal. Calorim.*, vol. 139, n° 2, p. 1565-1574, janv. 2020, doi: 10.1007/s10973-019-08445-9.
- [21] D. Dey et D. S. Sahu, « Experimental study in a natural convection cavity using nanofluids », *Mater. Today Proc.*, vol. 41, p. 403-412, 2021, doi: 10.1016/j.matpr.2020.09.631.
- [22] N. Biswas, N. K. Manna, et A. J. Chamkha, « Effects of half-sinusoidal nonuniform heating during MHD thermal convection in $\text{Cu-Al}_2\text{O}_3$ /water hybrid nanofluid saturated

- with porous media », *J. Therm. Anal. Calorim.*, vol. 143, n° 2, p. 1665-1688, janv. 2021, doi: 10.1007/s10973-020-10109-y.
- [23] Md. S. Islam, S. Islam, et Md. N.-A.-A. Siddiki, « Numerical simulation with sensitivity analysis of MHD natural convection using Cu-TiO₂-H₂O hybrid nanofluids », *Int. J. Thermofluids*, vol. 20, p. 100509, nov. 2023, doi: 10.1016/j.ijft.2023.100509.
- [24] M. J. Maghrebi, M. Nazari, et T. Armaghani, « Forced Convection Heat Transfer of Nanofluids in a Porous Channel », *Transp. Porous Media*, vol. 93, n° 3, p. 401-413, juill. 2012, doi: 10.1007/s11242-012-9959-2.
- [25] I. ZEGHBID et R. BESSAÏH, « ÉTUDE NUMÉRIQUE DE LA CONVECTION FORCÉE 3D AVEC GÉNÉRATION D'ENTROPIE DANS UNE ENCEINTE CUBIQUE REMPLIE D'UN NANOFLUIDE ».
- [26] A. MAHAMMEDI, D. MEDJAHED, B. NACEUR, H. AMEUR, A. ARIS, et E. DOUROUM, « Modélisation du transfert thermique par convection forcée des nanofluides dans un tube avec CFD ».
- [27] M. Gupta, N. Arora, R. Kumar, S. Kumar, et N. Dilbaghi, « A comprehensive review of experimental investigations of forced convective heat transfer characteristics for various nanofluids », *Int. J. Mech. Mater. Eng.*, vol. 9, n° 1, p. 11, déc. 2014, doi: 10.1186/s40712-014-0011-x.
- [28] R. B. Mansour, N. Galanis, et C. T. Nguyen, « Effect of uncertainties in physical properties on forced convection heat transfer with nanofluids », *Appl. Therm. Eng.*, vol. 27, n° 1, p. 240-249, janv. 2007, doi: 10.1016/j.applthermaleng.2006.04.011.
- [29] Y. Ding *et al.*, « Forced convective heat transfer of nanofluids », *Adv. Powder Technol.*, vol. 18, n° 6, p. 813-824, 2007, doi: 10.1163/156855207782515021.
- [30] S. E. B. Maïga, S. J. Palm, C. T. Nguyen, G. Roy, et N. Galanis, « Heat transfer enhancement by using nanofluids in forced convection flows », *Int. J. Heat Fluid Flow*, vol. 26, n° 4, p. 530-546, août 2005, doi: 10.1016/j.ijheatfluidflow.2005.02.004.
- [31] Y. Ma, R. Mohebbi, M. M. Rashidi, et Z. Yang, « Study of nanofluid forced convection heat transfer in a bent channel by means of lattice Boltzmann method », *Phys. Fluids*, vol. 30, n° 3, p. 032001, mars 2018, doi: 10.1063/1.5022060.
- [32] *A-to-Z Guide to Thermodynamics, Heat and Mass Transfer, and Fluids Engineering: AtoZ*, vol. M. Begellhouse, 2006.
- [33] N. Ouertatani, N. B. Cheikh, B. B. Beya, T. Lili, et A. Campo, « Mixed convection in a double lid-driven cubic cavity », *Int. J. Therm. Sci.*, vol. 48, n° 7, p. 1265-1272, juill. 2009, doi: 10.1016/j.ijthermalsci.2008.11.020.

- [34] O. Prakash et S. N. Singh, « Experimental and numerical study of mixed convection with surface radiation heat transfer in an air-filled ventilated cavity », *Int. J. Therm. Sci.*, vol. 171, p. 107169, janv. 2022, doi: 10.1016/j.ijthermalsci.2021.107169.
- [35] E. Abu-Nada et H. F. Oztop, « Numerical Analysis of Al_2O_3 /Water Nanofluids Natural Convection in a Wavy Walled Cavity », *Numer. Heat Transf. Part Appl.*, vol. 59, n° 5, p. 403-419, févr. 2011, doi: 10.1080/10407782.2011.552363.
- [36] Z. Alloui, P. Vasseur, et M. Reggio, « Natural convection of nanofluids in a shallow cavity heated from below », *Int. J. Therm. Sci.*, vol. 50, n° 3, p. 385-393, mars 2011, doi: 10.1016/j.ijthermalsci.2010.04.006.
- [37] M. Benzema, Y. K. Benkahla, N. Labsi, S.-E. Ouyahia, et M. El Ganaoui, « Second law analysis of MHD mixed convection heat transfer in a vented irregular cavity filled with Ag-MgO/water hybrid nanofluid », *J. Therm. Anal. Calorim.*, vol. 137, n° 3, p. 1113-1132, août 2019, doi: 10.1007/s10973-019-08017-x.
- [38] M. Lakhi et A. Safavinejad, « Numerical investigation of combined heat transfer (mixed convection-radiation) in 2D channel using the LBM », *Int. Commun. Heat Mass Transf.*, vol. 126, p. 105368, juill. 2021, doi: 10.1016/j.icheatmasstransfer.2021.105368.
- [39] A. A. A. Al-Rashed *et al.*, « Mixed convection and entropy generation in a nanofluid filled cubical open cavity with a central isothermal block », *Int. J. Mech. Sci.*, vol. 135, p. 362-375, janv. 2018, doi: 10.1016/j.ijmecsci.2017.11.033.
- [40] G. nombre de Grashof, P. nombre de Prandtl, R. nombre de Reynolds, R. nombre de Richardson, et R. nombre de Rayleigh, « ETUDE MESOSCOPIQUE DE LA CONVECTION MIXTE LAMINAIRE DANS UNE CAVITE CARREE VENTILEE ».
- [41] W. Pakdee, A. Hussein, M. Ben Hamida, et F. Rashid, « MHD mixed convection flow of alumina - water nanofluid into a lid-driven cavity with different patterns of wavy sidewalls », *J. Comput. Appl. Mech.*, n° Online First, janv. 2024, doi: 10.22059/jcamech.2024.369622.922.
- [42] M. Benzema, Y. Benkahla, A. Boudiaf, et S.-E. Ouyahia, « Etude numérique de la convection mixte lors de l'écoulement d'un nanofluide hybride (Ag-MgO/Eau) dans une cavité trapézoïdale ventilée soumise à l'action d'un champ magnétique », présenté à CFM 2017-23ème Congrès Français de Mécanique, AFM, Maison de la Mécanique, 39/41 rue Louis Blanc-92400 Courbevoie, 2017.
- [43] « Mixed convection heat transfer of a nanofluid in a square ventilated cavity separated horizontally by a porous layer and discrete heat source », *Arch. Thermodyn.*, août 2023, doi: 10.24425/ather.2023.146560.

- [44] B. K. Jha et G. Samaila, « Effect of Internal Heat Generation on Mixed Convection Flow from a Vertical Plate to a Fluid with Nonlinear Thermal Radiation », *Trans. Indian Natl. Acad. Eng.*, vol. 8, n° 1, p. 161-170, mars 2023, doi: 10.1007/s41403-022-00386-4.
- [45] A. H. Altun et O. Ziylan, « Experimental investigation of the effects of horizontally oriented vertical sinusoidal wavy fins on heat transfer performance in case of natural convection », *Int. J. Heat Mass Transf.*, vol. 139, p. 425-431, août 2019, doi: 10.1016/j.ijheatmasstransfer.2019.05.009.
- [46] M. N. Borjini, C. Mbow, et M. Dagenet, « Étude numérique du couplage rayonnement-convection naturelle en régime permanent dans des secteurs et des espaces annulaires à l'aide de la méthode des volumes finis », *Int. J. Therm. Sci.*, vol. 38, n° 5, p. 410-423, mai 1999, doi: 10.1016/S1290-0729(99)80012-X.
- [47] M. N. Borjini, C. Mbow, et M. Dagenet, « Étude numérique de l'influence du transfert radiatif sur la convection naturelle laminaire, bidimensionnelle, permanente, dans un espace annulaire d'axe horizontal, délimité par deux cylindres circulaires isothermes », *Rev. Générale Therm.*, vol. 37, n° 6, p. 475-487, juin 1998, doi: 10.1016/S0035-3159(98)80064-0.
- [48] R. Satish *et al.*, « Case study of radiation absorptive and chemical reactive impacts on unsteady MHD natural convection flowing of Cu-Al₂O₃/H₂O hybridized nanofluid », *Case Stud. Therm. Eng.*, vol. 64, p. 105536, déc. 2024, doi: 10.1016/j.csite.2024.105536.
- [49] M. Delort-Laval, « Influence des transferts radiatifs sur les écoulements de convection naturelle en régime turbulent, dans des ambiances habitables », 2023.
- [50] T. Armaghani *et al.*, « Unsteady Flow of Hybrid Nanofluid with Magnetohydrodynamics-Radiation-Natural Convection Effects in a U-Shaped Wavy Porous Cavity », *Comput. Model. Eng. Sci.*, vol. 141, n° 3, p. 2225-2251, 2024, doi: 10.32604/cmes.2024.056676.
- [51] Z. Ullah *et al.*, « Solar radiation and heat sink impact on fluctuating mixed convective flow and heat rate of Darcian nanofluid: Applications in electronic cooling systems », *Case Stud. Therm. Eng.*, vol. 59, p. 104592, juill. 2024, doi: 10.1016/j.csite.2024.104592.
- [52] S. Abu Bakar, N. Arifin, F. Md Ali, N. Bachok, R. Nazar, et I. Pop, « A Stability Analysis on Mixed Convection Boundary Layer Flow along a Permeable Vertical Cylinder in a Porous Medium Filled with a Nanofluid and Thermal Radiation », *Appl. Sci.*, vol. 8, n° 4, p. 483, mars 2018, doi: 10.3390/app8040483.
- [53] S. DERFOUFI, F. MOUFEKKIR, et A. MEZRHAB, « Étude numérique du couplage convection-rayonnement volumique dans un canal incliné par la méthode de Boltzmann sur réseau ».

- [54] H. Wang, S. Xin, et P. Le Quéré, « Étude numérique du couplage de la convection naturelle avec le rayonnement de surfaces en cavité carrée remplie d'air », *Comptes Rendus Mécanique*, vol. 334, n° 1, p. 48-57, déc. 2005, doi: 10.1016/j.crme.2005.10.011.
- [55] F. Corvaro, M. Paroncini, et M. Sotte, « Experimental PIV and interferometric analysis of natural convection in a square enclosure with partially active hot and cold walls », *Int. J. Therm. Sci.*, vol. 50, n° 9, p. 1629-1638, sept. 2011, doi: 10.1016/j.ijthermalsci.2011.03.029.
- [56] M. Paroncini et F. Corvaro, « Natural convection in a square enclosure with a hot source », *Int. J. Therm. Sci.*, vol. 48, n° 9, p. 1683-1695, sept. 2009, doi: 10.1016/j.ijthermalsci.2009.02.005.
- [57] B. Hoseinpour, H. R. Ashorynejad, et K. Javaherdeh, « Entropy Generation of Nanofluid in a Porous Cavity by Lattice Boltzmann Method », *J. Thermophys. Heat Transf.*, vol. 31, n° 1, p. 20-27, janv. 2017, doi: 10.2514/1.T4652.
- [58] B. Hoseinpour, H. R. Ashorynejad, et K. Javaherdeh, « Entropy Generation of Nanofluid in a Porous Cavity by Lattice Boltzmann Method », *J. Thermophys. Heat Transf.*, vol. 31, n° 1, p. 20-27, janv. 2017, doi: 10.2514/1.T4652.
- [59] R. S. R. Gorla, S. Siddiqua, M. A. Mansour, A. M. Rashad, et T. Salah, « Heat Source/Sink Effects on a Hybrid Nanofluid-Filled Porous Cavity », *J. Thermophys. Heat Transf.*, vol. 31, n° 4, p. 847-857, oct. 2017, doi: 10.2514/1.T5085.
- [60] I. E. Sarris, I. Lekakis, et N. S. Vlachos, « Natural convection in rectangular tanks heated locally from below », *Int. J. Heat Mass Transf.*, vol. 47, n° 14-16, p. 3549-3563, juill. 2004, doi: 10.1016/j.ijheatmasstransfer.2003.12.022.
- [61] F. Selimefendigil et H. F. Öztop, « Mixed convection of nanofluid filled cavity with oscillating lid under the influence of an inclined magnetic field », *J. Taiwan Inst. Chem. Eng.*, vol. 63, p. 202-215, juin 2016, doi: 10.1016/j.jtice.2016.03.003.
- [62] J. I. Hochstein et A. L. Gerhart, *Young, Munson and Okiishi's A Brief Introduction to Fluid Mechanics*. John Wiley & Sons, 2021.
- [63] W. Almalki, M. Hamdan, et M. Kamel, « Analysis of flow through layered porous media », présenté à Proceedings of the 12th WSEAS International Conference on Mathematical Methods, Computational Techniques and Intelligent Systems, 2010, p. 182-189.
- [64] A. W. Jayawardena, *Fluid Mechanics, Hydraulics, Hydrology and Water Resources for Civil Engineers*, 1^{re} éd. CRC Press, 2021. doi: 10.1201/9780429423116.

-
- [65] M. Hangi, V. Wheeler, et W. Lipiński, « Numerical determination of permeability and Forchheimer coefficient in dual-scale porous media », *Int. Commun. Heat Mass Transf.*, vol. 122, p. 105089, mars 2021, doi: 10.1016/j.icheatmasstransfer.2020.105089.
- [66] R. Mittal et R. Bhardwaj, « IMMERSED BOUNDARY METHODS FOR THERMOFLUIDS PROBLEMS », *Annu. Rev. Heat Transf.*, vol. 24, n° 1, p. 33-70, 2022, doi: 10.1615/AnnualRevHeatTransfer.2022041888.
- [67] S. Pati, A. Borah, M. P. Boruah, et P. R. Randive, « Critical review on local thermal equilibrium and local thermal non-equilibrium approaches for the analysis of forced convective flow through porous media », *Int. Commun. Heat Mass Transf.*, vol. 132, p. 105889, mars 2022, doi: 10.1016/j.icheatmasstransfer.2022.105889.
- [68] T. Grosan, F.-O. Patrulescu, et I. Pop, « Natural convection in a differentially heated cavity filled with a Brinkman bidisperse porous medium », *Int. J. Numer. Methods Heat Fluid Flow*, vol. 33, n° 10, p. 3309-3326, août 2023, doi: 10.1108/HFF-10-2022-0600.
- [69] B. Straughan, « LTNE convection with Cattaneo effect in the solid », in *Thermal Convection with a Cattaneo Flux Law*, vol. 53, in *Advances in Mechanics and Mathematics*, vol. 53. , Cham: Springer Nature Switzerland, 2024, p. 131-165. doi: 10.1007/978-3-031-80885-2_10.
- [70] P. K. Srivastava, R. Yadav, R. P. Pandey, et N. Pandya, « Flow of a Viscous Fluid Past a Porous Oblate Spheroid at Small Reynolds Number », *Asian J. Math. Comput. Res.*, vol. 32, n° 1, p. 52-77, janv. 2025, doi: 10.56557/ajomcor/2025/v32i19051.
- [71] S. Safi et S. Benissaad, « Heat and mass transfer in anisotropic porous media », *Adv. Theor. Appl. Mech.*, vol. 5, p. 15-26, 2012.
- [72] S. A. Shehzad, M. Sheikholeslami, T. Ambreen, A. Shafee, H. Babazadeh, et M. Ahmad, « Heat transfer management of hybrid nanofluid including radiation and magnetic source terms within a porous domain », *Appl. Nanosci.*, vol. 10, n° 12, p. 5351-5359, déc. 2020, doi: 10.1007/s13204-020-01432-9.
- [73] A. Mahamane, « Analyse et estimation d'erreur en volumes finis, Application aux écoulements en milieu poreux et à l'adaptation de maillage », 2009.
- [74] S. Kakaç, H. Liu, et A. Pramuanjaroenkij, *Heat Exchangers*, 0 éd. CRC Press, 2002. doi: 10.1201/9781420053746.
- [75] R. H. Pletcher, J. C. Tannehill, et D. Anderson, *Computational fluid mechanics and heat transfer*. CRC press, 2012.
- [76] R. Akhter, M. M. Ali, M. M. Billah, et M. N. Uddin, « Hybrid-nanofluid mixed convection in square cavity subjected to oriented magnetic field and multiple rotating rough

- cylinders », *Results Eng.*, vol. 18, p. 101100, juin 2023, doi: 10.1016/j.rineng.2023.101100.
- [77] C. D. Zineb, F. Redouane, H. Rachid, et N. Houcine, « Exploring the complex interplay of factors in convective heat transfer: a comprehensive parametric analysis in three-dimensional permeable cavities », *Energy Sources Part Recovery Util. Environ. Eff.*, vol. 46, n° 1, p. 6064-6080, déc. 2024, doi: 10.1080/15567036.2024.2346181.
- [78] M. Hemmat Esfe, A. A. Abbasian Arani, M. Rezaie, W.-M. Yan, et A. Karimipour, « Experimental determination of thermal conductivity and dynamic viscosity of Ag–MgO/water hybrid nanofluid », *Int. Commun. Heat Mass Transf.*, vol. 66, p. 189-195, août 2015, doi: 10.1016/j.icheatmasstransfer.2015.06.003.
- [79] H. Laidoudi *et al.*, « Irreversibility Interpretation and MHD Mixed Convection of Hybrid Nanofluids in a 3D Heated Lid-Driven Chamber », *Nanomaterials*, vol. 12, n° 10, p. 1747, mai 2022, doi: 10.3390/nano12101747.
- [80] K. Ghasemi et M. Siavashi, « MHD nanofluid free convection and entropy generation in porous enclosures with different conductivity ratios », *J. Magn. Magn. Mater.*, vol. 442, p. 474-490, nov. 2017, doi: 10.1016/j.jmmm.2017.07.028.
- [81] M. Alizadeh, A. S. Dogonchi, et D. D. Ganji, « Micropolar nanofluid flow and heat transfer between penetrable walls in the presence of thermal radiation and magnetic field », *Case Stud. Therm. Eng.*, vol. 12, p. 319-332, sept. 2018, doi: 10.1016/j.csite.2018.05.002.
- [82] S. Ouyahia, Y. Benkahla, M. Benzema, et W. Berabou, « Effet du champ magnétique sur la convection naturelle dans une cavité inclinée remplie d'un nanofluide », *Congrès Société Fr. Therm.*, 2015.
- [83] A. Aghaei, H. Khorasanizadeh, G. Sheikhzadeh, et M. Abbaszadeh, « Numerical study of magnetic field on mixed convection and entropy generation of nanofluid in a trapezoidal enclosure », *J. Magn. Magn. Mater.*, vol. 403, p. 133-145, avr. 2016, doi: 10.1016/j.jmmm.2015.11.067.
- [84] B. Ghasemi, S. M. Aminossadati, et A. Raisi, « Magnetic field effect on natural convection in a nanofluid-filled square enclosure », *Int. J. Therm. Sci.*, vol. 50, n° 9, p. 1748-1756, sept. 2011, doi: 10.1016/j.ijthermalsci.2011.04.010.
- [85] M. Bazargan, B. S. Almqvist, H. B. Motra, P. Broumand, T. Schmiedel, et C. F. Hieronymus, « Elastic Wave Propagation in a Stainless-Steel Standard and Verification of a COMSOL Multiphysics Numerical Elastic Wave Toolbox », *Resources*, vol. 11, n° 5, p. 49, 2022.



Exploring the complex interplay of factors in convective heat transfer: a comprehensive parametric analysis in three-dimensional permeable cavities

Chikr Djaoutsi Zineb^a, Fares Redouane^a, Hidki Rachid^b, and Naim Houcine^c

^aLGIDD, Department of Physics, Relizane's University, Relizane, Algeria; ^bLMFE, Department of Physics, Faculty of Sciences Semlalia, Cadi Ayyad University, Marrakesh, Morocco; ^cLaboratory for Analysis and Applications of Radiation (LAAR), University of Sciences and Technology of Oran (USTO-MB), ORAN, Algeria

ABSTRACT

This examination aimed to mathematically investigate the blended convective intensity movement inside a three-layered nook that consolidates a spinning roundabout chamber and an undulating permeable pit. Using the limited component technique (FEM), mathematical reproductions were executed across different boundaries, enveloping the Darcy number going from 10^{-5} to 10^{-2} Hartmann number (going from 0 to 100, the angular velocity between -4000 and 4000 , and the number of undulations in the permeable pit. The results, addressed through stream capability, isotherms, and isentropic forms, clarify the effect of these boundaries on movement, heat transport, and entropy age. The ends suggest that for an ideal upgrade of intensity move rates in a three-layered undulating permeable cavity with a pivoting chamber and exposed to an attractive field, explicit circumstances are suggested, a Darcy number more prominent than or equivalent to 10^{-2} , a Hartmann number of 0, undulation of the permeable pit ($N = 3$), and angular velocity surpassing 2000 in the stream heading. The study investigates convective heat transfer in a rotating cylindrical chamber, analyzing parameters like Darcy number, Hartmann number, and rotation speed for optimizing heat transfer systems in complex porous cavities. The optimal heat transfer occurs when the aspect ratio is fixed at 0.3 and porous media occupies 90% of the channel. A 150% enhancement in the Nusselt number can be observed when the cylinder's rotational speed increases. The study also found that increasing angular velocity and Darcy number positively affects the Nusselt number. The research aims to understand the interplay of factors affecting convective heat transfer and fluid dynamics and offer practical implications for optimizing heat transfer systems. The novelty of this study lies in its thorough examination of convective heat transfer in a rotating cylinder-shaped porous cavity, focusing on factors like Darcy number, Hartmann number, rotation speed, and pit geometry. It provides insights for optimizing heat transfer efficiency in complex cavities and explores the impact of external magnetic fields and fluid dynamics. The study investigates the influence of parameters like Hartmann number, angular speed, and Darcy number on thermal and flow characteristics in a porous cavity filled with $\text{Fe}_3\text{O}_4/\text{MWCNT}$ -water nanofluid, highlighting their importance for optimizing heat transfer efficiency.

ARTICLE HISTORY

Received 22 December 2023
Revised 15 April 2024
Accepted 17 April 2024

KEYWORDS

Undulation cavity; FEM; porous media; MHD; entropy; natural convection; heat transfer



Thermal radiation impact of MHD nanofluid natural convection in a special cavity

Chikr Djaoutsi Zineb¹ · Fares Redouane¹ · Aissani Abdelkader² · Rachid Hidki³ · Naim Houcine⁴

Received: 18 July 2024 / Accepted: 1 October 2024
© Akadémiai Kiadó, Budapest, Hungary 2024

Abstract

This study investigates the convection phenomena of magnetohydrodynamic (MHD) nanofluids, focusing on the effects of thermal radiation and nanoparticle shape factors. Using the finite element method, we analyze various parameters, including radiation parameter, nanoparticle volume fractions, Hartmann number, and Rayleigh number. Our findings reveal that the presence of nanoparticles significantly enhances both average and local Nusselt numbers compared to other nanoparticle shapes. This research contributes to understanding heat transfer mechanisms in nanofluid systems, with implications for engineering applications. The novelty of this study is to analyze the impact of thermal radiation, MHD, and nanoparticles on natural convection in a cavity. It provides insights into the interplay of parameters and heat transfer processes. The control-volume finite element method allows detailed examination of flow and heat transfer characteristics.

Keywords Nanofluid · Natural convection · Nanoparticles · Thermal radiation · Nusselt number

List of Symbols

C_p	Strength of magnetic field Specific heat ($Jkg^{-1}K^{-1}$)
Ha	Hartman number
K	Permeability m^2
Nu	Nusselt number
Pr	Prandtl number
T	Temperature (K)
p	Pressure term
Ra	Rayleigh number
m	Shape factor of nanoparticles
u, v	Velocity components (m/s)
q_{rad}	Radiative heat flux
B_0	Magnetic field intensity
F_c	Forcheimer coefficient
k	Thermal conductivity ($W.m^{-1}.K^{-1}$)

R	Radiation parameter
Gr	Grashof number
P	The dimensionless fluid pressure
x, y	Coordinates (m)
F_c	Forchheimer coefficient
k_{nf}^*	Mean absorption coefficient

Greek letters

α	Effective thermal diffusivity of porous medium and fluid ($m^2.s^{-1}$)
σ	Electrical conductivity ($\Omega. M$)
β	Volumetric thermal expansion coefficient of the fluid (K^{-1})
θ	Non-dimensional temperature
λ	Thermal expansion coefficient (K^{-1})
ρ	Density ($kg.m^{-3}$)
ϵ	Porosity
ϕ	Volume fraction
μ	Dynamic viscosity ($kg.m^{-1}.s^{-1}$)
σ^*	Stefan–Boltzmann constant

Subscripts

avg	Average
c	Cold
F	Fluid
h	Hot
loc	Local
nf	Nanofluid

✉ Fares Redouane
redouane.fares@univ-relizane.dz

¹ LGIDD, Department of Physics, Relizane's University, 48000 Relizane, Algeria

² University of Sciences and Technology of Oran (USTO-MB), 31000 Oran, Algeria

³ Laboratory of Inorganic Materials for Sustainable Energy Technologies (LIMSET), University Mohammed VI Polytechnic, Ben Guerir 43150, Morocco

⁴ Laboratory for Analysis and Applications of Radiation (LAAR), University of Sciences and Technology of Oran (USTO-MB), 31000 Oran, Algeria

Article

Mathematical Entropy Analysis of Natural Convection of MWCNT—Fe₃O₄/Water Hybrid Nanofluid with Parallel Magnetic Field via Galerkin Finite Element Process

Djelloul Ghali ¹, Fares Redouane ², Roubi Abdelhak ³, Amine Belhadj Mohammed ², Chikr Djaouti Zineb ², Wasim Jamshed ^{4,*}, Mohamed R. Eid ^{5,6}, Sayed M. Eldin ⁷, Awad Musa ^{8,9} and Nor Ain Azeany Mohd Nasir ¹⁰

- Laboratoire de Physique Plasmas des Matériaux Conducteur et Leur Applications—LPMCA, University of Sciences and Technology of Oran (USTO-MB), Bir El Djir 31000, Algeria
- Laboratory of Industrial Engineering and Sustainable Development—LGIED, Ahmed Zabana University, Relizane 48000, Algeria
- Laboratory of Industrial Technology Study and Research, University Saad Dahlab, Bldg 09000, Algiers
- Department of Mathematics, Capital University of Science and Technology (CUST), Islamabad 44000, Pakistan
- Department of Mathematics, Faculty of Science, New Valley University, Al-Kharga 72511, Al-Wadi Al-Gadid, Egypt
- Department of Mathematics, Faculty of Science, Northern Border University, Azar 1321, Saudi Arabia
- Center of Research, Faculty of Engineering, Future University in Egypt, New Cairo 11835, Egypt
- Department of Physics, College of Science and Humanities in Al-Qadisiyah, Prince Sultan bin Abdulaziz University, Al-Qadisiyah 11912, Saudi Arabia
- Department of Physics, College of Science, Sudan University of Science and Technology, Khartoum 11115, Sudan
- Department of Mathematics, Centre for Defense Foundation Studies, Universiti Pertahanan Nasional Malaysia, Kuala Lumpur 57000, Malaysia

* Correspondence: wasim8@hotmail.com

Abstract: Heat transfer in a symmetrical cavity with two semi-cylinders was explored in this study. Several parameters, such as $(10^3 \leq Ra \leq 10^4)$, $(10^{-3} \leq Da \leq 10^{-2})$, $(0.02 \leq \phi \leq 0.08)$, $(0.2 \leq \epsilon \leq 0.8)$, and $(0 \leq Ha \leq 100)$ were selected and evaluated in this research. The outcome of the magnetic field and the temperature gradient on the nanofluid flow is considered. The geometric model is therefore described using a symmetry technique. The flow issue for the governing equations has been solved using the Galerkin finite element method (G-FEM), and these solutions are presented in dimensionless forms. The equations for energy, motion, and continuity were solved using the application of the COMSOL Multiphysics® software computer package. According to the results, there is a difference in the occurrence of the magnetic parameter and an increase in heat transmission when the right wall is recessed inward. The heat transmission is also significantly reduced when the right wall is exposed to the outside. The number of Nusselt goes directly proportional to the number of nanofluids in the environment. In contrast, all porous media with low Darcy and Hartmann numbers, high porosity, and low volume fraction have high Nusselt numbers. It is found that double streamlines for the hot side and single cooling for Darcy, Rayleigh, and Hartmann numbers. A cold isotherm at various physical parameters is needed in the top cavity. Rayleigh's number and a solid volume fraction raise Darcy's number, increasing heat transmission inside the cavity and thermal entropy determines entropy components.

Keywords: hybrid nanofluid; symmetrical cavity; heat transfer; entropy; magnetic field; galerkin finite element method

1. Introduction

In recent years, experts have been intrigued by Magnetohydrodynamics (MHD) heat transfer due to its vast application in various industries. Boilers, cooling systems, thermal

Vehicle Engine Cooling System: Review Research

Fares Redouane^{1,*}, Chikr Djaouti Zineb¹, and Hidki Rachid²

- LGIDD, Department of Physics, Relizane's University, Relizane, B.P. 48000, Algeria
- Laboratory of Inorganic Materials for Sustainable Energy Technologies (LIMSET), Mohammed VI Polytechnic University (UMPV), Lot 660, Hay Moulay Rachid, 43150 Ben Guerir, Morocco

This study reveals that nano-refrigerants can improve the overall performance of these systems, particularly when used as nanoparticles in conjunction with a base refrigerant. The results show that nano refrigerants outperform base liquids in warm conductivity, and the size of the nanoparticles affects this conductivity. The thickness of nano-refrigerants shows a vertical pattern as the volume of particles increases, while it decreases with temperature increases. Traditional models, such as the Hamilton-Crosser and Einstein models, fail to accurately predict the warm conductivity and consistency of nanofluids when temperature is considered. Even a small amount of nanoparticles can significantly improve the base liquid's conductivity. The use of nano-fluids results in an improved convective intensity transfer coefficient for all volume concentrations of nanoparticles compared to water under different working conditions. This study delves into the characterization of nanofluids for vehicle engine cooling systems, focusing on their thermal properties and heat transfer capabilities. Through an analysis of thermal conductivity, heat transfer coefficients, viscosity, and nanoparticle size, the research aims to optimize the design and implementation of nanofluid-based cooling systems to enhance engine performance and fuel efficiency. By investigating the impact of different nanoparticles and concentrations on these properties, the study provides insights into the potential of nanofluids to improve cooling efficiency in automotive applications. The findings highlight the importance of understanding the relationship between nanoparticle characteristics and thermal properties for the effective utilization of nanofluids in vehicle cooling systems. By synthesizing findings from previous studies, this review aims to provide insights into the potential benefits of utilizing nanofluids in enhancing cooling efficiency and overall engine performance in automotive systems. The analysis underscores the importance of considering nanoparticle characteristics in optimizing nanofluid formulations for effective heat transfer in vehicle cooling systems.

KEYWORDS: Vehicle Engine Cooling, Nano-Refrigerants, Nanoparticles, Nanofluids, Cooling Systems.

CONTENTS

1. Introduction	625
2. Nanofluids for Improving Thermal Conductivity	626
3. Utilization of Nanofluids as Coolants in a Vehicle Radiator	628
4. The Effect of the Nanoparticle's Variable Volume Fraction	629
5. Performance of Certain Nanofluids Based on Metal Oxide Nanoparticles	631
5.1. Zinc Oxide Nano-Fluids	631
5.2. Aluminum Oxide Nano-Fluids	633
5.3. Copper and Copper Oxide Nanofluids	633
5.4. Iron Oxide Nano-Fluids	633
5.5. Titanium Dioxide Nano-Fluids	634
5.6. Silicon Dioxide Nano-Fluids	634
6. Conclusion	634
Acknowledgments	634
References and Notes	635

* Author to whom correspondence should be addressed.
Email: faresredouane@univ-relizane.dz
Received: 20 January 2024
Accepted: 16 May 2024

J. Nanofluids 2024, Vol. 13, No. 3 2169-432X/2024/3/625\$013 doi:10.1166/jon.2024.2172 625

Symmetry 2022, 14, 2312. <https://doi.org/10.3390/sym14112312> <https://www.mdpi.com/journal/symmetry>



Numerical Heat Transfer, Part B: Fundamentals

An International Journal of Computation and Methodology

ISSN: (Print) (Online) Journal homepage: www.tandfonline.com/journals/unhb20

Numerical investigation of MHD nanofluid natural convection in a cavity considering thermal radiation and nanoparticle shape factor effects using CVFEM

Abdelkader Aissani, Fares Redouane, Rachid Hidki, Zineb Chikr Djaouti & Omar Henini

To cite this article: Abdelkader Aissani, Fares Redouane, Rachid Hidki, Zineb Chikr Djaouti & Omar Henini (12 Mar 2025): Numerical investigation of MHD nanofluid natural convection in a cavity considering thermal radiation and nanoparticle shape factor effects using CVFEM, Numerical Heat Transfer, Part B: Fundamentals, DOI: 10.1080/10407790.2024.2391495

To link to this article: <https://doi.org/10.1080/10407790.2024.2391495>

Published online: 12 Mar 2025.

Submit your article to this journal

View related articles

View Crossmark data



Results in Engineering 28 (2025) 107032

Contents lists available at ScienceDirect

Results in Engineering

journal homepage: www.sciencedirect.com/journal/results-in-engineering

Research paper

Exploring natural convection dynamics of TiO₂-water nanofluid in complex geometries via finite volume methodology

Abdelmadjid Lounis^{1,*}, Fares Redouane^{1,2}, Zineb Chikr Djaouti^{1,3,4}, Wasim Jamshed^{5,6}, Mohamed R. Eid⁷, Bindar M. Fadil⁸, Kamel Guedri⁹, Syed M. Hussain¹⁰

- Sidal Research, University of Khemis Miliana, B.P. 46000 Khemis Miliana, Algeria
- LGIDD, Department of Physics, Relizane's University, B.P. 48000 Relizane, Algeria
- Department of Mathematics, Capital University of Science and Technology (CUST), Islamabad 44000, Pakistan
- Department of Computer Engineering, Beirut University, Beirut, Lebanon, Turkey
- Center for Scientific Research and Development, Northern Border University, Azar 13213, Saudi Arabia
- Department of Mathematics, Faculty of Science, New Valley University, Al-Kharga, Al-Badai Al-Gadid 72511, Egypt
- Mechanical Engineering Department, College of Engineering and Architecture, Umm Al-Qura University, P. O. Box 5555, Makkah 21555, Saudi Arabia
- Department of Mathematics, Faculty of Science, Umm Al-Qura University, Makkah 21555, Saudi Arabia

ARTICLE INFO

Keywords:
Natural convection
Nanofluids
Thermal radiation
TiO₂ water geometry
Finite volume methodology

ABSTRACT

As the demand for efficient heat transfer systems increases, nanofluids enhanced with nanoparticles have become viable options due to their exceptional thermal properties. This paper investigates the thermal and hydrodynamic behavior of TiO₂-water nanofluids within an isosceles trapezoidal cavity, focusing on the impact of various physical and geometric parameters on natural convection. The research employs a numerical approach based on the finite volume method (FVM) to analyze the equations governing fluid flow and heat transfer. By varying the Rayleigh number, the fractional size of nanoparticles, and the configuration of the heat sources, the authors aim to elucidate the complex interactions that govern heat transfer in such geometries. This study combines several physical influences, such as thermal radiation, magnetohydrodynamics, and nanofluid behavior, to provide detailed information about flow patterns and heat transfer features. Its originality stands out by examining how these factors work together in complex shapes, which are not often covered in current research. This contribution to the improvement of thermal management systems in engineering, it is useful for intricate geometries because it divides the computational area into control volumes, applies conservation laws, and solves algebraic equations step by step to determine temperature, velocity, and flow parameters. The study reveals an improvement in convective heat transfer efficiency. The Rayleigh number also affects cooling performance, with an increase in the average temperature within the cavity indicating enhanced cooling. Flow patterns within the cavity show larger recirculation cells as the Hartmann number increases. Thermal radiation effects increase heat transfer rates by up to 15% at higher parameters. The research identifies optimal conditions for maximizing heat transfer and outperforms traditional heat transfer fluids.

1. Introduction

Because of its many and extensive uses, the pressing need to explore nanofluids in closed systems like cavities and containers has attracted considerable attention. Extensive technology in renewable energy employ nanofluids to enhance heat transfer in electronic refrigeration systems. As a result, scientists use better computer simulations and precise imaging techniques to study how heat moves and to make systems work better without using traditional methods. Similar to other

recent research, several studies have demonstrated a significant improvement in thermal performance when compared to conventional fluids, making them an intriguing option for technical and industrial designs. For the first time, Choi and Eastman [1] used the word "nanofluid" in 1995 at the Argonne National Laboratory in the United States. Scientific and engineering professionals have been interested in using nanofluids as potential coolants for the past two decades. Compared to conventional fluids, nanofluids have higher thermal conductivities because they are suspensions of particles with nanometric sizes. Actually, it involves dispersing nanometric-sized particles into commonly

* Corresponding author.
E-mail address: researcher702@yahoo.com (Z.C. Djaouti).

<https://doi.org/10.1016/j.rineng.2025.107032>

Received 5 July 2025; Received in revised form 19 August 2025; Accepted 31 August 2025

Available online 31 August 2025

2590-128X/© 2025 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

Full Terms & Conditions of access and use can be found at
<http://www.tandfonline.com/action/journalInformation?journalCode=unhb20>



Research paper

Exploring natural convection dynamics of TiO₂-water nanofluid in complex geometries via finite volume methodology

Abdelmadjid Lounis^a, Fares Redouane^b, Zineb Chikr Djaouti^{b,*}, Wasim Jamshed^{c,d},
 Mohamed R. Eid^{e,f}, Bandar M. Fadhl^g, Kamel Guedri^g, Syed M. Hussain^h

^a Djillali Bouaouana University of Khemis Miliana, B.P. 44000 Ain Defla, Algeria

^b LGIDD, Department of Physics, Reliance's University, R.P. 48000 Reliance, Algeria

^c Department of Mathematics, Capital University of Science and Technology (CUST), Islamabad 44000, Pakistan

^d Department of Computer Engineering, Biruni University, Topkapı, Istanbul Turkey

^e Center for Scientific Research and Entrepreneurship, Northern Border University, Arar 73213, Saudi Arabia

^f Department of Mathematics, Faculty of Science, New Valley University, Al-Kharga, Al-Wadi Al-Gadid 72511, Egypt

^g Mechanical Engineering Department, College of Engineering and Architecture, Umm Al-Qura University, P. O. Box 5555, Makkah 21955, Saudi Arabia

^h Department of Mathematics, Faculty of Science, Islamic University of Madinah, Madinah 42351, Saudi Arabia

ARTICLE INFO

Keywords:

Natural convection
 Nanofluids
 Thermal radiation
 TiO₂ wavy geometry
 Finite volume methodology

ABSTRACT

As the demand for efficient heat transfer systems increases, nanofluids enhanced with nanoparticles have become viable options due to their exceptional thermal properties. This paper investigates the thermal and hydrodynamic behavior of TiO₂-water nanofluids within an isosceles wavy rectangular cavity, focusing on the impact of various physical and geometric parameters on natural convection. The research employs a numerical approach based on the finite volume method (FVM) to analyze the equations governing fluid flow and heat transfer. By varying the Rayleigh numbers, the fractional size of nanoparticles, and the configuration of the heat source, the authors aim to elucidate the complex interactions that govern heat transfer in such geometries. This study combines several physical influences, such as thermal radiation, magnetohydrodynamics, and nanofluid behavior, to provide detailed information about flow patterns and heat transfer features. Its originality stands out by examining how these factors work together in complex shapes, which are not often covered in current research. This contributes to the improvement of thermal management systems in engineering. It is useful for intricate geometries because it divides the computational area into control volumes, applies conservation laws, and solves algebraic equations step by step to determine temperature, velocity, and flow parameters. The study reveals an improvement in convective heat transfer efficiency. The Rayleigh number also affects cooling performance, with an increase in the average temperature within the cavity indicating enhanced cooling. Flow patterns within the cavity show larger recirculation cells as the Hartmann number increases. Thermal radiation effects increase heat transfer rates by up to 15% at higher parameters. The research identifies optimal conditions for maximizing heat transfer and outperforms traditional heat transfer fluids.

1. Introduction

Because of its many and extensive uses, the pressing need to explore nanofluids in closed systems like cavities and containers has attracted considerable attention. Extensive technologies in renewable energy employ nanofluids to enhance heat transfer in electronic refrigeration systems. As a result, scientists use better computer simulations and precise imaging techniques to study how heat moves and to make systems work better without using traditional methods. Similar to other

recent research, several studies have demonstrated a significant improvement in thermal performance when compared to conventional fluids, making them an intriguing option for technical and industrial designs. For the first time, Choi and Eastman [1] used the word "nanofluid" in 1995 at the Argonne National Laboratory in the United States. Scientific and engineering professionals have been interested in using nanofluids as potential caloporters for the past two decades. Compared to conventional fluids, nanofluids have higher thermal conductivities because they are suspensions of particles with nanometric sizes. Actually, it involves dispersing nanometric-sized particles into commonly

* Corresponding author.

E-mail address: researcher702@yahoo.com (Z.C. Djaouti).

<https://doi.org/10.1016/j.rineng.2025.107032>

Received 5 July 2025; Received in revised form 19 August 2025; Accepted 31 August 2025

Available online 31 August 2025

2590-1230/© 2025 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).



