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Propulsion mechanics and Applications

(Propulsion mécanique et applications)

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Propulsion Mechanics

Summary

Course supplement

Recap. of propulsion mechanics

Propulsion Mechanics

CHAPTER I

GENERAL DEFINITIONS AND CHARACTERISTICS
PROPULSIVE SYSTEMS

I.1- CONCEPT OF PROPULSION

Propulsion (or reaction) engines are based on the principle of reaction discovered by Heron of Alexandria in the first century BCE, and formulated by Newton in the 17th century: for each action there corresponds a reaction of equal intensity, but directed in the opposite direction. The reactors are therefore only devices intended to eject gases at high speed, so that it results in a "thrust" which is transmitted to the aircraft and forces it to move in the opposite direction to the direction according to which it is exercised. The propulsion mechanism does not require ambient air as a medium but it does involve the presence of at least two bodies. Indeed, for land vehicles the reaction (propelling effect) is exerted by a fixed support which is the ground. While for space vehicles the reaction is due to a mobile support which is the ambient air and gases ejected in the case of planes, and only the gases ejected in the case of rockets in space (vacuum). Propulsion therefore results in a transfer of momentum between the two bodies. To clarify this point, we consider the case of an airplane in horizontal and uniform flight. The 2nd Newton's law gives us:

$$\sum \mathbf{F_{ext}} = \mathbf{m.\gamma}$$

Thus, the horizontal component of the resultant force applied to the aircraft is zero. Also, it is clear that to propel the plane we must overcome drag (resistance opposed by the movement of a body in the air). One way to cancel out the effect of drag is to mount a propeller on the plane. The force produced by the propeller is sufficient to compensate for the effect of this resistance and push the aircraft upstream (fig.I.1).

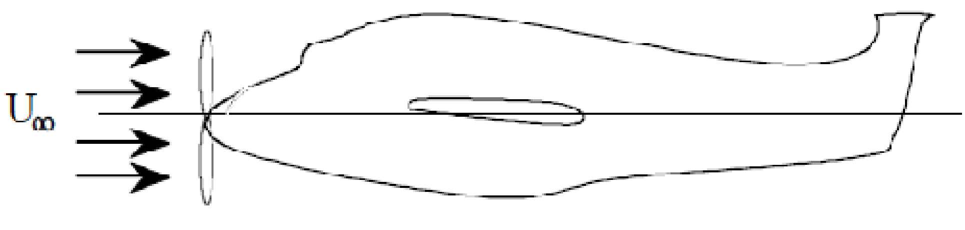

Figure I.1- Propeller propulsion.

The propeller can develop the previous force by assigning momentum to the ambient air surrounding the system. Therefore the condition of uniformity of motion implies that the variations in horizontal momentum, due to propeller thrust and drag forces, must be equal and opposite.

So although the air was disturbed by the passage of the plane, its momentum remained constant. This is illustrated by the example in figure (I.2) below:

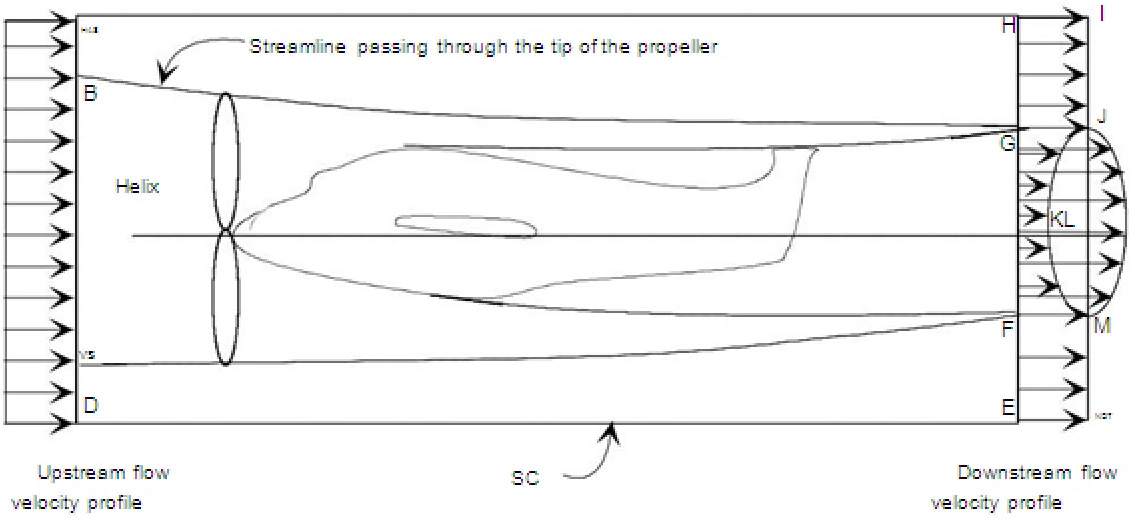

Figure I.2- Illustration of the conservation of momentum before and after the passage of the plane.

Considering the control surface (SC) the speed profile which is uniform upstream of the flow becomes distorted downstream because of:

- The mounting of the propeller on the plane and its communication to the fluid a quantity of movement, hence the excess speed in the interval GF (schematized by the arc JLM);

- Air resistance (due to the shape of the plane and friction on the exterior surface) which causes a speed deficit in an interval which is approximately equal to GF (schematized by the arc JKM).

So, by way of illustration, we see that the two previous deformations compensate for each other and the momentum is conserved before and after the passage of the plane.

We will now define the different propulsion systems:

I.2- DIFFERENT TYPES OF JET ENGINES

I.2.1- PISTON ENGINE

These engines are still used in light aircraft and in the relatively low power range (150 - 400 hp). They are economically cheaper (construction cost and fuel consumption) and their maintenance is easy. The propeller is driven by a piston engine. It comprises, depending on the case, from 2 to 5 blades whose section is identical to that of an airplane wing. When the engine turns, it drives the propeller which "screws into the air" and draws in a large mass of air at the front and pushes it backwards.

In this type of engine, the classic four strokes - intake, compression, explosion, exhaust -take place one after the other in the same place, the cylinder, whose volume and communication ports vary during the cycle.

These engines are more powerful and lighter than their terrestrial analogues. Therefore, wall thicknesses and part dimensions are reduced and construction materials are alloys and other special metals.

I.2.2- ROCKET ENGINE

It is the simplest and fastest engine (for speeds > 1500 km/h). The combustion gases are ejected at high speed through the nozzle (fig.I.3). Its advantage is that it can operate indifferently in the atmosphere and in space (vacuum), since it carries with it the oxidizer and fuel necessary for the generation of gases.

For all other types of jet engines, it is the air which plays the role of oxidizer, which allows you to carry only fuel (generally kerosene) on board the vehicle.

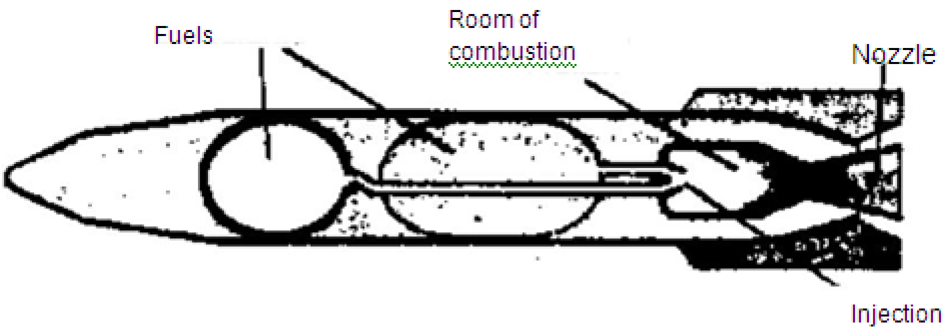

Figure I.3- Rocket engine.

I.2.3- RAMJET

This engine (fig.I.4) can only operate with the air + fuel system above 1000 km/h (which requires adding another engine to the aircraft to reach the speed necessary for the operation of the ramjet). The ramjet is lighter than other engines since it does not have a turbocharger. Also, the effect of constraints linked to the thermal resistance of the turbine blades disappears. Such a type of engine is particularly well suited to hypersonic speeds, where it competes (in the atmosphere) with the rocket engine, if a high speed guarantees sufficient air pressure.

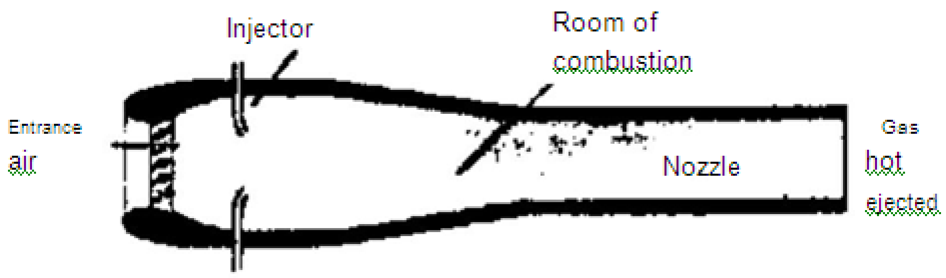

Figure I.4- Ramjet.

I.2.4- PULSOREACTOR

When you want to fly at subsonic speeds with a ramjet, the air pressure entering the combustion chamber is no longer sufficient. Also, the researchers have imagined to sequentially close the air inlet, to admit only "puffs", in order to ensure, momentarily at least, the pressure necessary for the combustion of the fuel. They had invented the pulse-reactor (fig.1.5).

However, the latter has the same disadvantage as the ramjet, namely that it can only operate from a few speed. Also, its efficiency is low, it exhibits vibrations and its noise level is quite high.

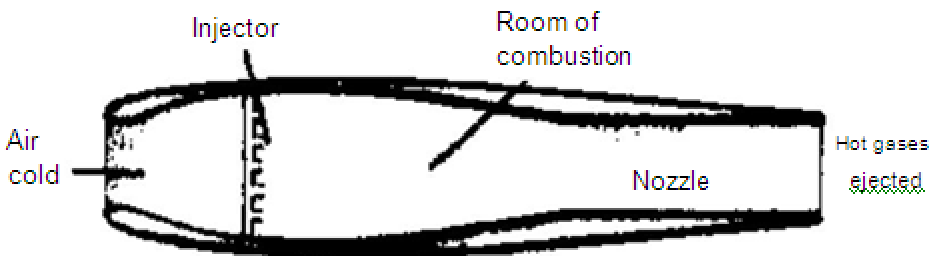

Figure I.5- Pulse-reactor.

I.2.5- PROPULSIVE TURBOENGINE

The assembly includes a compressor, often of the axial type, a combustion chamber and a turbine, mounted on the same shaft as that of the compressor, which ensures the expansion of the burnt gases. The latter being at high temperature can be used to reproduce power in a direct (nozzle) or indirect (propeller, other mechanical systems) way.

These types of engines can produce sufficient thrust for airplanes to take off.

Their thermodynamic cycle is modified, especially at high altitude, because, on the one hand, the ambient air becomes very cold; and on the other hand, the increase in the overall compression ratio (the dynamic pressure at the inlet no longer retains its terrestrial value).

This category of engines is subdivided into two parts: land and aeronautical propulsion turbine engines.

I.2.6- LAND PROPULSION TURBOENGINE

It includes the same main elements (compressor, combustion chamber and turbine) and its role is to drive land vehicles (cars, boats); thus certain modifications are introduced to meet particular specifications. For example, in the case of cars, the axial compressor is replaced by a centrifugal compressor in order to reduce the volume of the propulsion system (fig.I.6).

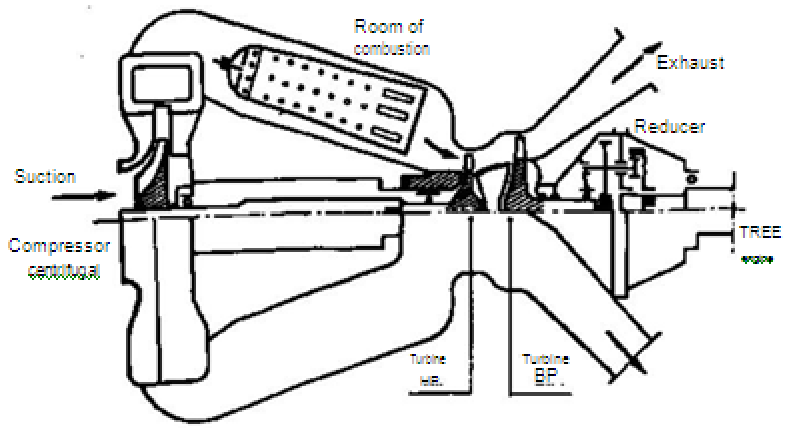

Figure I.6- Turbo motor for car.

I.2.7- AERONAUTICAL PROPULSION TURBOENGINE

We also find the three main elements mentioned above and in addition a nozzle which directly produces thrust by expanding the hot gases. There are several versions of these propulsion systems, we distinguish:

I.2.8- TURBOJET

To solve the problem of the ramjet (as well as the pulsejet) researchers have imagined other types of engines capable of creating sufficient air pressure themselves, thanks to a compressor driven by a turbine, itself actuated by the flow of burnt gases ejected by the nozzle.
This is the well-known principle of the turbojet (fig.I.7), valid for speeds between 600 and 2200 km/h. After partial expansion in the turbine, the ejected gases complete their expansion during their ejection (and not exhaust) through the nozzle, thus releasing the pressure energy which determines the thrust of the engine.

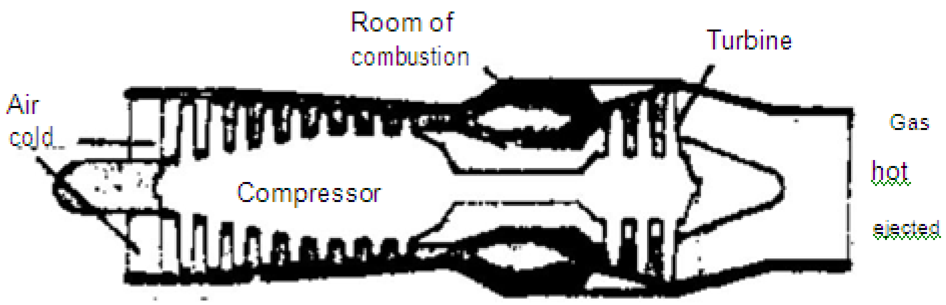

Figure I.7- Turbo jet.

I.2.9- DOUBLE FLOW TURBOJET

It is a variant of the turbojet engine very common throughout the world. In this case the compressor is split into two elements: the high pressure element, which supplies the combustion chamber, and the low pressure element, which sends air to mix directly into the combustion chamber. Combustion gas nozzle (fig.I.8). The low pressure element ("cold body", as opposed to the "hot body" which is the rest of the reactor) is a large diameter compressor, in other words a "blower" which surrounds the reactor. This arrangement increases the efficiency, which is all the better as the "dilution rate" (cold air/hot air quantity ratio) is higher, up to an optimum.

Dual flow is suitable for moderate flight Mach numbers (ÿ 0.8) and allows both lower fuel consumption and reduced noise.

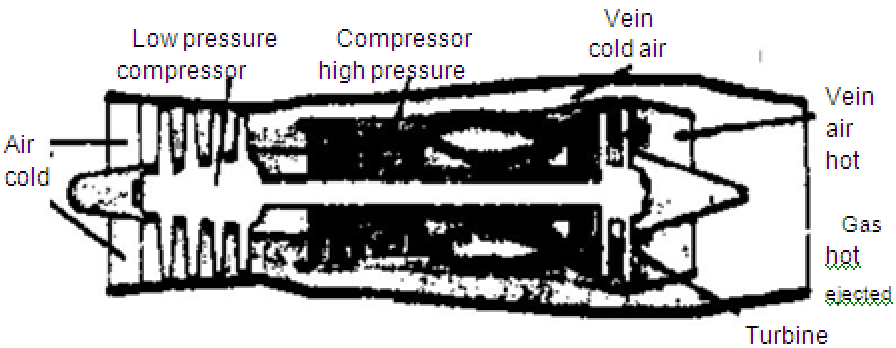

Figure I.8- Double flow turbojet.

I.2.10- DOUBLE BODY TURBOJET

To increase the compression ratio of the compressor, it is practically impossible to increase the number of stages beyond a dozen (for aerodynamic reasons, the compression ratio of the last stages is low). This motor is then made up of two axial compressors extending from one another but rotating at different speeds. Each is driven by a particular turbine, the two turbines being coaxial (fig.I.9).

Such a solution is essential if we want to obtain high compression ratios with acceptable yields at all speeds.

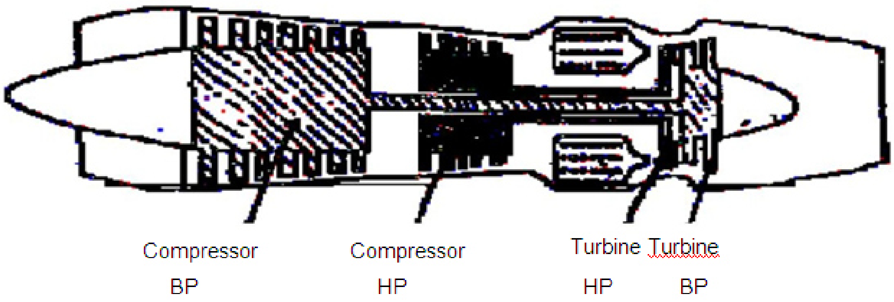

Figure I.9- Twin-body turbojet.

I.2.11- DOUBLE FLOW TURBOJET WITH AFTERCOMBUSTION

This type of turbojet is generally used in military aircraft. We sought to obtain additional thrust, even for a relatively short duration, in order to improve takeoff and allow acceleration of the aircraft during use. The gases leaving the turbine still contain oxygen, it is therefore possible to inject fuel again and burn it so as to increase the ejection speed and therefore the thrust in this part of the nozzle (fig.I.10). High temperatures can be reached (up to 1700°).

Post-combustion can provide 30 to 40% of the total thrust with a slightly longer nozzle, but at the cost of very high consumption, which can reach 5 times that of the reactor without reheat. It should therefore only be used for a very short time, not exceeding a few minutes.

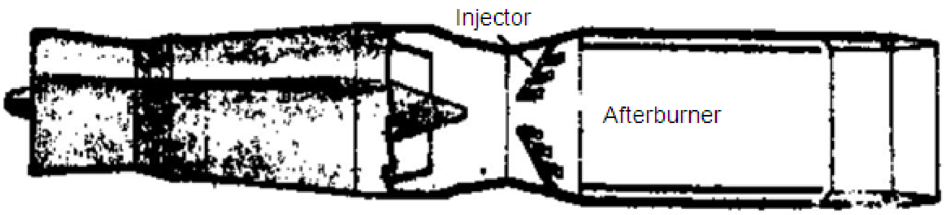

Figure I.10- Double flow turbojet with post-combustion (reheat).

I.2.12- TURBOPROPELLER

Until now, in all types of aeronautical propulsion engines mentioned, the propulsion energy was essentially due to the thrust provided by the ejected gases. But we can also recover most of the energy from the turbine to operate a propeller or a rotor: this is the case of the turboprop (fig.I.11 and I.12), operating at speeds of order of 480 to 640 km/h and powers exceeding 500 hp.

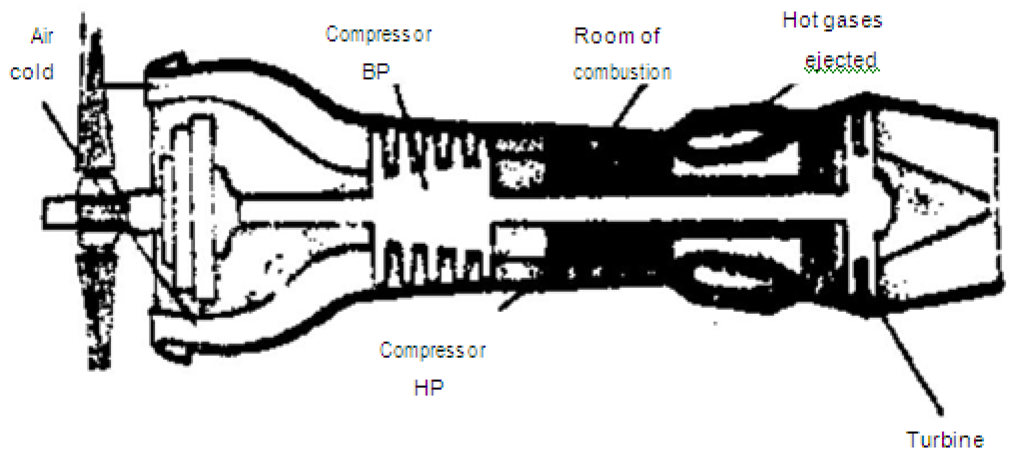

Figure I.11- Turboprop.

Thus the turboprop transforms the heat energy of a fuel (generally kerosene) mainly into mechanical energy and the remaining usable part is transformed into kinetic energy.

Mechanical energy ensures the rotation of a propeller which creates traction, the remaining kinetic energy is used in the form of additional thrust.

In this case, we say that the turboprop combines (in a way) the propulsion mode of the piston engine and that of the jet engine. The propeller is equipped with a speed reducer on the compressor shaft whose speed exceeds that of the propeller several times because the latter cannot exceed a higher rotation speed. Indeed, above a flight Mach number of 0.5, the efficiency of the propeller collapses, because the ends of the blades dissipate the energy received in the form of shock waves in all directions.

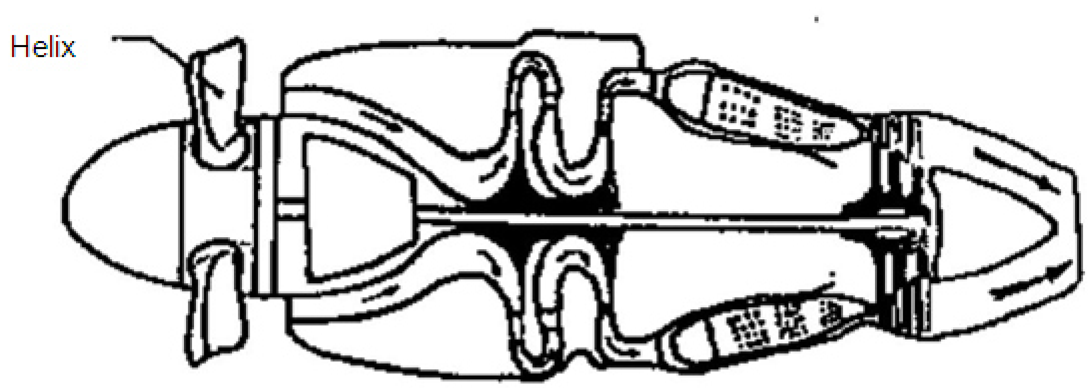

Figure I.12- Turboprop with centrifugal compressor.

CHAPTER II
GLOBAL THERMODYNAMIC STUDY OF A TURBOJET

A turbojet is characterized mainly by its thrust and its efficiency:

II.1- THRUST OF A TURBOJET ENGINE

The backward ejection of accelerated gases through the nozzle produces a force (thrust) that propels the system upstream. The equation for this thrust can be obtained by applying the momentum theorem to a control volume (vc), of section A, suitably chosen around the propulsion system (fig.II.1).

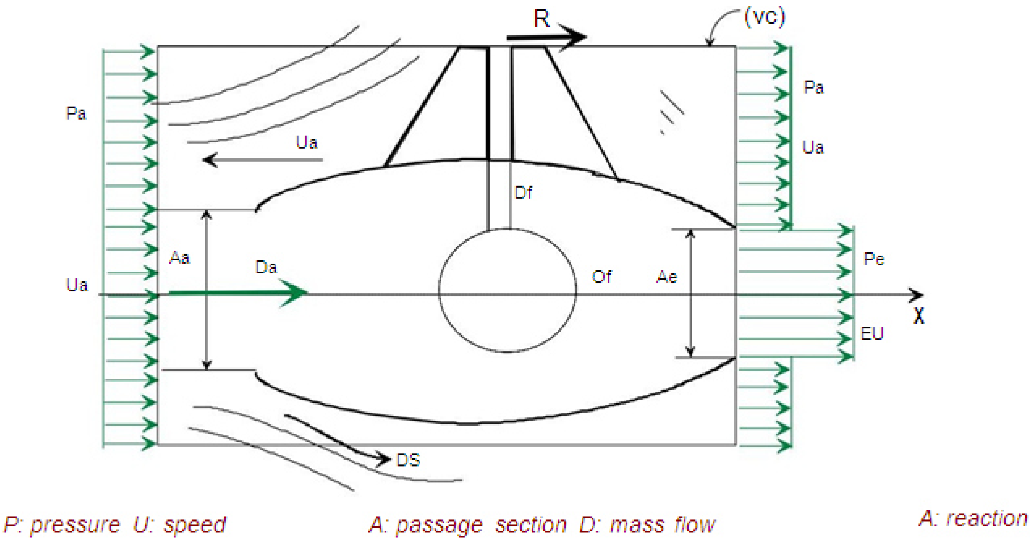

Figure II.1- Diagram of a turbojet in flight.

Let us apply the equations of continuity and quality of motion to the system by considering stationary flow.

Continuity equation:

$$\frac{\partial}{\partial t}\int_V \rho dV+\int_S \rho(\vec{q}.\vec{n})dS=0$$

S: control surface

V: control volume

ρ:volumic mass

q:speed

By projecting this equation along the X axis we will have:

$$\rho_a U_a A + \rho_e U_e A_e + \rho_a (A-A_e) U_a + D_s - D_f = 0$$

$$\Rightarrow D_e + D_s - D_f - \rho_a U_a A_e = 0 \qquad (a)$$

On the other hand we have, still according to the continuity equation:

$$D_e = D_a + D_f \qquad (2.1)$$

Momentum equation:

$$\frac{\partial}{\partial t}\int_V \rho\vec{q}dV+\int_S \rho\vec{q}(\vec{q}.\vec{n})dS=\sum\vec{F}_{ext}=\int_V \rho\vec{f}_v dV+\int_S \vec{P}dS+\vec{R}$$

F_{ext} : External forces

f_v :Volume or gravity forces

R: Reaction forces

P: Pressure forces

By projecting it along the X axis we will have:

$$\sum F_{ext} = P_a A - P_e A_e - P_a (A-A_e) + R = R + A_e (P_a - P_e) \qquad (b)$$

By combining (a), (b) and (2.1) we will have:

$$R = A_e (P_a - P_e) + U_e D_e - U_a D_a \qquad (2.2)$$

Hence the thrust of the turbojet:

$$|F| = A_e (P_a - P_e) + D_a [(1+f)U_e - U_a] \qquad (2.3)$$

$$f = D_f / D_a \qquad (2.4)$$

f being the mixing or richness ratio (fuel flow rate / intake air flow rate).

Remarks:

1- Equation (2.3) is established according to the turbojet diagram (fig.II.1) and depends on the control volume (vc) chosen. Indeed, if we choose a (vc) equal with the interior surface of the turbojet, i.e. which does not take into account the outgoing flow Ds then the thrust equation will be:

2- $$|F| = P_e A_e - P_a A + D_a [(1+f)U_e - U_a] \quad (2.3b)$$

The thrust due to the pressure difference is generally negligible compared to that due to the difference in speeds.

2- Equation (2.3) gives the expression for the thrust of a turbojet in flight with a speed uniform Ua. At the fixed point or at takeoff Ua is almost zero, in this case the thrust is called gross thrust, it is given by:

$$|F| = A_e (P_e - Pa) + D_a (1+f)U_e \quad (2.5)$$

The gross thrust is therefore greater than the thrust of the turbojet in flight. This is due to the variation in efficiency between the bell of the bench and the sleeve at the fixed point and the fact that Ma=0 makes the thrust dependent only on the ejection speed. On the other hand, the altitude negatively influences the thrust due to the fact that the turbojet in flight stirs a quantity of air less important.

3- When Pe = Pa, we say that the ejection nozzle is "adapted". In this case the turbojet no longer benefits from the pressure surge, but the expansion being almost isentropic; its usable rate is maximum.

4- When Pe = Pa, we say that the ejection nozzle is "adapted". In this case the turbojet no longer benefits from the pressure surge, but the expansion being almost isentropic; its usable rate is maximum.

5- Equation (2.3) tells us that the thrust increases as much as Pe increases and Ua decreases. But the increase in Pe means that the relaxation is not maximum because the growth in Pe causes the decrease in Ue. So the increase in Pe at the expense of Ue. Also the decrease of Ua involves increasing the inlet section in order to suck in the quantity of air necessary to combustion.

6- In establishing the thrust equation, resistance was not taken into account aerodynamic (drag). Momentum theory is not capable of being used to further study the factors that affect thrust.

II.2- EFFICIENCY OF A TURBOJET ENGINE

We can generally define the efficiency of a propulsion device as the ratio between what we earn and what we spend. In propulsion mechanics we define several types of returns. In this paragraph we will present the expressions of the yields applicable to the turbojet and the ramjet. * In the expressions of the different efficiencies we neglect the term of thrust due to the pressure difference Ae (Pe-Pa) in front of the other terms in equation (2.3). In other words: the ejection nozzle is suitable.

II.2.1- THERMAL EFFICIENCY

It is defined as the rate of kinetic energy supplied to the turbojet (power kinetic or propulsive) compared to the rate of energy consumed by combustion (thermal power expended).

$$\eta_{th} = D_a\,[(1+f)U_e^2 - U_a^2]/2\,D_f\,Q_R \qquad (2.6)$$

QR being the heat of reaction or lower calorific value **(LCV)** of the fuel (for the Kerosene QR = 46.103 KJ/Kg)..

$$\eta_{th} = [(1+f)U_e^2 - U_a^2]/2\,f\,Q_R \qquad (2.7)$$

η_{th} itself includes the gain in gas temperature as well as the efficiency of the components (compressors, turbine, etc.). It generally ranges between 25% and 40%.

II.2.2- PROPULSIVE EFFICIENCY

It is defined as the rate of energy converted into propulsive energy (power useful) compared to the rate of kinetic energy supplied to the turbojet (propelling power).

$$\eta_p=2\, F.U_a \;/\; [(1+f)U_e^2 - U_a^2] \quad (2.8)$$

II.2.3- THERMOPROPULSIVE OR OVERALL EFFICIENCY

It is defined as the useful power compared to the thermal power expended.

$$\eta_{thp}=\eta_g=\eta_{th}.\,\eta_p= F.U_a \;/\; D_f.Q_R \quad (2.9)$$

The overall yield is generally between 20% and 35%.

* In the case where f <<1, we can then use the following approximate formulas:

$$\eta_{th} \approx (U_e^2 - U_a^2)/2\, f\, Q_R \quad (2.10)$$

$$\eta_p \approx [2.U_a(U_e-U_a)]/(U_e^2-U_a^2)=2/(1+U_e/U_a) \quad (2.11)$$

$$\eta_g \approx U_a\ (U_e-U_a)/f.Q_R \quad (2.12)$$

Or:

$$\eta_g \approx 2.\, \eta_{th}\, .(U_a/U_e)/[1+(U_a/U_e)] \quad (2.13)$$

Remarks:

1- The relation (2.13) tells us that ÿg strongly depends on the speed ratio **Ua/Ue**.

2- If **Ua = Ue** then formula η_{th} = 0 and η_p = 1. Or F = 0 so there is no thrust on this vehicle which is moving with speed Ua ,and therefore no air resistance which implies that the environment is empty. We therefore conclude that it is impossible to have η_p =**1** in atmosphere.

By combining the equations above we obtain the following relationship:

$$\eta_p \approx 2/[2+F/ (D_a\, U_e)] \quad (2.14)$$

If F is imposed then we have an interest in increasing the product Da Ua to have a η_p pupil. Hence a large flow of air sucked in and consequently a larger diameter of the blower. That is to say, the dilution rate (cold air/hot air ratio) increases. But we cannot increase the diameter of the blower indefinitely (size, weight, birds). However, Ua can be increased (generally Ua > 700 Km/h).

II.3- THERMODYNAMIC CYCLE

In this paragraph we consider the thermodynamic cycle of a turbojet simple flow in order to determine the thermodynamic aspect of this propulsion system.

II.3.1- DEFINITION OF A THERMODYNAMIC CYCLE

It is any energy process which uses, at varying temperatures, several forms of energy (thermal, mechanical, etc.) and converts them towards each other while exchanging part with the environment.

II.3.2- STUDY OF THE THEORETICAL CYCLE OF A TURBOJET IN FLIGHT

In all that follows, we will use the numbering indicated in figure II.2 below.

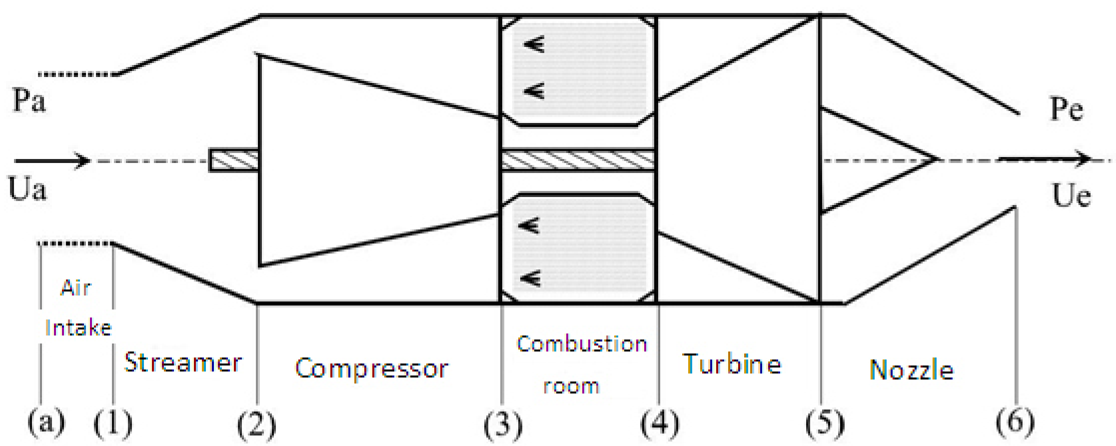

Figure II.2- Schematic representation of the elements of a turbojet.

During the flow of fluid through the turbojet, the air undergoes the following processes:

- From (a) to (1): the air which has the flight speed at point (a) reaches the inlet of the diffuser after a certain acceleration or deceleration (Ch.III, $III.1.1);
- From (1) to (2): the air speed decreases in the diffuser and in the piping system up to the compressor inlet;
- From (2) to (3): the air is compressed in the compressor;
- From (3) to (4): the air is heated by the combustion of fuel or fuel oil (generally kerosene) in the combustion chamber;
- From (4) to (5): the air is expanded in the turbine to produce the power necessary to drive the compressor.
- From (5) to (6): the air speed increases in the nozzle (another expansion) up to the ejection section.

To understand the thermodynamic cycle of the fluid in the turbojet we will consider at the beginning the ideal case (theoretical cycle) in which all transformations will be considered adiabatic and reversible except in the combustion chamber. In the latter, the transformation will be assumed as a simple heating without friction.

Also, it is assumed that the flow speed is negligible in sections (2), (3), (4), and (5). Finally, we consider that the fluid is an ideal gas (GP).
The thermodynamic cycle satisfying these conditions is represented by the TS (temperature-entropy) diagram in Figure II.3.

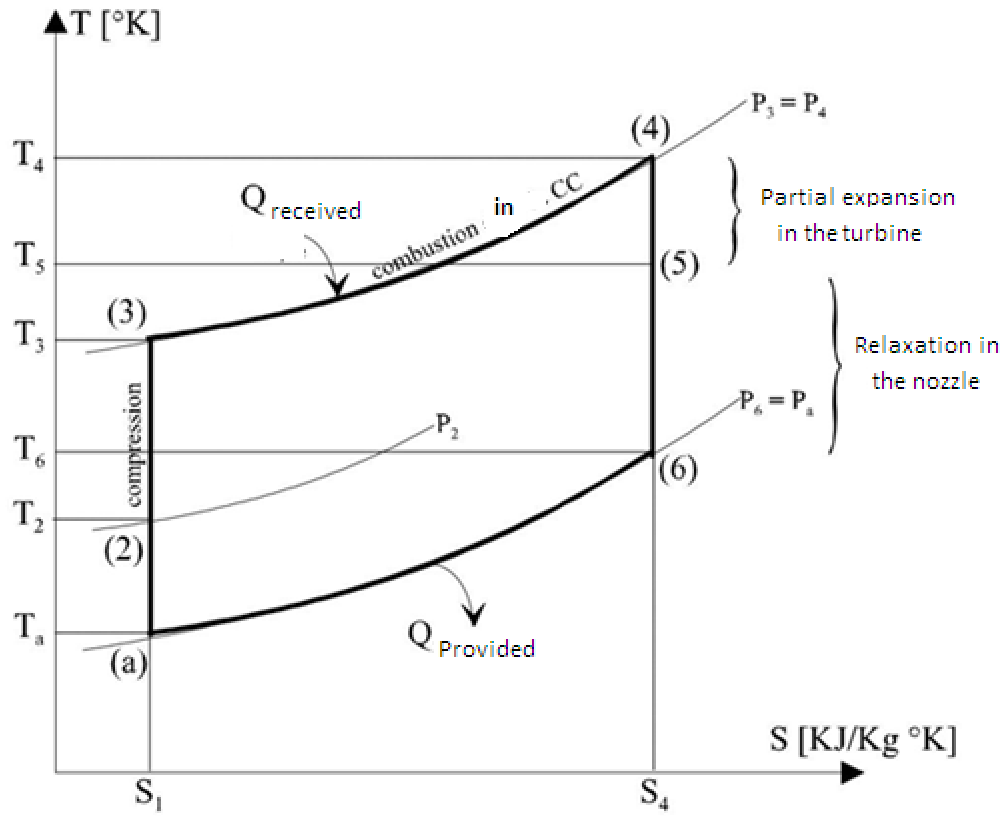

Figure II.3- TS diagram of the theoretical cycle of a turbojet.

We can see that the pressure increases from point (a) to point (1) then from (1) to (2) because of the deceleration of the air relative to the machine (cruising flight).

Since the velocity in (2) is assumed to be zero and the deceleration is isentropic, then P2 is the stopping pressure for states (a), (1) and (2). Likewise, T2 is the stopping temperature for these states. The power consumed in compressing the air from (2) to (3) must be provided by expanding the air from point (4) to point (5) in the turbine. Thus, if the mass flow rates of the compressor and the turbine are equal, we have:

$$\mathbf{h_3 - h_2 = h_4 - h_5}$$

h being the enthalpy of the fluid.

Also, if the specific heat Cp is constant we will have the following equality:

$$\mathbf{T_3 - T_2 = T_4 - T_5}$$

Finally, the decrease in enthalpy from (5) to (6) is proportional to the square of the escape velocity:

$$\Delta h_{56} = q_e^2$$

II.3.3- STUDY OF THE REAL CYCLE OF A TURBOJET IN FLIGHT

The differences between the real cycle and the theoretical cycle are:

- No element of the turbojet is truly reversible, but it is reasonable to assume they are adiabatic.
- The combustion chamber is not a simple stove and the composition of the fluid varies during combustion.
- The fluid speeds are not negligible in the different parts of the turbojet.
- The flow rates of the compressor and the turbine can be different because the fuel is added on the one hand, and the air can be sucked on the other hand between the two elements (for example to cool the turbine).

We thus represent (fig.II.4) the TS diagram of the real cycle of the turbojet.

The process begins with atmospheric air at pressure Pa and enthalpy ha.

Since the air is in relative motion to the machine (in flight), the stopping enthalpy for air h_0 a must be greater than the static enthalpy ha. Additionally, since there is no exchange of work and heat between states (a) and (2), the shutdown enthalpy must be the same for states (a), (1) and (2).

It is often acceptable to consider that the external deceleration of the air upstream of the turbojet is isentropic (unless a shock wave is formed somewhere in the previous path), hence the representation of states (a) and (1) on the same isentropic hence the equality of P0a and P_{01}.

Then, the air undergoes yet another deceleration from (1) to (2) but with friction. Hence the increase in entropy and the decrease in stopping pressure P_{02} compared to P_{01}

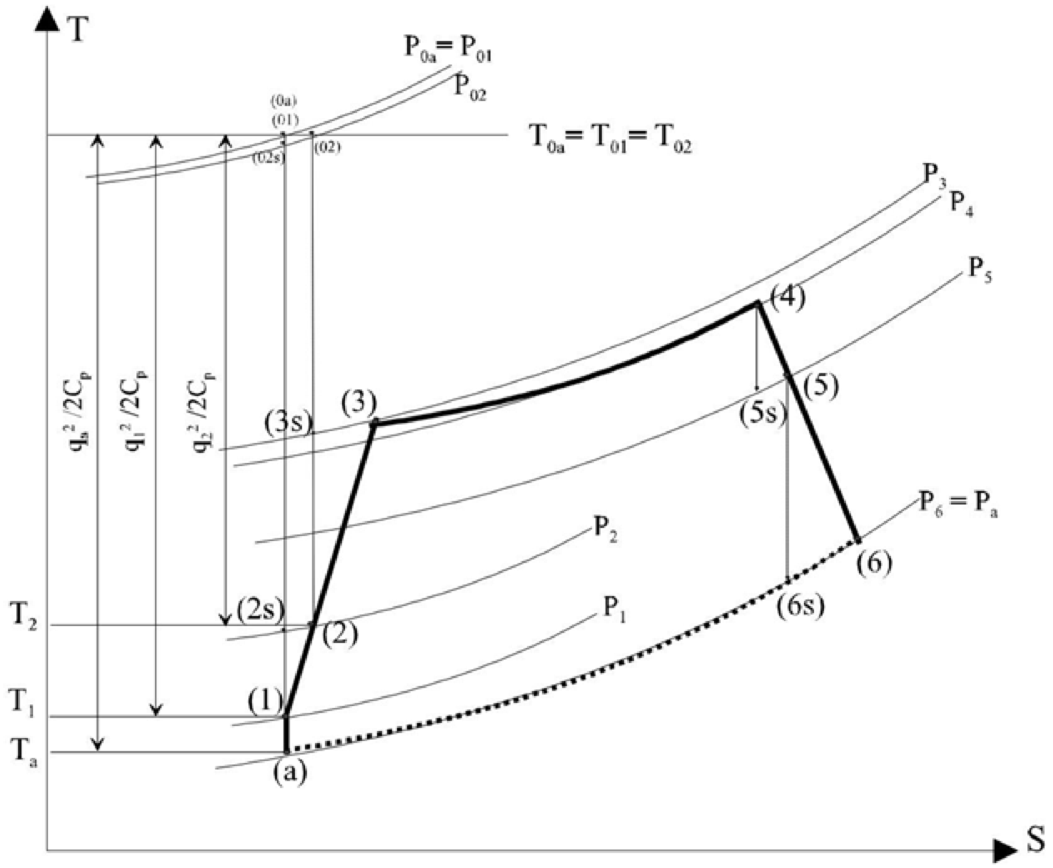

Figure II.4- TS diagram of the real cycle of a turbojet.

From state (2) to (3), the air is compressed with an increase in entropy which is due to the irreversibility of the compression process.

The state (3s) is defined as a state that is obtained if the air is compressed isentropic ally up to the pressure P_3 which exists at the compressor outlet. The rotor of the latter provides the fluid with a positive work W_{23} because $h_{03} > h_{02}$.

Between states (3) and (4), air mixes with the fuel and combustion takes place. Since the addition of fuel injected into the air does not considerably change the essential characteristics of the fluid, the thermodynamic evolution of the previous mixture is represented on the same diagram. The stopping pressure P_{04} must be lower than P_{03} because of the flow of the fluid with friction and the release of heat in the combustion chamber which causes an increase in entropy.

From state (4) to (5), the fluid expands through the turbine. The process also takes place with an increase in entropy which is due to the irreversibility of the relaxation. The fluid transfers work to the turbine rotor and $h_{05} < h_{04}$, therefore $W_{45} < 0$.

Finally, the fluid further expands from state (5) until it exits the turbojet in state (6). The process also takes place with friction.

Since there is no work or heat exchange in the ejection nozzle, the shutdown enthalpy is the same for states (5) and (6). The outlet pressure P_6 is generally equal to the atmospheric pressure Pa. If the gas flow in the ejection section is supersonic, the pressure P6 may be different from Pa.

Assuming that compression and expansion are almost adiabatic processes we can reasonably estimate the performance of the turbojet. Thus, it is useful to define adiabatic efficiencies for the turbojet elements as follows:

$$\eta_d = \frac{h_{02s} - h_a}{h_{02} - h_a} \qquad (2.15) \qquad\qquad \eta_c = \frac{h_{03s} - h_{02}}{h_{03} - h_{02}} \qquad (2.16)$$

$$\eta_T = \frac{h_{04} - h_{05}}{h_{04} - h_{05s}} \qquad (2.17) \qquad\qquad \eta_R = \frac{h_{05} - h_6}{h_{05} - h_{6s}} \qquad (2.18)$$

In addition to these 4 adiabatic efficiencies, a 5th efficiency for the combustion chamber noted **Y_{ch}** is often used. In general, previous returns fall within the following ranges:

$0{,}70 < \eta_d < 0{,}90$………………………… Input diffuser (strongly depends on M_a)

$0{,}85 < \eta_c < 0{,}90$……………………….. Compressor

$0{,}97 < \eta_{Ch} < 0{,}99$……………………… Combustion room

$0{,}90 < \eta_T < 0{,}95$……………………… Turbine

$0{,}95 < \eta_R < 0{,}98$…………………… Nozzle

We will now determine two important parameters characterizing a turbojet: the thrust per unit of mass flow of the air passing through the turbojet as well as the specific fuel consumption per unit of thrust.

* In the case where Pe = Pa , the thrust per unit mass flow is given by:

$$F/D_a = (1+f)q_e - q_a \qquad (2.19)$$

This formula provides us with a relationship between the thrust and the dimensioning of the turbojet.

* The specific fuel consumption per unit of thrust is given by:

$$C_s = D_f/F = f/[(1+f)q_e - q_a] \qquad (2.20)$$

To calculate these two important parameters it is first necessary to determine qe and f. Using the Saint-Venant equation of energy conservation in the exhaust nozzle, the gas velocity is calculated by:

$$(W+Q)_{56} = h_{06} - h_{05} = h_6 - h_5 + 0,5(q_6^2 - q_5^2)$$

By combining with the relation (2.18) we will have

$$q_e^2 = 2\,\eta_R\,(h_{05} - h_{65})$$

And if the fluid properties are constant, we obtain the following relation:

$$q_e^2 = 2C_p\,T_{05}\,\eta_R\,[1-(p_6/P_{05})^{\gamma-1/\gamma}] \qquad (2.21)$$

To relate the previous pressure ratio and the ejection speed to the flight speed, the ambient conditions P_a and T_a, the efficiencies of the turbojet elements, the temperature maximum in the machine T04 and the pressure ratio for the compressor P_{03}/P_{02}, it is It is appropriate to use the following identity:

$$P_{05}/P_6 = (P_{05}/P_{04})(P_{04}/P_{03})(P_{03}/P_{02})(P_{02}/P_a)(P_a/P_6) \quad (2.22)$$

As the second and fifth gears are close to unity (load losses negligible in the combustion chamber and the nozzle is suitable), we will determine the first and fourth ratio in (2.22). For that:

* **Diffuser:** The energy equation gives us according to the cycle in figure (II.4):

$$(W+Q)_{a1} = h_{01} - h_{0a} \Rightarrow T_{01} = T_{0a}$$

$$(W+Q)_{12} = h_{02} - h_{01} \Rightarrow T_{02} = T_{01}$$

So: $$T_a + \frac{q_a^2}{2c_p} = T_1 + \frac{q_1^2}{2c_p} = T_2 + \frac{q_2^2}{2c_p}$$

$$\frac{T_{01}}{T_a} = \frac{T_{02}}{T_a} = 1 + \frac{\gamma - 1}{2} M_a^2 \qquad (2.23)$$

Combining (2.15) and (2.23) we will have:

$$\frac{T_{02s}}{T_a} = 1 + \eta_d \frac{\gamma - 1}{2} M_a^2 \qquad (2.24)$$

And using the isentropic relationship between states (a) and (02s) we obtain the ratio of pressures between these two states:

$$\frac{P_{02}}{P_a} = \left[1 + \eta_d \frac{\gamma-1}{2} M_a^2\right]^{\frac{\gamma}{\gamma-1}} \qquad (2.25)$$

* **Compressor:** Between the compressor inlet and outlet the energy equation gives:

$$W_c = h_{03} - h_{02} \qquad (2.26a)$$

By combining with relation (2.16) we will have:

$$W_c = \frac{c_p T_{02}}{\eta_c}\left[\left(\frac{P_{03}}{P_{02}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \qquad (2.26)$$

* **Turbine:** Likewise for the turbine we have:

$$W_T = h_{04} - h_{05} \qquad (2.27a)$$

By combining with relation (2.17) we will have:

$$W_T = c_p T_{04} \eta_T \left[1 - \left(\frac{P_{05}}{P_{04}}\right)^{\frac{\gamma-1}{\gamma}}\right] \qquad (2.27)$$

If the mass flow rates passing through the compressor and the turbine are equal and the losses mechanical between the latter are negligible, we will then have the following relationship:

$$\frac{P_{05}}{P_{04}}=\left\{1-\frac{T_a}{\eta_c\eta_T T_{04}}\left(1+\frac{\gamma-1}{2}M_a^2\right)\left[\left(\frac{P_{03}}{P_{02}}\right)^{\frac{\gamma-1}{\gamma}}-1\right]\right\}^{\frac{\gamma}{\gamma-1}} \tag{2.28}$$

We can therefore calculate, according to the formulas above, the ratio P_{05}/P_{06} . It then remains to calculate T_{05} in relation (2.21) to finally determine **qe** . Combining (2.27a) and (2.27) we will have:

$$T_{05}=T_{04}\left\{1-\eta_T\left[1-\left(\frac{P_{05}}{P_{04}}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\} \tag{2.29}$$

*** Combustion chamber:**

The mixing ratio f can be expressed as a function of the total temperatures and the heat of formation of the gases as follows: We consider ideal combustion (adiabatic and at constant pressure) with **ÿ_{ch}** = 1 and negligible flow velocities. If we consider a control volume combined with the combustion chamber, the application of the integral form of the energy equation gives us, neglecting the enthalpy of the fuel:

$$D_f Q_R = (D_a + D_f)h_{04} - D_a h_{03}$$

Where h_{04} is the enthalpy of the burned gases.

$$f = \frac{h_{04}-h_{03}}{Q_R-h_{04}} \tag{2.30}$$

If Cp is constant then equation (2.30) becomes:

$$f = \frac{\frac{T_{04}}{T_{03}}-1}{\frac{Q_R}{c_p T_{03}}-\frac{T_{04}}{T_{03}}} \tag{2.31}$$

Using equations (2.26a) and (2.23) we can express T_{03} as a function of T_a. Thus, equations (2.21) .. (2.31) allow us to calculate the thrust per unit of flow F/Da as well as

the specific consumption Cs (air being considered as an ideal gas and Cp and ÿ are constant throughout the turbojet).

Below we represent the performance of a turbojet with the following data:

T_{04} = 1300°K Ta = 218.65 °K Pa = 4000 Pa η_d = 0.879

η_c= 0.88

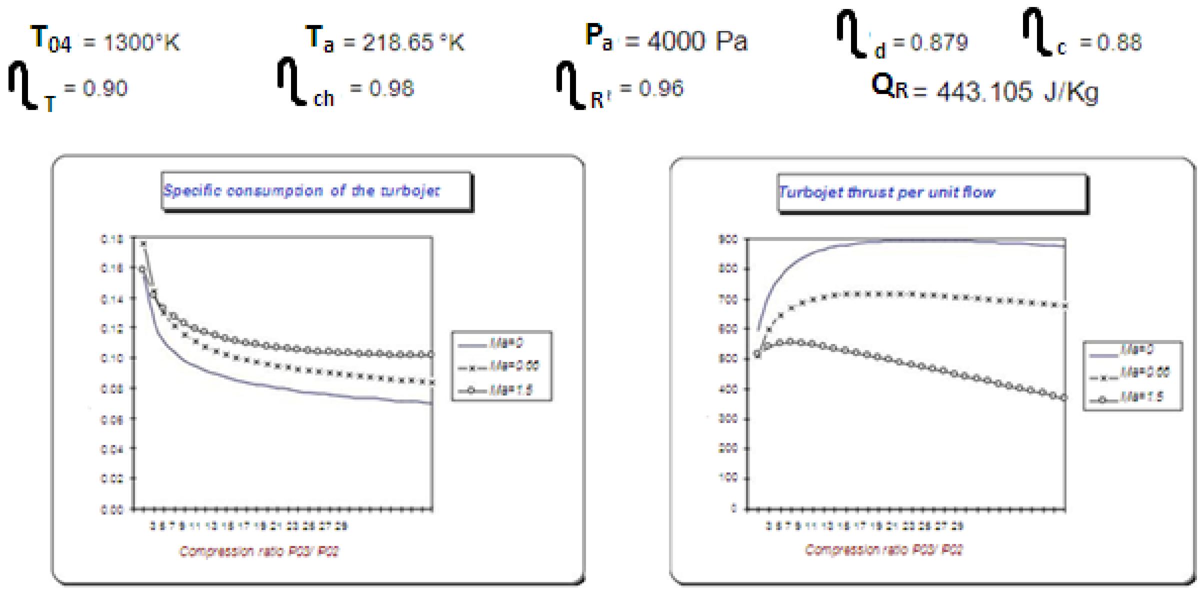

Figure II.5- Variation of specific consumption and thrust depending on the compression ratio for different Mach numbers [5].

We first notice that for a given Mach number, there exists a compression ratio which gives us a minimum specific consumption. This depends on the efficiency of the different components of the turbojet, the maximum temperature T_{04} as well as flight speed.

Several factors, other than specific consumption, must be considered in choosing the compression ratio. For example, a large compression ratio which seems a good solution for low flight speeds requires a powerful machine to produce the corresponding thrust, thus the thrust per unit airflow is relatively

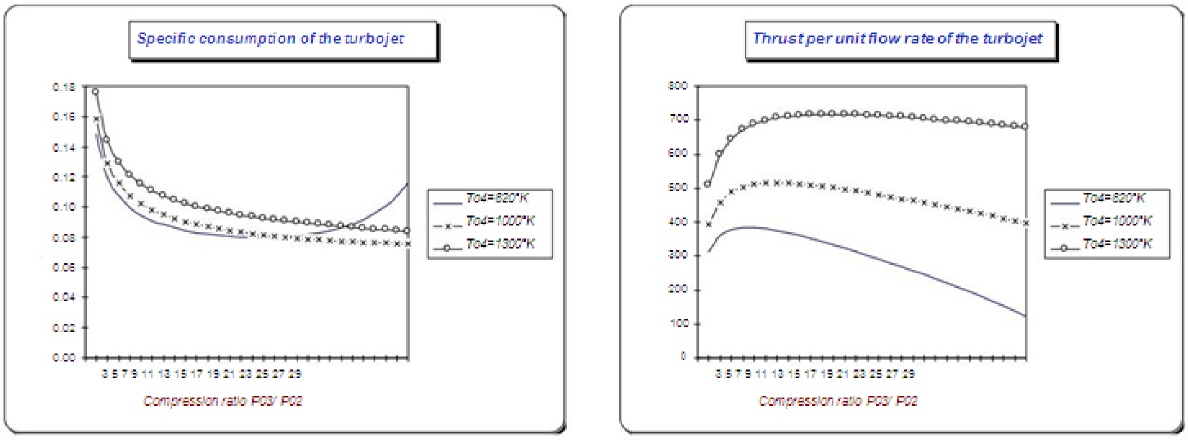

Figure II.6- Variation of specific consumption and thrust depending on the compression ratio for different turbine inlet temperatures (for Ma = 1.5) [5].

These figures show the effect of increasing the maximum temperature of the T_{04} cycle on the specific consumption and the specific thrust. For minimum fuel consumption the compression ratio decreases considerably with T_{04}, and the same for the specific thrust. In addition, it can be noted that reducing the temperature at the turbine inlet from 1300 °K to 1000 °K leads to a small reduction for the minimum specific consumption. Also, the reduction of T_{04} leads, however, to an increase in the minimum specific consumption for high compression ratios. It should be emphasized that the variation in performance shown in the figures depends considerably on the efficiencies of the different components as well as the flight Mach number.

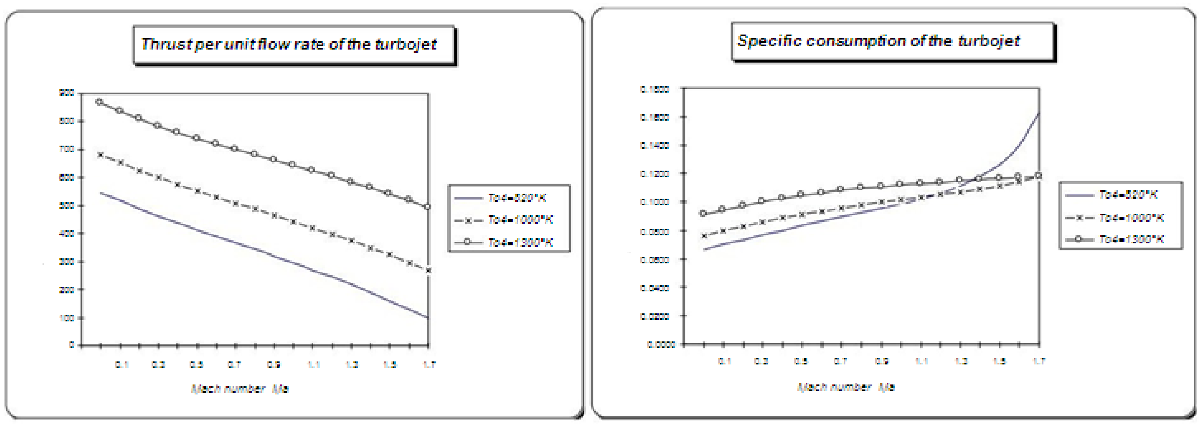

Figure II.7- Variation of specific consumption and thrust as a function of Mach number for different turbine inlet temperatures [5].

We first notice that for each given temperature T_{04} , there exists a maximum Mach number for which there will be no fuel burning in the air.

Conversely, the two figures show us that for each given Mach number, the operation of the turbojet is advantageous for low temperatures T_{04} since the specific consumption is low. However, this results in a relatively low specific thrust and consequently a high bulkiness of the machine. but the larger the turbojet, the greater the flow and drag.

In conclusion, the operation of a turbojet at high speeds is often based on the maximum temperature tolerable by the turbine blades. Cruising operation must therefore use a low temperature which leads, on the one hand, to a long lifespan of the machine and, on the other hand, to low specific consumption. Choosing the best machine size and temperature for economical operation while cruising requires great attention in analyzing drag and penalizing the size of large machines as well as their low fuel consumption.

Also, we notice that the thrust of the turbojet per unit of flow decreases linearly with the Mach number until its speed reaches the same value as that of the ejected gases. That being said, of course the theoretical case; but actually, there is a pre-compression in the sleeve which causes the thrust, instead of continuing to decrease, to gradually increase and only decrease quite far after the speed of sound.

We therefore see that the services expected of an engine are very diverse, not to say opposite. When climbing, for example, what matters is thrust so that the aircraft reaches as quickly as possible the altitude where its aerodynamic performance is optimal. In subsonic flight (waiting period, diversion), it is, on the contrary, the reduced thrust and the specific consumption which prevail.

We then notice that the motorization of a supersonic is a very complex matter. The different phases of flight require such dissimilar propulsion modes that in reality, not one, but two, or even three types of reactors are required: a single flow for cruising, a double flow for takeoff and a hybrid of the previous two during climb and descent.

Faced with these requirements, engineers from the "Société Nationale d'Etudes et de Construction de Engine d'Avion (SNECMA)" first considered equipping the Future Supersonic
Transport Aircraft (ATSF) with double-flow reactors. This was relatively advantageous in that,the dual flow offers satisfactory take-off and climb performance. In addition, with such

an engine, the take-off noise should, in principle, be significantly lower. However, wind tunnel studies, carried out in cooperation with aerospace and the National Aeronautical Studies and
Research Office (ONERA) showed that the creation of such a reactor, although simple in design, raised an insoluble problem in the form of dilemma. Indeed, either it was too big then, or it had a reasonable size, but it was then too noisy.

Currently, studies are being carried out which aim at a reactor which will encompass the role of a single flow and double flow in the volume of a single and even to replace the two compressors (HP and LP) and the two turbines (HP and LP) by a unique compressor-turbine system.

CHAPTER III
ENERGY STUDY OF PASSAGES INPUT AND OUTPUT

III.1- AIR INLET

The function of the air inlet is to slow the relative flow of air upstream of the turbojet up to a certain internal Mach number imposed for compressor operation (M 0.4). It is mainly composed of an air capture zone and a divergent pipe (diffuser). The gradual opening of this pipeline must satisfy the following conditions:

- Low total pressure loss.
- The flow must be uniform at the compressor inlet because distortion of the velocity profile at the compressor inlet can cause aerodynamic rollover which leads to blade failure due to the resulting vibrations.
- A small footprint since the available space is limited.
- Low external drag.

We note that it is more difficult to obtain good performance for the compressing elements (diffuser, compressor) than for the expanding elements (turbine, nozzle). This is due to the tendency of the boundary layer to detach in the case of a flow with a unfavorable pressure gradient (i.e. pressure increases during flow).

III.1.1- EXTERNAL FLOW CONFIGURATION

The flow configuration in the air capture zone upstream of the diffuser depends on the flight speed and the mass flow requested by the turbojet. For a subsonic flow we distinguish 2 shapes of streamlines (fig.III.1). The air capture section of case (b) is larger than that of case (a). This is due to the requirement for a very large mass flow at the time of takeoff (fig.III.1.b) to produce sufficient thrust for the vehicle to climb; and in order to admit the necessary quantity of air, the flow must be accelerated externally at takeoff. On the contrary, in the case of cruising flight (fig.III.1.a) the flow undergoes an external deceleration. For speeds given in sections (a) and (2) and knowing that the two preceding external processes are isentropic, the external acceleration of the flow in case (b) will increase the speed and decrease the pressure at the diffuser inlet. Therefore, the increase in pressure inside the diffuser becomes greater than that which corresponds to the external deceleration in case (a). If the pressure increase by external acceleration is large enough, the diffuser may have aerodynamic overturn due to separation of the boundary layer.

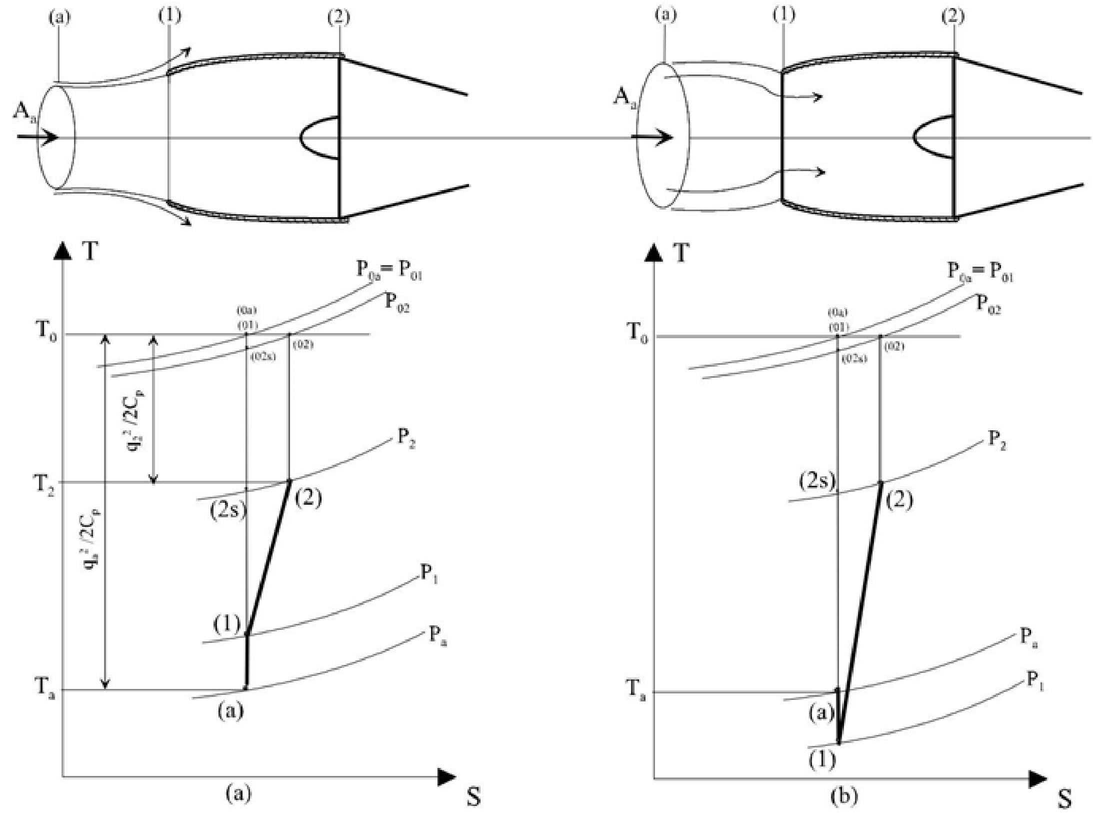

Figure III.1- Configuration of subsonic flow in a turbojet diffuser.

(a): High speed or low mass flow.

(b): Low speed or high mass flow.

We therefore conclude that the inlet section of the diffuser must be chosen such that it minimizes the external acceleration during takeoff and produces an external deceleration during the cruise flight.

III.1.2- INTERNAL FLOW CONFIGURATION

The flow in the diffuser still undergoes an irreversible deceleration due to the friction between the walls and the fluid. The type of flow to be obtained in a given diffuser depends largely on the rate of increase of the cross section. If this section does not expand rapidly in the direction of the flow the boundary layer does not take off and the flow will behave well in the diffuser.

Whereas if the divergence rate of the diffuser exceeds a certain limit, the flow will suffer separations and an aerodynamic rollover which are transient or permanent depending on the case (fig. III.2).

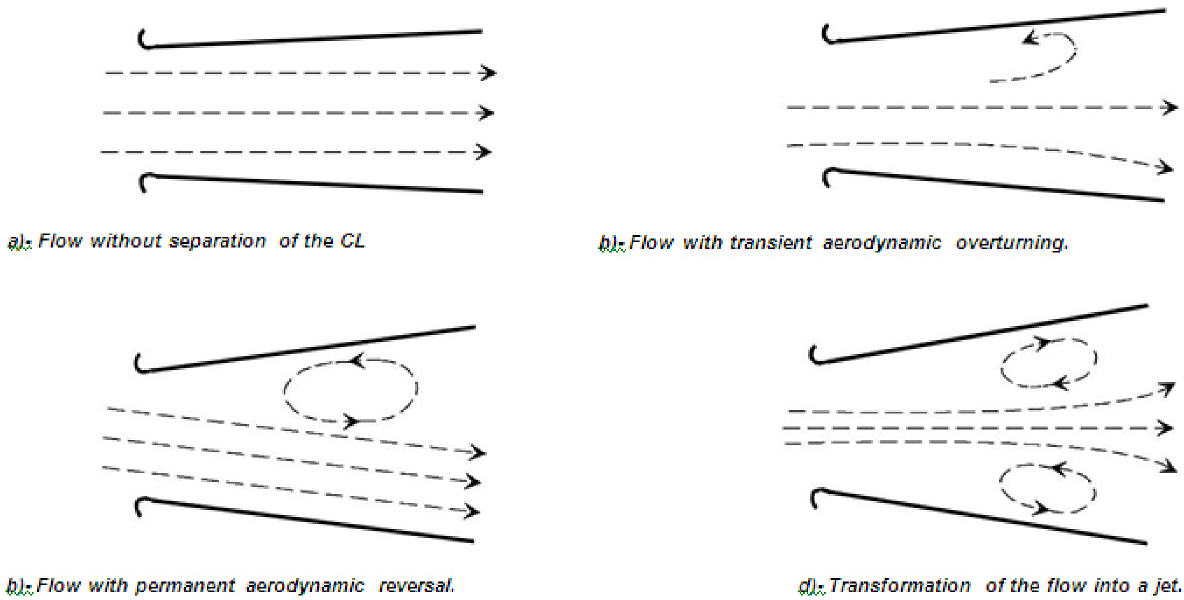

Figure III.2- Different types of flows in diffusers.

In case (d) the diffuser plays no role in compressing the fluid and the flow passes through this inlet element as a jet.

To have good flow behavior in a diffuser, the divergence angle must not exceed the critical value which is indicated in figure (III.3).

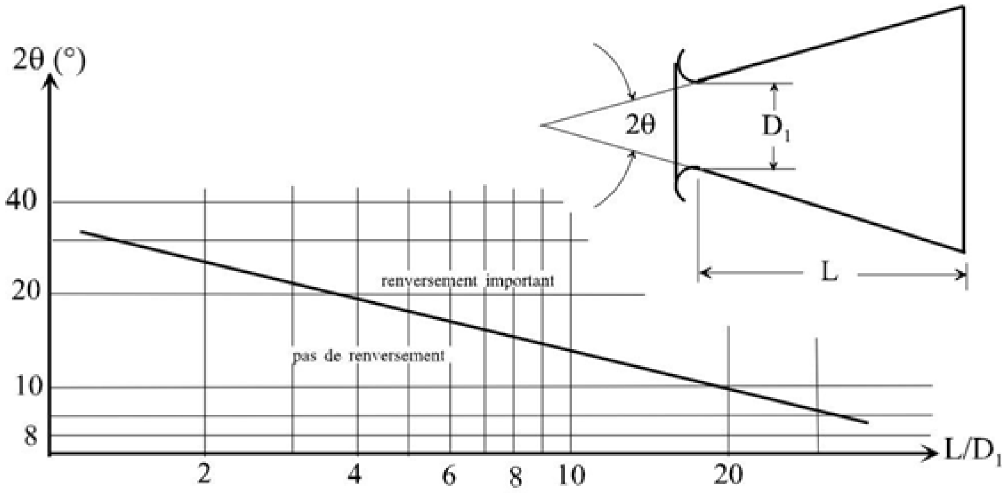

Figure III.3- Limit of appearance of aerodynamic rollover [1].

III.1.3- PERFORMANCE OF A DIFFUSER

Among the parameters defining the relative performances of a diffuser we distinguish:

a- Adiabatic efficiency:

Referring to figure (III.1.a) we define the adiabatic efficiency of the diffuser by:

$$\eta_d = \frac{h_{02s} - h_a}{h_{02} - h_a} \approx \frac{T_{02s} - T_a}{T_{02} - T_a} \tag{3.1}$$

The state (02s) is defined as a state that can be reached if the air is compressed isentropic ally up to the total pressure which exists at the outlet of the diffuser. We have :

$$\frac{T_{02s}}{T_a} = \left(\frac{P_{02}}{P_a}\right)^{\frac{\gamma-1}{\gamma}}$$

And by combining with relation (2.23), equation (3.1) then becomes:

$$\eta_d = \frac{\left(\frac{P_{02}}{P_a}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{\gamma-1}{2} M_a^2} \tag{3.2}$$

b- **Ratio of total pressures:** It is defined by:

$$r_d = \frac{P_{02}}{P_{0a}} \tag{3.3}$$

$$\frac{P_{02}}{P_a} = \frac{P_{02}}{P_{0a}}\frac{P_{0a}}{P_a} = \frac{P_{02}}{P_{0a}}\left(\frac{T_{0a}}{T_a}\right)^{\frac{\gamma}{\gamma-1}} = \frac{P_{02}}{P_{0a}}\left[1+\frac{\gamma-1}{2}M_a^2\right]^{\frac{\gamma}{\gamma-1}}$$

By combining this relationship with equations (3.2) and (3.3) we will have:

$$\eta_d = \frac{\left(1+\frac{\gamma-1}{2}M_a^2\right)r_d^{\frac{\gamma-1}{\gamma}} - 1}{\frac{\gamma-1}{2}M_a^2} \tag{3.4}$$

The variations of rd and ÿd as a function of the Mach number Ma are shown in the figure (III.4).

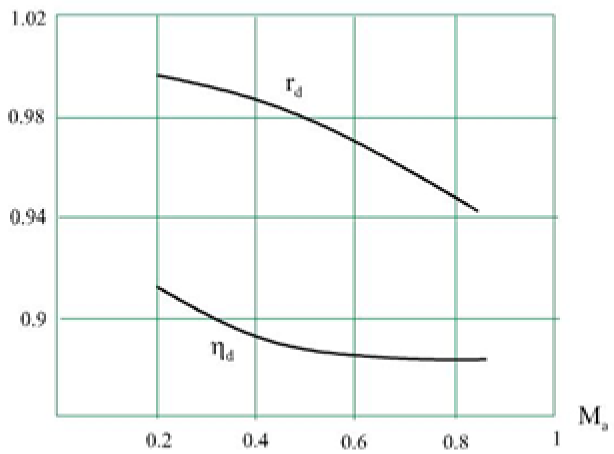

Figure III.4- Performance of a typical subsonic diffuser [1].

III.2- AIR OUTLET PIPE (TUYERE)

The role of the nozzle is to produce the thrust necessary for propulsion by expanding the burnt gases of high temperature and pressure. It can be simply convergent, convergent-divergent or convergent with the pass just at the exit.

The convergent part is given a rounded profile ensuring rapid convergence up to the neck, the divergent part consisting of a truncated cone with an opening of 7 to 8° at the top. By using nozzles with adjustable sections, the performance of the turbojet is improved.

III.2.1- FLOW STUDY

A nozzle operates in the best conditions when it is sized "adapted", that is to say when the ejection pressure Pe is equal to the ambient pressure Pa. In this case the flow is not disturbed nor inside, nor downstream of the nozzle.

*** If Pe < Pa:** A recompression wave (oblique wave) is produced at the outlet, causing a rise in temperature. If the Pa/Pe ratio is large enough, the temperature increase can be high enough that the local speed of sound becomes greater than that at the tip of the nozzle. The recompression wave then moves up the divergent. If its compression ratio decreases (as well as its temperature and its speed), it then stops before the pass in a section where its speed is equal to that of the current. We then obtain a stationary recompression wave. If the wave reaches the neck, the entire flow becomes subsonic (since behind the wave M<1): the nozzle then functions as with an incompressible fluid, i.e. there is an expansion in the convergent and compression in the divergent.

*** If Pe > Pa:** In general, the jet expands at the exit and creates a depression wave whose speed is lower than that of sound at the end of the nozzle. But P_e can take different very close values of Pa at the end of the nozzle which reveal some important phenomena (fig.III.5).

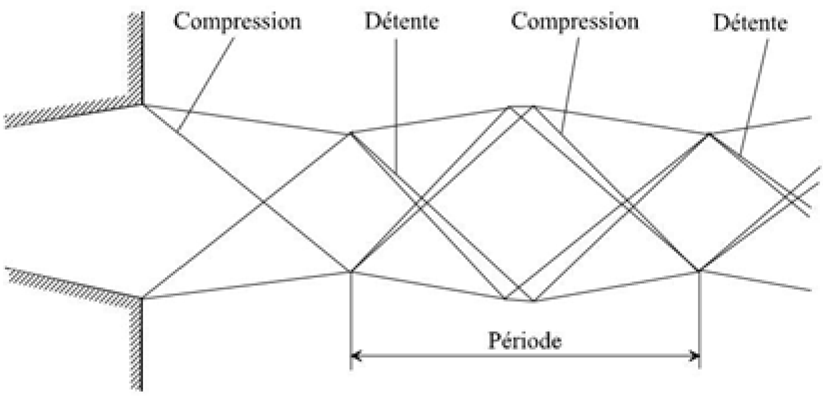

Figure III.5- Appearance of recompression and expansion waves at the exit of the nozzle.

In addition, we note that the risk of rapid separation of the jet and the appearance of straight and oblique shock waves difficult to control would be felt at low altitude.

III.2.2- PERFORMANCE OF A NOZZLE

The expansion process in the nozzle is also adiabatic like that of compression in the diffuser. This is due to the fact that the heat transfer per unit of mass of the fluid is much lower than the enthalpy difference between the inlet and outlet of the nozzle. On the other hand, this transfer does not have time to take place given that the speed is very high. The expansion in the nozzle can be described, as a first approximation, by the equations of a one-dimensional flow.

We define the adiabatic efficiency in relation to the isentropic of a nozzle by:

$$\eta_R = \frac{h_{05} - h_6}{h_{05} - h_{6s}} \qquad (3.5)$$

Where the state (6s) is the one obtained if the gases are accelerated isentropic ally to the outlet pressure (fig.III.6).

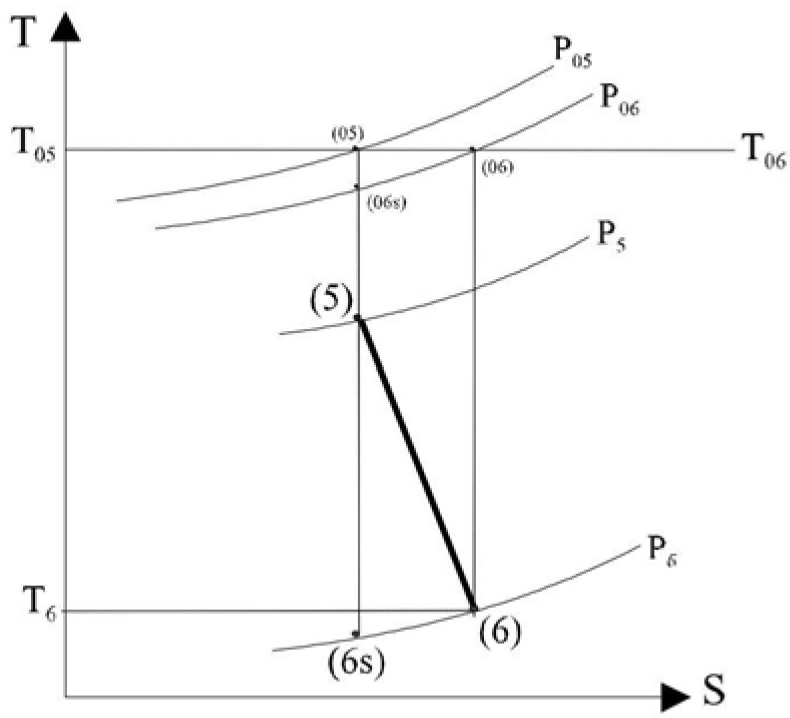

Figure III.6- Definition of fluid states in a nozzle.

The ejection speed is given by the following relationship:

$$q_e = \sqrt{2\frac{\gamma R}{\gamma - 1} T_{05} \eta_R \left[1 - \left(\frac{P_6}{P_{05}} \right)^{\frac{\gamma-1}{\gamma}} \right]} \tag{3.6}$$

By introducing the Mach number we will have:

$$M_6^2 = \frac{2}{\gamma - 1} \eta_R \left[1 - \left(\frac{P_6}{P_{05}} \right)^{\frac{\gamma-1}{\gamma}} \right] \left\{ 1 - \eta_R \left[1 - \left(\frac{P_6}{P_{05}} \right)^{\frac{\gamma-1}{\gamma}} \right] \right\}^{-1} \tag{3.7}$$

Mass flow can be determined by:

$$D = \rho_6 A_6 q_6 = \rho_6 A_6 M_6 \sqrt{\frac{\gamma R T_{05}}{1 + \frac{\gamma - 1}{2} M_6^2}} \tag{3.8}$$

* If the relaxation is complete: P_{06} = Patm

$$\frac{A_6}{A_5} = \frac{P_{05} M_5}{P_{06} M_6} \left[\frac{1 + \frac{\gamma - 1}{2} M_6^2}{1 + \frac{\gamma - 1}{2} M_5^2} \right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{3.9}$$

The P_{05}/P_{06} ratio can be obtained by:

$$\frac{P_{05}}{P_{06}} = \frac{P_{05}}{P_6} \frac{P_6}{P_{06}} = \frac{P_{05}}{P_6} \left\{ 1 - \eta_R \left[1 - \left(\frac{P_6}{P_{05}} \right)^{\frac{\gamma-1}{\gamma}} \right] \right\}^{\frac{\gamma}{\gamma-1}} \tag{3.10}$$

P_{03}/ P_{02} ... using the same We can express these relationships according to T_{04}, reasoning as i n T , the previous chapter.

CHAPTER IV
ENERGY STUDY OF TORQUE COMPRESSOR-TURBINE

IV.1- COMPRESSOR STUDY

Axial compressors are machines used to compress air into it communicating a speed through the moving vans and decelerating it through the fixed vanes. The radial movement of the fluid is very small compared to its axial movement. This kind of compressor does not require cooling; Therefore compression takes place without exchange of heat with the outside. The fluid is usually air which undergoes a change in density important. The compression ratio of an axial compressor can reach values ≥ 10 in creating as many floors as needed (7,8,...). The compression ratio per stage (≈ 1.16) is lower than that of a centrifugal compressor due to the absence of the centrifugal effect. Each stage includes a grid of fixed blades and another mobile one (fig.IV.1).

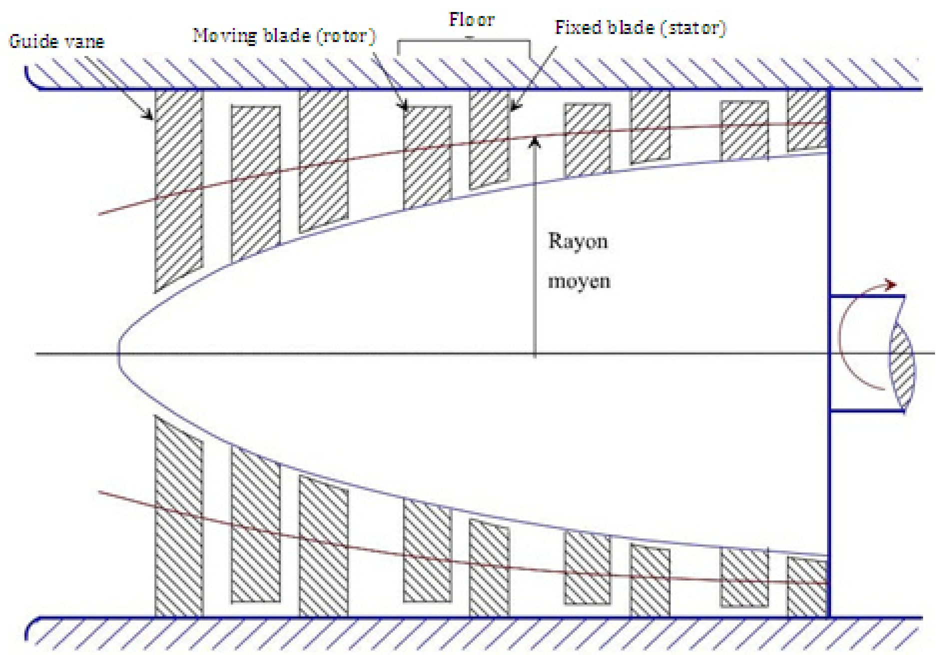

Figure IV.1- Diagram of an axial compressor.

We are thus led to first study a stage which can be considered as an elementary compressor.

IV.1.1- AIR FLOW IN AN ELEMENTARY COMPRESSOR

Let us designate by P1 , T1 , q1 the characteristics of the air at the entrance to the stage, i.e. at the exit of the previous floor (so they are assumed to be known). and by P_{03} , T_{03} , q_3 the characteristics air at the exit of the stage (fig.IV.2).

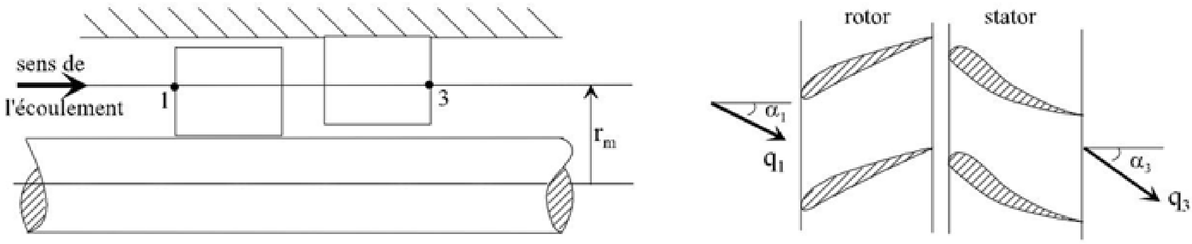

Figure IV.2- Flow in an elementary axial compressor.

The energy equation is written, between points (1) and (3), as follows:

$$(W+Q)_{13} = h_3 - h_1 + \frac{1}{2}(q_3^2 - q_1^2)$$

So

$$W_{13} = h_3 - h_1 + \frac{1}{2}(q_3^2 - q_1^2) \qquad (4.1)$$

In axial compressors, we ensure that the air is with the same conditions at the entrance to the next floor and at the entrance to the floor studied.

SO **$q_3 = q_1$** and **$\alpha_3 = \alpha_1$** .

Equation (4.1) then becomes:

$$W_{13} = h_3 - h_1 \qquad (4.2)$$

a- Air flow in fixed channels:

The fixed channels are limited by the rotor blades (fig.IV.3).

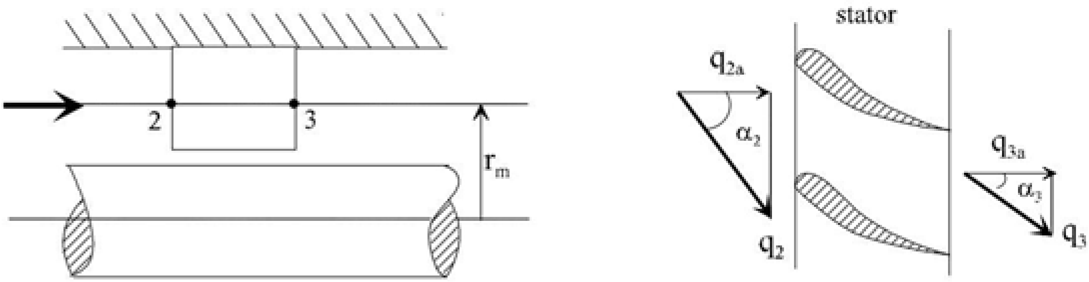

Figure IV.3- Flow in fixed channels.

Since there is no exchange of heat or work, the energy equation gives us:

Formula

Since $h_3 > h_2$ (compression) then $q_2 > q_3$ hence a slowdown in speed in the fixed channels. In axial compressors, the axial components of the speeds are generally equal ($q_{2a} = q_{3a}$). Figure (IV.4) shows that there is a slowdown in speed and a change in its direction which translates into quantity $\mathbf{\Delta q_u}$

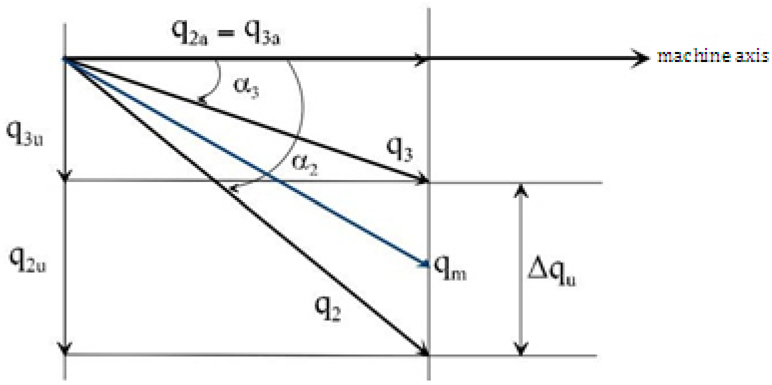

Figure IV.4- Velocity triangle in fixed channels.

Noticed:

The value of Δq_u which is generally very small. These results in the fact that the Gas compression is difficult to achieve compared to expansion. If Δq_u which is very high, we risk of having a separation of the air vein along the walls of the canal as well as losses important.

b- Air flow in the mobile channels:

The study of flow in moving channels (fig.IV.5) essentially depends on the choice of reference (linked either to the rotor or to the stator). For an observer bound to the rotor, everything happens as if air is flowing through fixed channels. The relative speed goes from the value w_1 to w_2 . The energy equation gives us:

Formula

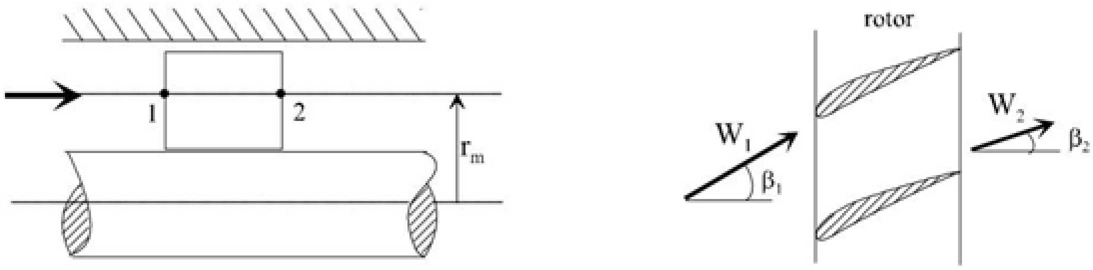

Figure IV.5- Flow in mobile channels.

Since $h_2 > h_1$ then $w_1 > w_2$ hence a slowdown in speed. If the axial speeds are equal: $w_{1a} = w_{2a}$ then the slowing down of the flow is shown schematically by figure (IV.6) below:

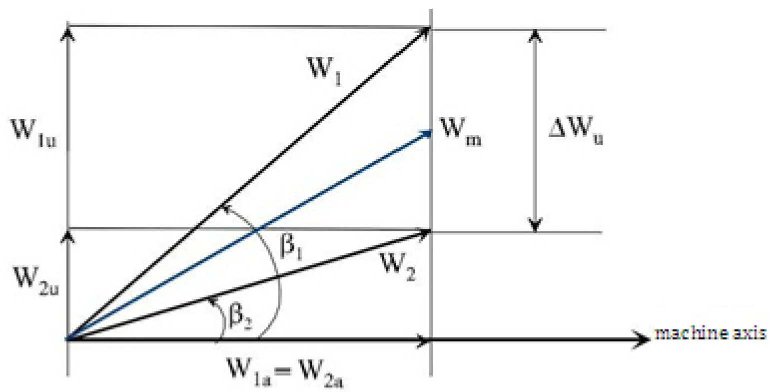

Figure IV.6- Triangle of relative speeds in the rotor.

We also have for an observer linked to the stator:

$$(W+Q)_{12} \quad = \quad h_2 - h_1 + \frac{1}{2}(q_2^2 - q_1^2)$$

So:

$$W_{12} \quad = \quad h_2 - h_1 + \frac{1}{2}(q_2^2 - q_1^2) \tag{4.5}$$

Combining relations (4.3) and (4.5) gives us:

$$W_{12} \quad = \quad h_3 - h_1 + \frac{1}{2}(q_3^2 - q_1^2)$$

If $q_3 = q_1$ then $W_{12} = h_3 - h_1$ so there is no work exchanged in the fixed channels. The work exchanged with the moving blades is equal to the increase in enthalpy throughout the stage.

IV.1.2- SPEED TRIANGLE FOR ONE FLOOR

In order to simplify the study of the flow in the compressor, we consider only periodic stage machines (identity of the speed diagrams for the floors). The latter gives the fluid speed the same direction and the same value at the outlet only at the entrance to the floor.

We will draw the speed diagram (fig.IV.7) for an average radius. The management of relative velocities must be tangent (or nearly so) to the skeleton of the blade at its leading edge.

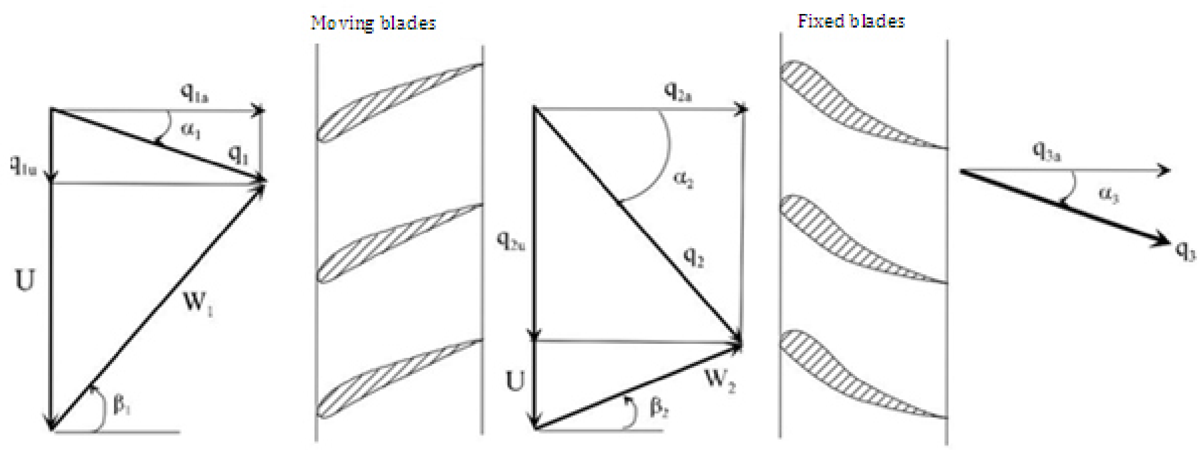

Figure IV.7- Velocity triangle in a floor for an average radius.

Since the drive speed U is constant for axial machines (displacement negligible radial of the fluid particles), the triangles of the velocities at the entrance and exit of the mobile channel can be superimposed (fig.IV.8).

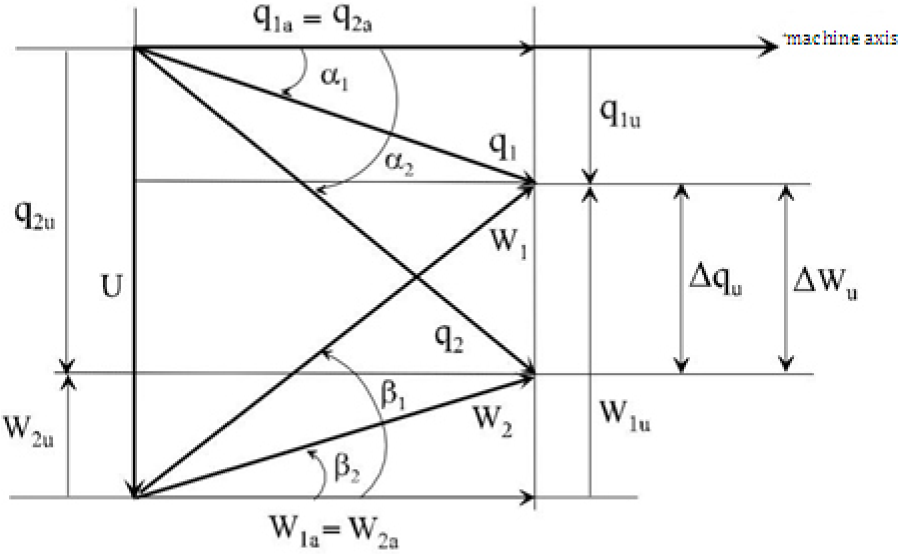

Figure IV.8- Superposition of the speed triangles in the rotor.

IV.1.3- THERMODYNAMIC STUDY OF A FLOOR

Since the compressor is not cooled, the compression process through a stage is adiabatic with friction (fig.IV.9).

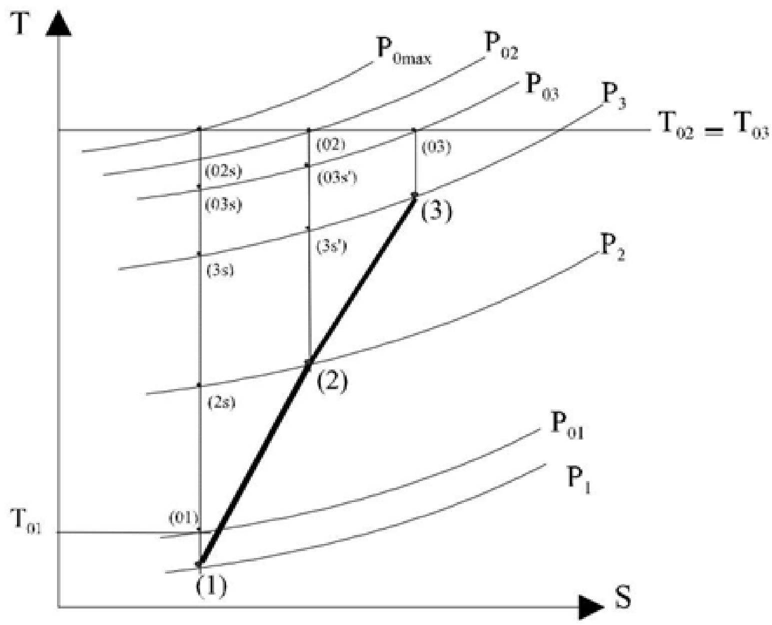

Figure IV.9- Compression through a stage.

If the process was isentropic, the final isentropic stopping pressure, for the same work carried out on the fluid, would be P_{0max}. But because of friction losses in the rotor and the stator, we have:

$P_{03} < P_{02} < P_{0max}$. If we define the efficiency of a stage by:

$$\eta_{et} = \frac{h_{03s} - h_{01}}{h_{03} - h_{01}} \tag{4.6}$$

Using isentropic relations we will have:

$$\frac{P_{03}}{P_{01}} = \left[1 + \eta_{et} \frac{\Delta T_0}{T_{01}}\right]^{\frac{\gamma}{\gamma - 1}} \tag{4.7}$$

Or

$$\Delta T_0 = T_{03} - T_{01}$$

IV.1.4- DYNAMIC STUDY OF A FLOOR

To determine the efforts (elementary forces of pressure and friction) exerted on the blades of a compressor, we consider a control volume which contains only a blade (fig.IV.10). It is assumed that the flow is uniform in the inlet and outlet sections of the volume of control (vc), and that the surfaces are divided such that they coincide with the current surfaces.

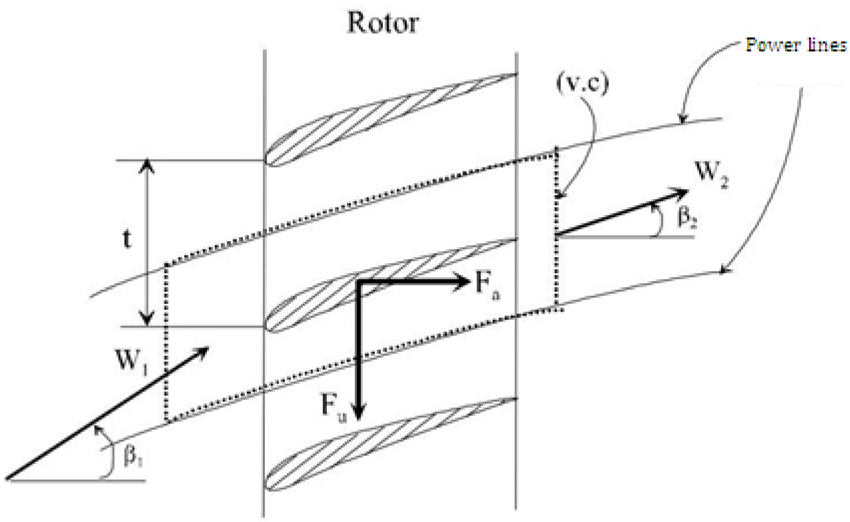

Figure IV.10- Forces exerted by the blade on the fluid.

Applying the momentum equation in the tangential direction gives the effort per unit length of blade:

$$F_u = D(w_{1u} - w_{2u}) \tag{4.8}$$

With

D: mass flow per unit length passing in the inter-blade channel. The continuity equation applied to the control volume gives:

$$D = \rho_1 t w_1 \cos\beta_1 = \rho_2 t w_2 \cos\beta_2 \tag{4.9}$$

Likewise in the axial direction:

$$F_a + (P_1 - P_2)t = D(w_{2a} - w_{1a}) \tag{4.10}$$

Since the relative axial velocities are the same:

$$w_{1a} = w_{2a} = w_a$$

Then (4.10) becomes:

$$F_a = (P_2 - P_1)\, t \qquad (4.11)$$

* We can relate the forces exerted by the fluid on the blade to the forces of drag (Dr) and lift (Li) as follows (fig.IV.11).

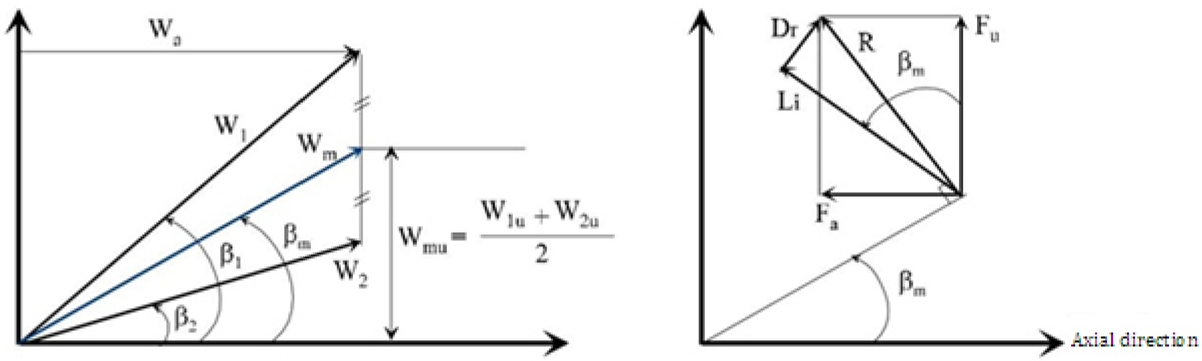

Figure IV.11- Definition of blade forces.

From the figure above, we can easily establish the following relationships:

$$\begin{aligned} Li &= F_u \cos\beta_m + F_a \sin\beta_m \\ Dr &= F_u \sin\beta_m - F_a \cos\beta_m \end{aligned} \quad \text{où} \quad \begin{aligned} \sin\beta_m &= \frac{w_{mu}}{w_m} \\ \cos\beta_m &= \frac{w_a}{w_m} \end{aligned} \qquad (4.12)$$

We conventionally define the coefficients of lift CL and drag CD as follows:

$$C_L = \frac{2\,Li}{\rho b\, w_m^2} \quad \text{et} \quad C_D = \frac{2\,Dr}{\rho b\, w_m^2} \qquad (4.13)$$

Where b is the width of the blade (or span). The tangential force Fu resulting from the pressure distribution on the rotor blades where the pressure on the intrados is higher than that on the extrados. So the effect on dawn in the tangential direction is directed in the opposite direction to the peripheral speed and the fluid tries to break the rotor which must be driven by an external torque.

Using the momentum equation and choosing a control volume which contains only the rotor, we obtain the expression of the torque applied to the fluid:

$$\tau = D(r q_{2u} - r q_{1u}) \qquad (4.14)$$

So the power required to drive the stage is:

$$P_{et} = \tau.\Omega \qquad (4.15)$$

With:

Ω: rotor rotation speed and **P_{et}** : power absorbed by the compression stage.

The power supplied to the fluid is:

$$P_f = D.\Omega(r q_{2u} - r q_{1u})$$

Or

$$U = r\Omega$$

Hence:

$$P_f = D.U(q_{2u} - q_{1u}) \qquad (4.16)$$

The work done by the rotor on the fluid is:

$$W = \frac{P_f}{D} = U(q_{2u} - q_{1u}) \qquad (4.17)$$

IV.1.5- SOME IMPORTANT PARAMETERS

a- **Manometric coefficient:** It is the ratio between the work received per unit of mass of the fluid and the square of a reference speed. We generally choose U as the speed of reference for a given mass flow.

$$\mu = \frac{W_f}{U^2} = \frac{h_3 - h_1}{U^2} \tag{4.18}$$

μ :varies between 0.25 and 0.40 (very loaded compressor).

b- Degree of reaction: It is defined by:

$$\varepsilon = \frac{h_2 - h_1}{h_3 - h_1} \tag{4.19}$$

Or

$$\varepsilon = \frac{w_1^2 - w_2^2}{(q_2^2 - q_3^2) + (w_1^2 - w_2^2)} \tag{4.20}$$

If **ε** = 0.5 then the total increase in enthalpy takes place at 50% in the rotor.

c- Pressure limitation coefficient:

Since the passage between two blades is then diverging the boundary layer which develops on the walls opposes the gradient of unfavorable pressure. If this gradient exceeds a certain limit we obtain a separation of the boundary layer and an aerodynamic reversal of the flow. This limit is specified depending on the pressure coefficient which is defined by:

Formula

Where $\mathbf{W_i}$ is the relative velocity of the incident flow at point **i** where the boundary layer begins to develop. ΔP is the increase in static pressure from points i to the point where the Kp is evaluated. In general, constructors take: $0.4 < Kp < 0.8$

IV.2- STUDY OF THE TURBINE

Axial turbines are machines that transform the pressure rise of the fluid into kinetic energy by expanding the high pressure hot gases exiting the chamber combustion. These expanded gases drive the rotor of the machine by communicating to it mechanical energy. It is often reasonable to consider this expansion as adiabatic because the fluid flows through the turbine at a high speed. A significant expansion rate can be achieved by connecting turbines which contain several floors (2 to 4).

IV.2.1- DESCRIPTION

Axial turbines, like axial compressors, consist of a nozzle (or distributor) and a rotor (fig.IV.12). The latter includes blade grids whose Tops are usually linked by a metal band. This envelope is used to reduce vibrations of the blades on the one hand, and control air leaks through the tops of the fins, on the other hand.

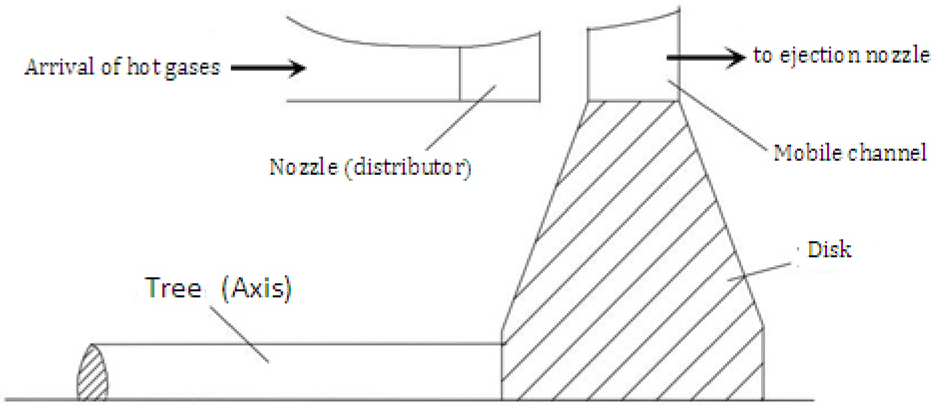

Figure IV.12- Turbine blades.

The moving blade grids are interspersed with fixed blade grids called distributors which are fixed on the frame of the machine. Since the pressure drop per stage is important, the height of the blades increases to facilitate the rapid expansion of the gases while keeping axial speed uniform across each stage. From a thermodynamic point of view, there are two classes of axial turbines.

a- Action turbine

In this type of turbine the total pressure drop (enthalpy) takes place in the fixed blades, while the moving blades are used only for changing the direction of speed.

b- Reaction turbine

In this type of turbine part of the pressure drop takes place in the blades fixed and the rest in the moving blades. The degree of reaction for a turbine is defined as the fraction of the entire drop of the static enthalpy (per stage) taking place in the rotor

$$\varepsilon = \frac{\Delta h_{rotor}}{\Delta h_{étage}} \qquad (4.22)$$

If $\varepsilon = 0.5$ then the enthalpy drop in the rotor and in the nozzle is the same. An action turbine is therefore a zero reaction machine.

IV.2.2- KINEMATIC STUDY OF THE FLUID THROUGH A STAGE

a- Action turbine:

Since in this type of turbine there is no enthalpy drop in the rotor then the energy equation imposes that $W_2 = W_3$. We will therefore have for a speed constant axial, the triangle of speeds at the level of the rotor (fig.IV.13).

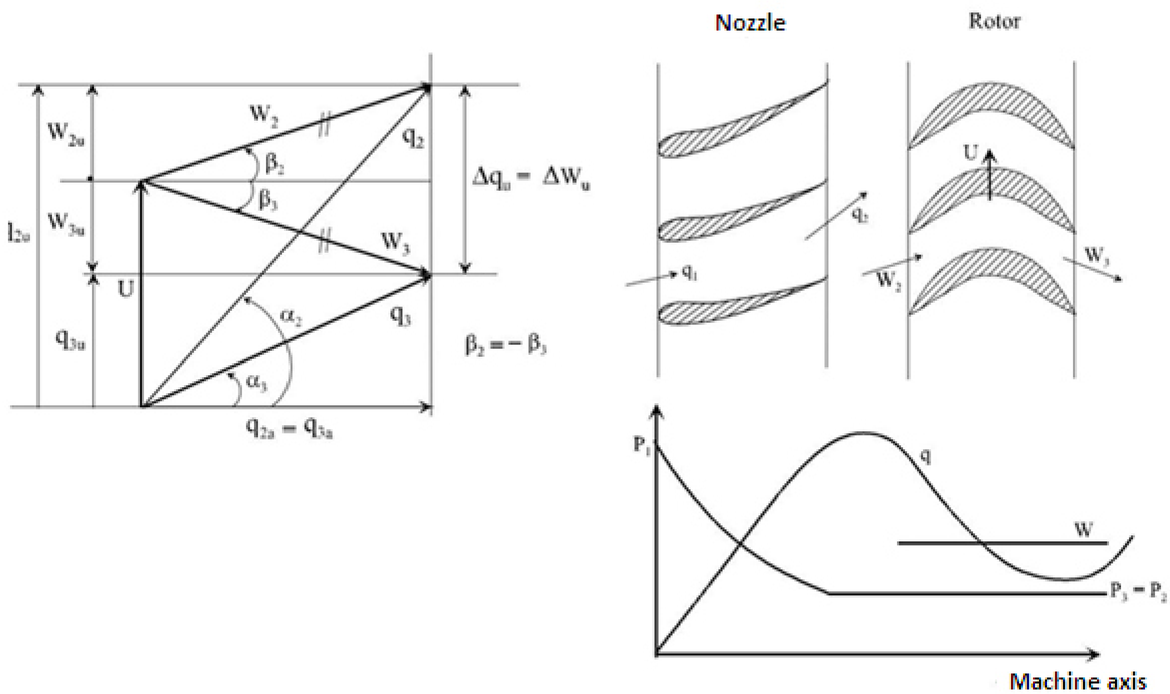

Figure IV.13- Velocity triangle and distribution of P, W and q through one stage of an action turbine.

b- Reaction turbine:

The shape of the blades and the corresponding speed triangles are shown in the figure (fig.IV.14):

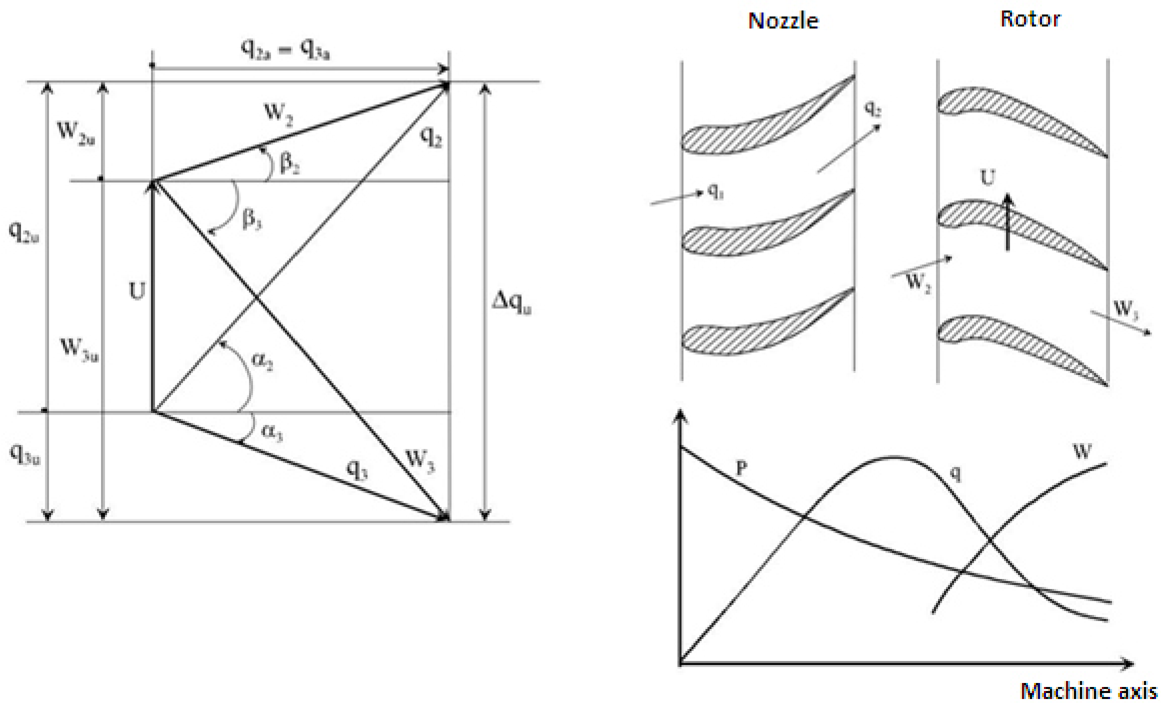

Figure IV.14- Velocity triangle and distribution of P, W and q through a stage of a reaction turbine.

IV.2.3- THERMODYNAMIC STUDY OF THE FLUID THROUGH A STAGE

The expansion of the gases in the turbine can be considered adiabatic because the flow rate of hot gases is considerably high. Taking into account the friction in the turbine elements, the expansion process is shown in the figure (IV.15).

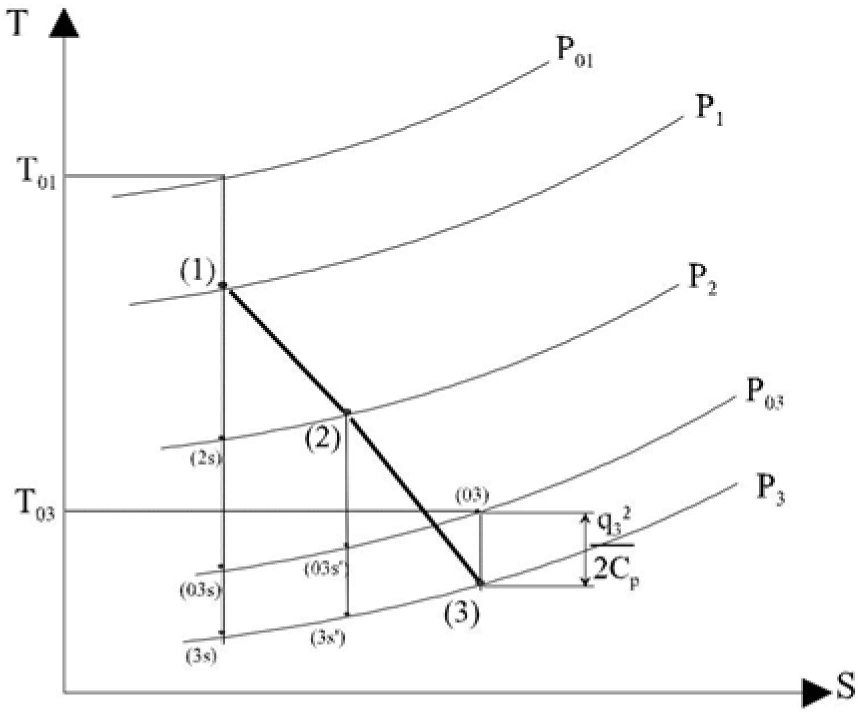

Figure IV.15- Expansion through one stage of an axial turbine.

We define the isentropic efficiency per stage as follows:

$$\eta_{st} = \frac{h_{01} - h_{03}}{h_{01} - h_{03s}} \qquad (4.23)$$

If the fluid is considered an ideal gas and Cp is constant the above relationship becomes:

$$\eta_{st} = \frac{1 - \frac{T_{03}}{T_{01}}}{1 \left(\frac{P_{03}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}} \qquad (4.24)$$

Hence the expansion rate of the stage:

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{1}{\eta_{st}} \frac{\Delta T_0}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}} \qquad (4.25)$$

Or

$$\Delta T_0 = T_{03} - T_{01}$$

IV.2.4- DYNAMIC STUDY OF THE FLOOR

In the rotor, the kinetic energy of the fluid is transformed into mechanical energy (rotational movement). In the moving frame, the relaxation causes the increase in the relative speed ($W_3 > W_2$). To determine the forces exerted on the blade, we choose a volume control around the blade and we apply the momentum theorem (fig.IV.16).

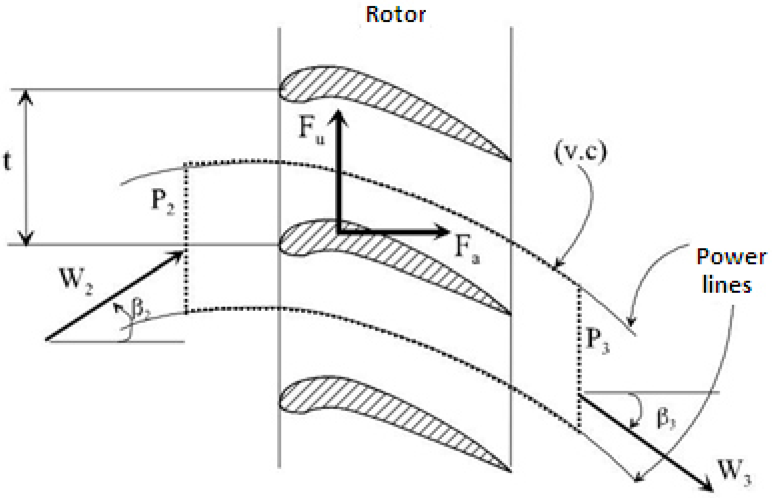

Figure IV.16- Forces exerted by the blade on the fluid.

In the same way as for the compressor stage we have along the tangential axis:

$$F_u \quad = \quad D(w_{3u} - w_{2u}) \tag{4.26}$$

and following the axial direction we have:

$$F_a = (P_2 - P_3)\,t + D(w_{3a} - w_{2a}) \tag{4.27}$$

If the axial component is constant then

$$w_{2a} = w_{3a} = w_a$$

So:

$$F_a = (P_2 - P_3)\,t \qquad (4.28)$$

IV.3- COUPLING THE TURBINE WITH THE COMPRESSOR

The performance problem of coupling the turbine with the compressor has a great importance for propulsion machines, which are forced to operate in conditions resulting in wide variation of thrust, pressure and inlet temperature as well as the Mach number. The coupling problem is relatively simple although the calculation can be long. There stationary performance of the machine is determined at each speed for two conditions:

- the mass flow rate of the turbine (D_t) must be equal to the sum of the mass flow rates of the compressor (D_c) and the flow rate of fuel injected into the combustion chamber (D_f), less flow losses in the compressor;

- the power developed by the turbine must be equal to that requested by the compressor. Given the Mach number, the ambient conditions, the efficiencies of the diffuser and of the nozzle, the air passage sections, the performance of the propulsion machine can be determined by compressor and turbine performance charts. In principle, the coupling calculation procedure is done as follows:

1- Select the working speed;

2-Assume a turbine inlet temperature T_{04};

3-Assume a compression ratio of the compressor;

4-Calculate the work of the Wc compressor per unit of mass;

5-calculate the pressure ratio, in the turbine, necessary to produce Wc; check on the charts if $D_c + D_f = Dt$. Otherwise, assume a new value of repeat steps (4), (5) and (6) until continuity is satisfied;

Now calculate the ratio of pressures through the nozzle from the ratios of pressure of the diffuser, compressor, combustion chamber and turbine; calculate the outlet section of the nozzle necessary to pass Dc calculated in step (6) with the pressure ratio calculated in step (7) and the shutdown temperature calculated. If the section calculated is different from the current output section, assume a new value of T_{04} (step 2) and repeat the procedure.

CHAPTER V

ENERGY STUDY FROM THE COMBUSTION CHAMBER

V.1- DESIGN AND DESCRIPTION OF THE COMBUSTION CHAMBER

The combustion chamber is the vital part of the turbojet engine. Its role is to establish the mixing compressed air with fuel and, through combustion, transforming its energy chemical into thermal (calorific) energy. The design of a combustion chamber must satisfy the following conditions:

- Complete combustion because the quality of this process directly affects yield thermal of the turbojet.

- A geometric configuration which ensures the stability of combustion over a large range of mixing ratio values f.

- Ability to re-ignite in different atmospheric conditions.

- Complete mixing of air and fuel as well as combustion products in order to to avoid the presence of hot spots, and to obtain a uniform temperature distribution when leaving the room.

- Flame protection from the outside.

- Maximum reduction in chamber volume and weight as these factors are essential in aircraft design.

Many of these conditions are incompatible and they require a lot of intuition and skill. A combustion chamber essentially consists of:

V.1.1- INJECTION SYSTEM

- Combustion occurs in the gas phase, it is then necessary to inject the fuel in a form close to the gaseous state. High pressure fuel undergoes swirling movement in the injector. The jet, which then takes the conical shape under the action of centrifugal force, bursts into thousands of droplets of 5 to 500 **μ**. Injection can be done directly in the direction of flow (fig.V.1), or against it fluent. The advantages of the latter are:

- For the same combustion intensity the chamber is shorter, hence the reduction in the length of the turbojet and therefore that of its weight.

- Combustion is more complete because the speed is reduced at the injector.

- It ensures better spraying following impacts on the fuel molecules.

- The disadvantage of this method is that the injector is in the flame.

- This difficulty can be overcome by the use of a thermally resistant material.

V.1.2- IGNITION SYSTEM

The ignition of the gas mixture is established not only by burners but also by a set of accessories that is located outside the room.

V.1.3- FLAME HANGING DEVICE

It is a swirling body which serves to homogenize the mixture and stabilize the flame given the high speed at the edges of the injectors. The injector is placed in its center.

V.1.4- GAS COOLING SYSTEM

The stoichiometric combustion of fuel oil results in temperatures around 2300 to 2500°K. This temperature cannot be supported by the combustion chamber (the accessories which surround it), nor by the blades of the turbine which are thermally stressed and mechanically. Currently the most efficient turbojet engines do not exceed 1700 °K while the most traditional turbojets can reach 1500 °K. We therefore have an interest to cool the combustion gases before entering the turbine to admissible values of the temperature.

The cooling system is made up of a jacket pierced with holes to create secondary and tertiary air flows (60 to 75% of the air coming from the compressor) which are used to cool the gases as well as the combustion chamber. The number of orifices can't exceed a certain value for reasons of resistance of the liner. Their shape can be circular or rectangular with rounded ends. Experience shows that the combination of two shapes of orifices gives better combustion, greater thrust and homogeneity of the temperature as a result of better dilution. This homogeneity then allows a more uniform distribution of stresses on the turbine blades and which will thus be favorable to its resistance.

A longitudinal section of the combustion chamber is shown schematically in the figure (V.1) below:

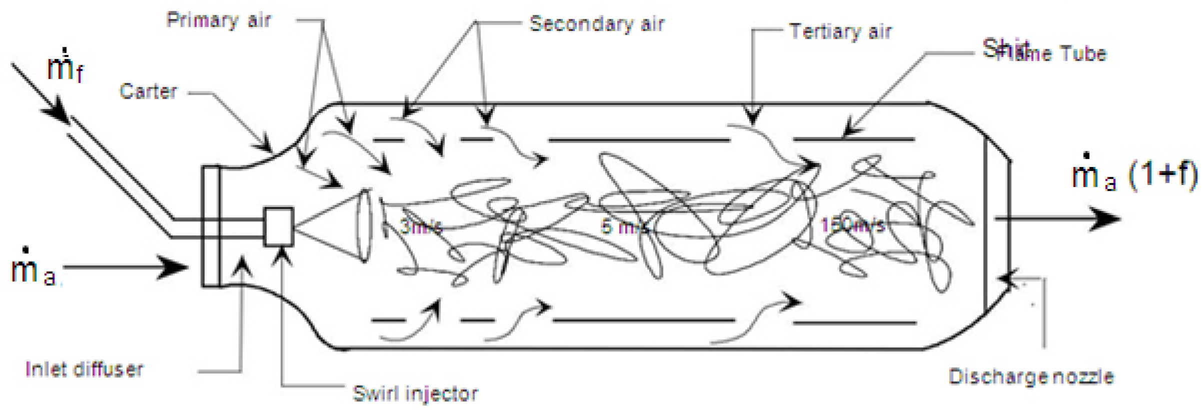

Figure V.1- Longitudinal section of a combustion chamber.

Regarding the types of rooms, we can cite the following three (fig. V.2):

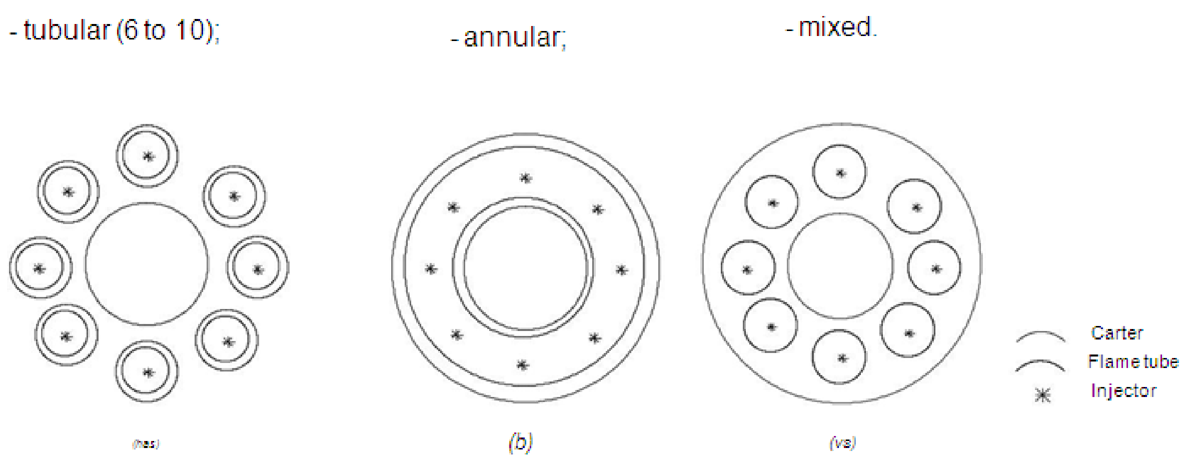

Figure V.2- Different types of combustion chambers.

a)- tubular; b)- annular; c)- mixed.

Noticed:

Tubular combustion chambers have the following advantages at the expense of both others:

- If a chamber is faulty, replacement is easily carried out;

- More economical since the replacement is partial;

- Better cooling since a large surface area is in contact with the air;

- Better homogeneity of the temperature at the exit of the chamber than the annular ones, but less than mixed rooms.

V.2- ENERGY BEHAVIOR OF THE COMBUSTION CHAMBER

To have a relative simplification of this complicated system, we assume that:

- Combustion takes place at constant pressure;

- The flow is stationary;

- The transformation is adiabatic.

-The fuel/air ratio called mixing ratio f is one of the most important parameters in combustion.

-Neglecting f in front of 1, the thermal balance of the combustion chamber is written:

$$D_f\,Q_R = c_p\,D_a\,(T_{04} - T_{03}) \tag{5.1}$$

$$f = \frac{D_f}{D_a} = \frac{c_p\,(T_{04} - T_{03})}{Q_R} \tag{5.2}$$

Cp being the average specific heat.

Q_R = Pci : lower calorific value of the fuel (oil).

In turbojet engines, kerosene of chemical formula is used as fuel.

C8H18. The stoichiometric chemical reaction which corresponds to combustion is written:

$$2C_8H_{18}+25\left(O_2+\frac{79}{21}N_2\right)\rightarrow 16CO_2+18H_2O+25\left(\frac{79}{21}\right)N_2$$

$$f=\frac{2(12.8+18)}{25\left(32+\frac{79}{21}.28\right)}=0.0664$$

This value of f leading to high temperatures at the exit of the combustion chamber (2500 °K), it is then necessary to dilute the mixture to have **f** ≈0.01.

For a fuel represented by **CnHm** we can calculate its **Pci** by the following approximate

$$Q_R \approx 2.3244\left(15900+15800\,\frac{1.008.m}{12.01.n}\right) \qquad \text{[KJ/Kg]} \qquad (5.3)$$

The ratio of specific heats ÿ depends not only on the temperature but also on the composition of the mixture (because R depends on the composition). This is illustrated by figure (V.3) below.

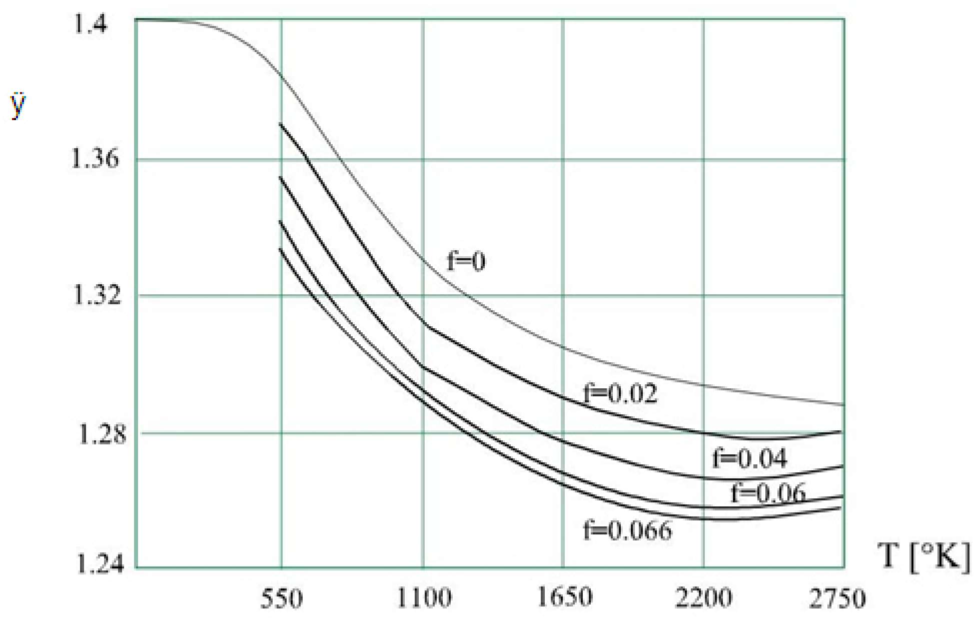

Figure V.3- Variation of γ as a function of temperature for kerosene [1].

According to the description given in (§V.1.4), the calculation of pressure losses in the chamber combustion can be simplified by the following modeling based on the organization internal of the room:

Initially, the flow not yet subjected to the heat input must bypass obstacles such as fuel inlet pipes, injectors, hook-up flame ...etc. The temperature of the fluid (that of the compressor outlet) is sufficiently low so that the effects of viscosity are significant. We therefore introduce a loss of aerodynamic load.

Secondly, the flow undergoes the heat input and the temperature reached allows the effects of viscosity to be significantly reduced, with the fluid approaching the perfect state when the temperature increases. We then introduce a second loss of load, only thermodynamic.
To determine the conditions at the exit of the first part of the chamber combustion (between 3 and 3') (fig.V.4), we express the pressure losses due to friction in the following dimensionless

$$K = \frac{2(P_{03} - P_{03'})}{\rho_3 q_3^2} \qquad (5.4)$$

Where K is an empirical number such that:

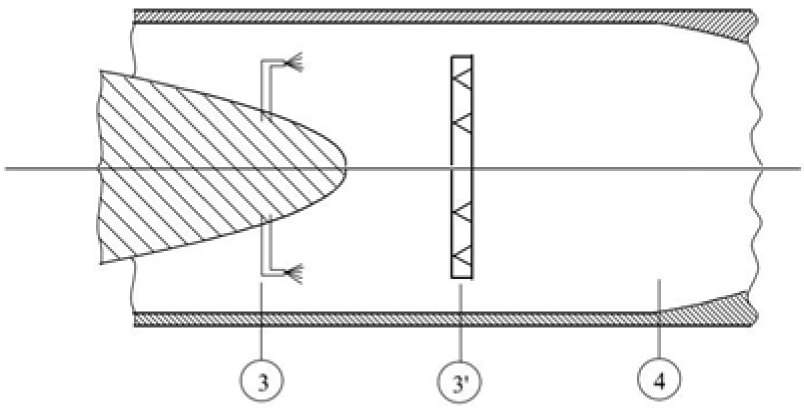

Figure V.4- Diagram of the turbojet combustion chamber(room).

By introducing dynamic pressure:

$$V_3 = \frac{1}{2}\rho_3 q_3^2 \qquad (5.5)$$

Equation (5.4) becomes:

$$\frac{P_{03'}}{P_{03}} = 1 - K\frac{V_3}{P_{03}} \qquad (5.6)$$

And by introducing the Mach number we will have:

$$\frac{P_{03'}}{P_{03}} = 1 - \frac{K}{2}\frac{\gamma M_3^2}{\left(1+\frac{\gamma-1}{2}M_3^2\right)^{\frac{\gamma}{\gamma-1}}} \qquad (5.7)$$

For a constant cross section, the continuity equation gives:

$$\rho_3 q_3 = \rho_{3'} q_{3'}$$

And using the ideal gas equation of state, we can write:

$$\frac{P_{3'}}{p_3} = \sqrt{\frac{T_{3'}}{T_3}}\frac{M_3}{M_{3'}} \qquad (5.8)$$

Since the flow is assumed adiabatic, then:

$$\frac{P_{3'}}{P_3} = \left[\frac{1+\frac{\gamma-1}{2}M_3^2}{1+\frac{\gamma-1}{2}M_{3'}^2}\right]^{\frac{1}{2}}\frac{M_3}{M_{3'}} \qquad (5.9)$$

$$\frac{P_{03'}}{P_{03}} = \left[\frac{1+\frac{\gamma-1}{2}M_{3'}^2}{1+\frac{\gamma-1}{2}M_3^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}\frac{M_3}{M_{3'}} \qquad (5.10)$$

The exit conditions for this part (between points 3 and 3') of the ideal room (theoretical) are obtained by solving equations (5.7) and (5.10) for P03'/P_{03} and $M_{3'}$. Their solution of this system is represented by figure (V.5) for the value K = 2.

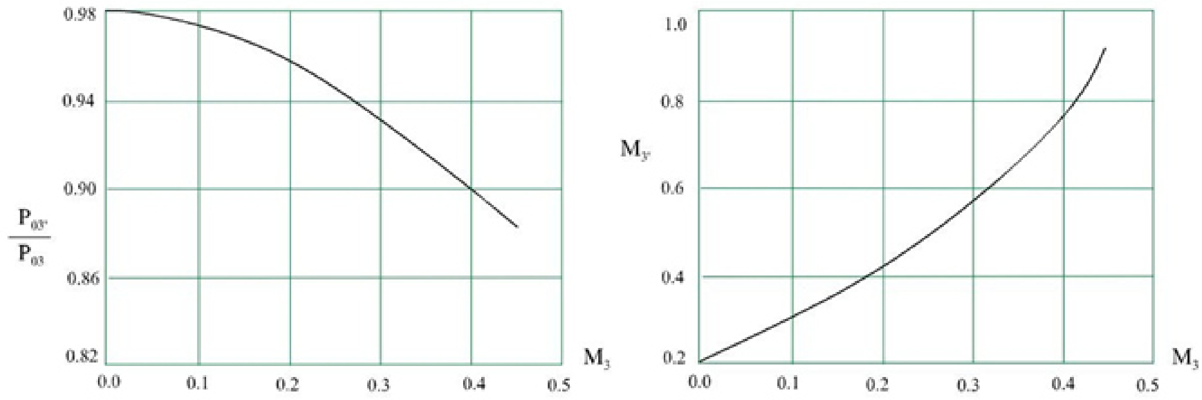

Figure V.5- Flow through an ideal flame tube (K=2, γ =1.4).

Now to determine the maximum temperature at the outlet of the chamber combustion we consider, in the second part of the combustion chamber, the equation energy in its integral form.

*** Energy equation for the combustion chamber (between points 3' and 4):**

By taking a control volume (vc) coincident with the combustion chamber, we will have:

$$\dot{Q} - \dot{W} = \int_{SC} \rho \, (\vec{q} . \vec{n}) \, e \, dS$$

$\dot{Q} = 0$	car pour une chambre de combustion idéale le processus est adiabatique.
$\dot{W} = 0$	car aucun travail n'est effectué au niveau de la chambre de combustion.
e :	énergie totale du fluide.
q :	vitesse du fluide.

If the flow variables are uniform in the inlet and outlet sections and if we neglects the potential energy g z, the energy equation becomes:

$$D h_{03'}\Big|_{air} + D(h + Q_R)\Big|_{fuel} = (D_a + D_f)\, h_{04}$$

h_{04} being the enthalpy of the gases leaving the chamber;

$$h_{03'}+f\left(h_f+Q_R\right)=\left(1+f\right)h_{04}$$

Or:

$$f = \frac{D_f}{D_a}$$

$\mathbf{h_f}$ being the enthalpy of the fuel.

Taking into account the efficiency of the room which expresses the fraction of the calorific value which is actually transformed into useful heat, the previous equation becomes:

$$h_{03'}+f\left(h_f+\eta_{ch}Q_R\right)=\left(1+f\right)h_{04} \qquad (5.11)$$

And neglecting h_f :

$$\eta_{ch}=\frac{\left(1+f\right)h_{04}-h_{03'}}{fQ_R} \qquad (5.12)$$

The temperature at the exit of the combustion chamber can be expressed by:

$$T_{04}=\frac{T_{03'}+f\eta_{ch}\frac{Q_R}{c_p}}{\left(1+f\right)} \qquad (5.13)$$

Cp being, of course, an average coefficient. In reality combustion does not take place at constant pressure . There exists a slight loss of pressure of the order of 3 to 5% of the inlet pressure to the chamber combustion.

Experience shows that η_{Ch} also depends on altitude. Indeed, for the three different types of chambers A, B and C (fig.V.6), the efficiency decreases at high altitude because the conditions at the inlet of the turbojet are less favorable.

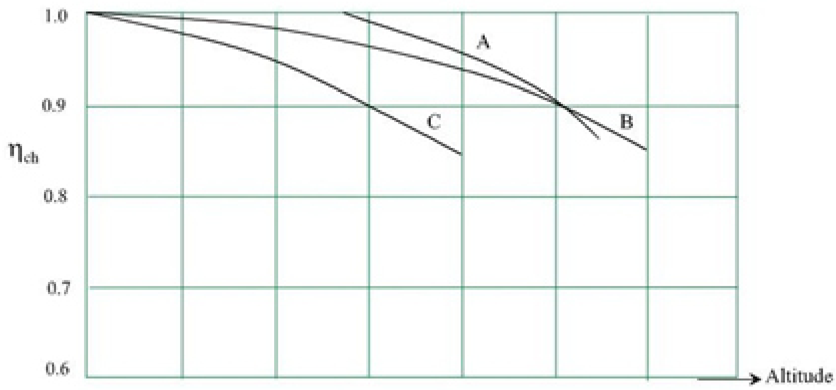

Figure V.6- Effect of altitude on combustion efficiency.

Now let's return to the procedure for determining static pressure drops and total through this region of the combustion chamber (between points 3' and 4). Since we assumes that the combustion is concentrated in this region, i.e. the composition and physical properties of the fluid change significantly due to the increase important of the temperature, the combustion equations must be written in such a way to introduce the mixing ratio f and the variations of , R, Cp , ... The continuity equation gives us:

$$\rho_{3'} q_{3'} (1+f) = \rho_4 q_4 \qquad (5.14)$$

Using the ideal gas equation of state, we will have:

$$\frac{P_4}{P_{3'}} = (1+f) \frac{q_{3'}}{q_4} \frac{R_4 T_4}{R_{3'} T_{3'}} \qquad (5.15)$$

$$\frac{P_4}{P_{3'}} = (1+f) \sqrt{\frac{\gamma_{3'} R_4 T_4}{\gamma_4 R_{3'} T_{3'}}} \frac{M_{3'}}{M_4} \qquad (5.16)$$

We can write:

$$\frac{T_{04}}{T_4} = 1 + \frac{\gamma_4 - 1}{2} M_4^2 \qquad \text{et} \qquad \frac{T_{03'}}{T_{3'}} = 1 + \frac{\gamma_{3'} - 1}{2} M_{3'}^2$$

Hence the ratio of static pressures:

$$\frac{P_4}{P_{3'}} = (1+f)\sqrt{\frac{\gamma_{3'} R_4 T_{04}}{\gamma_4 R_{3'} T_{03'}}}\frac{M_{3'}}{M_4}\left\{\frac{1+\frac{\gamma_{3'}-1}{2}M_{3'}^2}{1+\frac{\gamma_4-1}{2}M_4^2}\right\}^{\frac{1}{2}} \qquad (5.17)$$

Another approximate expression of this ratio can be obtained from the equation of quantity of movement in this region, neglecting friction forces:

$$(P_{3'} - P_4) A_{3'} = \rho_4 q_4^2 A_{3'} - \rho_{3'} q_{3'}^2 A_{3'}$$

So

$$\frac{P_4}{P_{3'}} \approx \frac{1 + \gamma_{3'} M_{3'}^2}{1 + \gamma_4 M_4^2} \qquad (5.18)$$

The Mach number M4 at the exit of the combustion chamber can be obtained by doing the equality between equations (5.17) and (5.18). Once determined, we can then calculate the ratio of total pressures using the following relationships:

$$\frac{P_{04}}{P_4} = \left(1+\frac{\gamma_4-1}{2}M_4^2\right)^{\frac{\gamma_4}{\gamma_4-1}} \qquad \text{et} \qquad \frac{P_{03'}}{P_{3'}} = \left(1+\frac{\gamma_{3'}-1}{2}M_{3'}^2\right)^{\frac{\gamma_{3'}}{\gamma_{3'}-1}}$$

From where

$$\frac{P_{04}}{P_{03'}}=\frac{1+\gamma_{3'}M_{3'}^{2}}{1+\gamma_{4}M_{4}^{2}}\frac{\left(1+\frac{\gamma_4-1}{2}M_4^2\right)^{\frac{\gamma_4}{\gamma_4-1}}}{\left(1+\frac{\gamma_{3'}-1}{2}M_{3'}^2\right)^{\frac{\gamma_{3'}}{\gamma_{3'}-1}}} \tag{5.19}$$

Equations (5.17), (5.18) and (5.19) allow us to obtain all the conditions flow at the exit of the combustion chamber once the entry conditions are known of this part of the chamber ($P_{03'}$, $M_{3'}$) and the maximum temperature T_{04}. Figure (V.7) illustrates the variations of $P_{04}/ P_{03'}$ and M_4 as a function of $M_{3'}$.

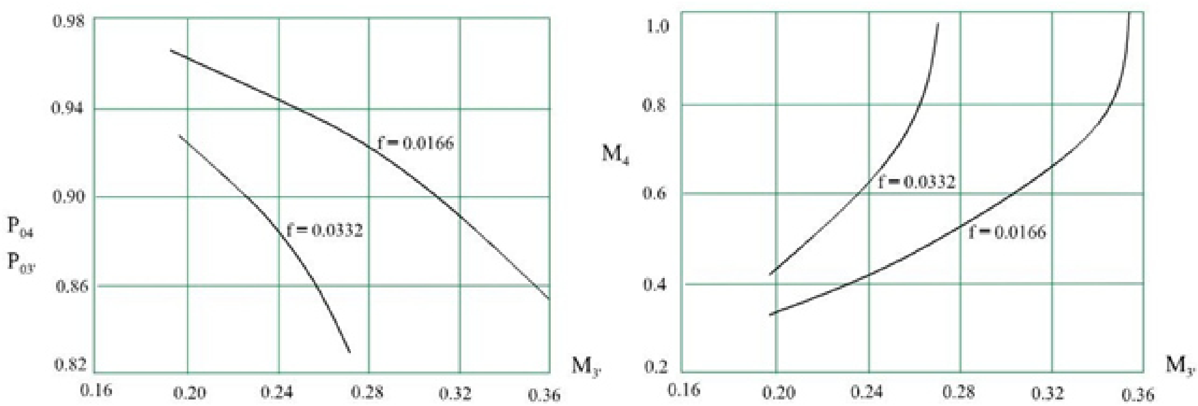

Figure V.7- Variations of the $P_{04}/ P_{03'}$ and M_4 ratio **as a function of $M_{3'}$.**

$T_{03'}$ = 555.5°K

T_{04} = 1710 °K for f = 0.0332

T_{04} = 1172.7 °K for f = 0.0166

These curves show us that the pressure loss is proportional to the number of Mach $M_{3'}$. The more the latter decreases, the more the total pressure loss decreases but this results in**.** inevitably a higher maximum cycle temperature T_{04} because the mixing ratio f is bigger.

Course supplement

Summary(Recap) of propulsion mechanics

R1. INTRODUCTION

We looked at the Standard Atmosphere, the environment in which aircrafts operate, at Bernoulli's equation and its relationship to airplane (or more specifically, wing) aerodynamics, and at some basic parameters that influence the aerodynamic performance of an airplane. In this chapter we will look at the way that we account for airplane propulsion; i.e., jet or propeller engines. This means we will be looking at the factors that affect things like airplane thrust and power. We will also find that the same factors that explain thrust can also be used to account for some of the drag on an airplane.

In looking at thrust, power, and drag we are interested in how these may vary with airplane speed and with altitude. We must have a basic understanding of these dependencies if we are to eventually use these in determining the performance of an airplane.

Airplane engines are, of course, the subject of entire engineering courses dealing with things such as internal combustion engines and air-breathing jet engines. We want to confine ourselves to a very simple approach to understanding how propulsion works without going into any more of the details than are absolutely necessary. Fortunately, we will be able to do this.

R2.1 Jet Engines

Jet engines come in a wide range of designs. Most are considered "turbine" engines because turbines are used to extract energy from the high speed exhaust flow to drive a "compressor" to compress the flow into the engine prior to fuel addition and combustion, but at very high speeds (hypersonic flow) it is possible to get compression through shock waves and a non-turbine jet engine called a "ramjet" is the result. However, we are going to restrict ourselves to subsonic, incompressible flight where turbines and compressors are always needed.

The most basic type of jet engine is called a "turbojet" and it consists basically of an inlet, followed by a compressor that increases the pressure (and lowers the speed) of the air before it enters the combustion chamber where fuel is added and ignited. After combustion a turbine extracts enough energy from the high energy (high speed) exhaust products to drive the compressor, and the flow then exits through the engine exhaust at high speed to provide thrust.

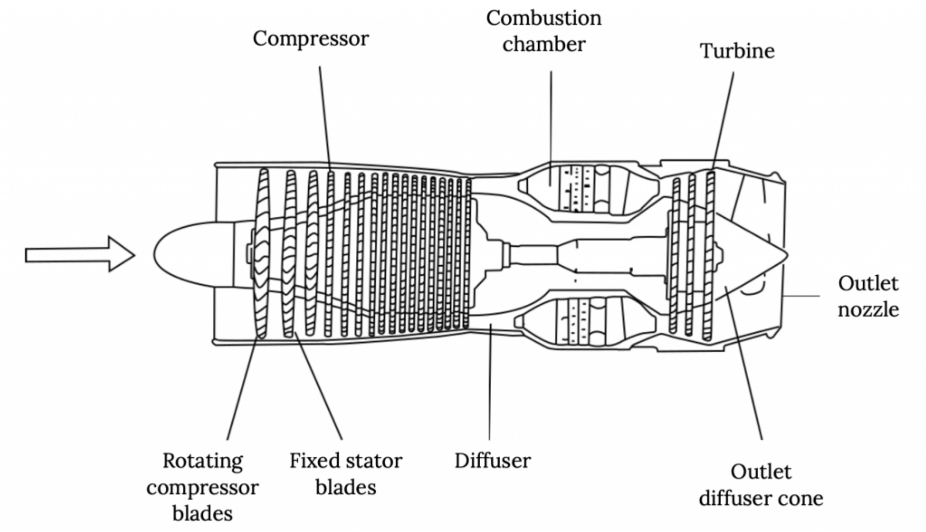

Figure R2.1: Turbo-Jet Engine Illustration

It turns out that a pure turbojet isn't a very efficient way to make thrust. It creates thrust through a very high speed exhaust and this is both very noisy and very loss prone. The high speed exhaust jet essentially rips its way through the surrounding air and this violent interaction between exhaust and atmosphere results in a lot of friction-like losses and makes a lot of noise.

We will look at jet propulsion in terms of momentum changes (energy per unit time) with the difference between the momentum in the exhaust and the engine inlet accounting for the thrust and, at first glance, it will seem that any way we can get a change in momentum is just as good as any other way, but that is not the case.

Momentum is essentially the mass multiplied by speed (velocity). This means that there are two ways to get a momentum change. One is to take a small amount of mass and accelerate it to a very high speed as is done in a turbojet engine. Another is to take a large amount of mass and accelerate it by a lesser amount. As it turns out, the latter way is the most efficient way to get thrust. It is sort of like comparing the effects of a large ceiling fan rotating slowly with those of a little "personal" fan. If you made two propeller driven air boats of the type used in swamps, one boat with a small propeller and the other with a large one, you would find that the boat with the larger prop would need less power to move at a given speed than the one with the small prop.

Fan-jets rely on this principle to provide more efficient thrust to an airplane than turbo-jets. In a fan-jet, the engine turbine or turbines drive both the compressor that works on the air going into the combustion chamber and a large fan that adds momentum to a large mass of air going around the engine core without being used to burn fuel. This fan or "bypass" air then mixes with the higher speed core, combustion products to give a high momentum total engine exhaust that derives its momentum from the large mass of the bypass flow and the high speed of the core flow.

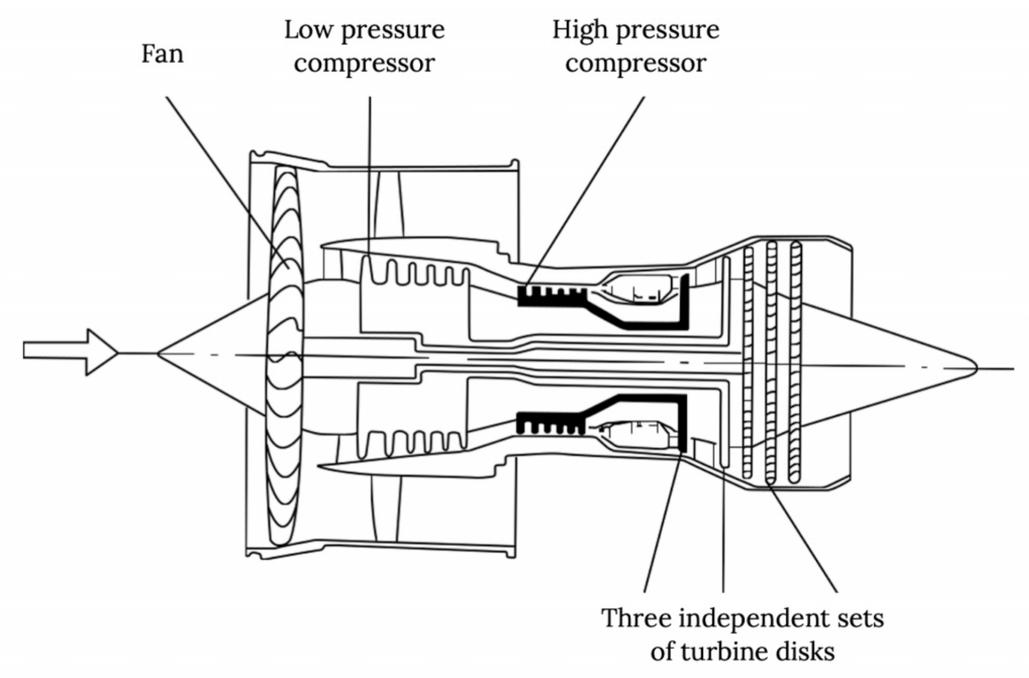

Figure R2.2: Illustration of a Turbo-Fan or Fan-Jet Engine

So, for a given amount of thrust we will need a given amount of momentum change of the air going through the engine between its entrance and exit. We will look at how this momentum change mathematically accounts for the thrust a little later. The point here is that the most efficient way to get this momentum change with minimal losses is to accelerate a large mass of air by a small amount (small change in speed). This means that the bigger the bypass ratio (the ratio of the bypass air mass to the mass of air going through the engine core) the more efficient the engine is. But there is a limit to this.

As the fan jet engine gets larger due to higher bypass ratio design, the engine enclosure (nacelle) also gets larger and it produces more drag. So at some point it makes more sense to replace the bypass "fan" with a large propeller. The result is the "turboprop" engine.

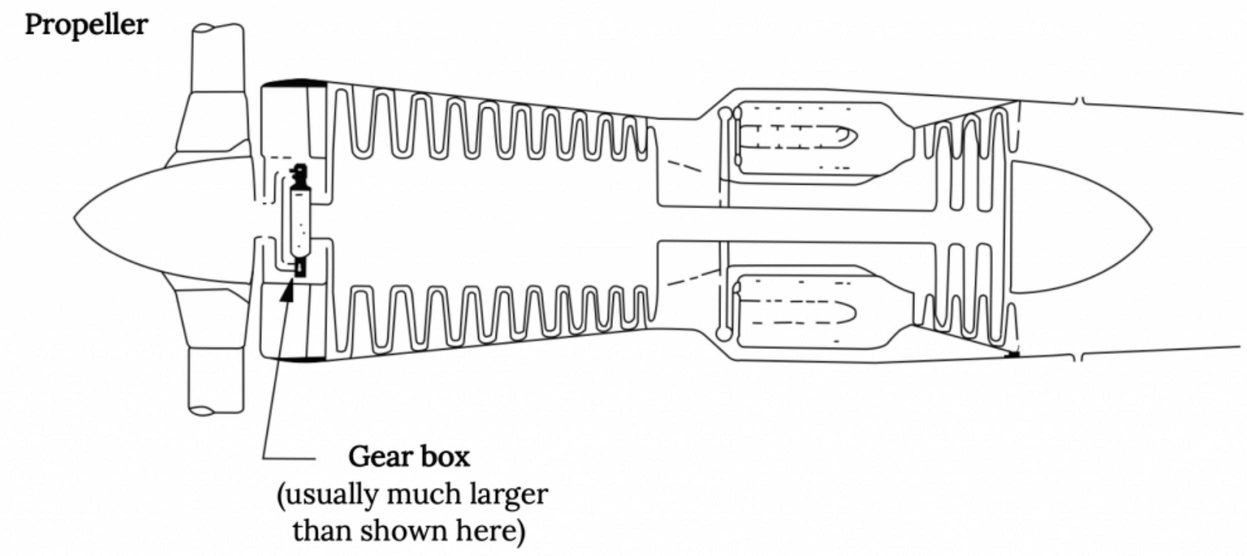

Figure R2.3: Turboprop Engine Illustration

In the turboprop engine the flow through the engine core is really not used to produce any significant thrust. The exhaust turbine is designed to take all of the energy it can from the exhaust to drive the propeller and all of the engine's thrust comes from the flow through the propeller. In a turboprop engine the amount of thrust that comes from the core flow is so negligible that, in some engine designs, the core flow actually goes "backwards".

We might wonder, if the turboprop engine is more efficient than the fan-jet, which is in turn more efficient than the turbojet, why fan-jets are the engine of choice for most airplanes today? The answer is in the desired speed of flight.

Just as there is a big drag rise on a wing as it approaches the speed of sound, there are drag type losses on a propeller blade when its speed approaches Mach one. In fact, for a given propeller rotation speed, the limit on practical diameter for the prop is determined by the radius at which the propeller blade section reaches its critical Mach number. And, since the airspeed seen by the propeller blades is a function of both their rotational speed and the speed of the airplane, this limits the speed of the aircraft. Propeller design can extend this speed range somewhat with things like swept blade tips but the turboprop will always impose limits on aircraft cruise speeds.

Also, it turns out that the rotational speeds needed for a turboprop propeller are an order of magnitude below those of those in an efficient turbine core and this necessitates speed reduction gears between the turbine and the prop and this introduces both noise and vibrations that are not found in the fan-jet.

R 2.2 Propeller Engines

So what is the difference between a turboprop and a propeller driven by an internal combustion engine? From the point of view of the thrust provided by the propeller, there isn't much difference. The difference is in the engine and the gearing that drives the propeller.

The turboprop is driven by a small turbine (jet) engine that sends as much of its energy as possible to the propeller through a driveshaft and reduction gear system. The IC engine propeller is attached to the driveshaft of an internal combustion engine that, like most automobile engines, uses the burning of gasoline or diesel fuel in a piston/cylinder type motor to turn the shaft.

Today most internal combustion driven propeller engines are found on smaller general aviation aircraft. This type of engine has provided reliable, affordable power for airplanes since the first flight of the Wright brothers in 1903. Over the years there have been many fascinating variations of IC engine used in airplanes, from the "rotary" engines of World War I in which the driveshaft was attached to the airplane and the propeller and engine actually rotated together around the shaft, to the massive piston engines of the 1940s and 1950s with dozens of cylinders arranged around the driveshaft like kernels on an ear of corn, to the four and six cylinder, car type but air cooled, engines usually found on today's GA airplanes. These many varieties of IC engines would make an interesting and exhausting study in themselves, but that is beyond the scope of this text.

As far as we will be concerned, a propeller engine is a propeller engine, whether driven by a turbine or an IC engine or a rubber band. We will merely be concerned with the "power" output by the engine and we will call this the "shaft power" regardless of the type of engine that drives the shaft.

R 2.3 Thrust and Power

This brings us to the main difference in the way we will talk about propulsion for jet and prop engines. For jet powered aircraft, whether turbojets or fanjets, we will characterize the propulsion properties of the airplane in terms of thrust. For propeller powered airplanes, whether the propeller is attached to an IC engine or a turbine, we will talk about performance in terms of power.

Power and thrust are merely two different ways of looking at aircraft propulsion and performance. They are directly related to each other through speed.

Power = (Thrust)(Velocity)

While we normally talk about jet propulsion in terms of thrust and propeller propulsion in terms of power, there is little reason beyond convention that we must do so. We could talk about the power of a jet engine and the thrust of a propeller and we sometimes do so. Perhaps one reason for this distinction is that we will later find it convenient to look at the variation of both power and thrust with velocity and we will find that it is common to assume that thrust is fairly constant with speed for a jet and power is fairly constant with speed for a propeller driven plane.

The units normally associated with power and thrust, respectively, are pounds and horsepower. Yes, these are "politically incorrect" units; nonetheless, they are far more widely used than Newtons and Watts, their SI equivalents. [Have you ever heard anyone talk about the power of their car engine in watts?] This, of course, means we need to learn how the unit of horsepower relates to basic units in the "English" system.

1 horsepower = 550 foot-pounds / second.

[A bit of engineering trivia: this conversion was used so often in the days of slide rule calculations that most slide rules had a special mark on them at the 550 location on the slide.]

R 2.4 Thrust and Conservation Laws

To find out how things like altitude and airspeed affect thrust and power we need to take a look at how the air goes through the propeller or the jet engine when an airplane is in flight and how the momentum of the air changes as it follows that path. To do this we will need to look at two "conservation laws", conservation of mass and conservation of momentum.

R 2.4.1 Mass Conservation

In its simplest concept mass conservation is often stated something like "mass cannot be either created or destroyed; i.e., it is constant or conserved". This is often accompanied by a qualifier noting that, in an atomic reaction, mass can actually be created (fusion reaction) or destroyed (fission reaction). This is an interesting way to look at mass if one is looking at the mass in the universe or in a closed container but it doesn't help us when talking about engines. We need to look at the conservation of mass in a flow; that is, in the air going through a room or a pipe or a propeller or a jet engine.

If we had a sealed room filled with air it would be simple to state that the amount of air in the room is a constant. We could have people and plants in the room with the chemical reactions that are part of human breathing and plant chemistry continually altering the chemical constituencies in the "air"; nonetheless, the total mass of the "air" would remain constant.

The picture changes when we add ventilation to the room, either by using a forced ventilation system such as an air conditioning or heating system or by simply opening windows and doors. With either system there would be new air coming into the windows, doors, or intake vents and old air going out of other windows, doors, or exhaust vents. If we had a room with only a forced air inlet and no exhaust, the mass of air in the room would increase as air came in through the inlet. To accommodate this increasing mass the room would either have to expand like a balloon or the pressure and density of the air in the room would have to increase. Note that, if we assumed that the air was "incompressible" it would be impossible to pump new air into the room without providing an exhaust for an equal amount of air to escape. This would require conservation in the mass of the air in the room.

So in the example of room ventilation, conservation of mass for the air in the room would simply mean that, as a mass of new air enters the room, the same amount of mass of air must leave the room. Room or window air conditioners work this way, taking a given mass flow rate from the room, sending it through cooling coils, and returning that same mass flow rate to the room after some heat was removed from the air.

This brings us to the subject of mass flow rate, often called "m-dot" and given the symbol of a lower case "m" with a dot on top of the letter to represent a time derivative of the mass, mass per unit time, **dm/dt**.

When we speak of a room with vents or doors and windows we must talk about mass flow rates, and we say that in order to have mass conservation we must have no change in the mass within the room per unit time, simply another way of saying that the amount of mass that goes in during a given time period must equal the amount of mass that goes out in that same time. This is stated as:

$$\mathbf{dm/dt = 0.}$$

In other words, the amount of mass in the room does not change with time.

We often put this in equation form, saying that

$$\mathbf{dm/dt = \Sigma\ \rho VA = 0.}$$

Here, we are saying that the mass flow rate is equal to the density of the air, multiplied by its speed, as it passes through an area of size "A". In other words, if air at sea level density is blowing through a window at a speed of 20 feet-per-second and if that window has an opening of 2 feet by 4 feet, we can calculate the mass of air per unit time that is passing through the window.

$$\mathbf{dm/dt = \rho VA = (0.002378\ sl/ft^3).(20\ ft/sec.).(2\ ft\ x\ 4\ ft) = 0.3805\ sl/sec.}$$

[Note here that the units of mass rate of flow have been found to be slugs per second. **In the SI system they would be found in** kilograms per second **and in a version of the "English" system often used in fields such as Mechanical Engineering the units of mass flow rate would be** pounds-mass per second**.]**

Now, if conservation of mass is met for the air in the room, the same mass of air per unit time must be going out of another opening or openings.

$$\mathbf{(dm/dt)_{in} + (dm/dt)_{out} = 0}$$

or

$$(\rho VA)_{in} + (\rho VA)_{out} = 0$$

So, if there is a single window letting in the air flow found above and the exit is through a door, we can use conservation of mass to determine the speed of the air going out the door.

$$(\rho VA)_{out} = -(\rho VA)_{in}\ .$$

Just as three factors, the size (area) of the window, the speed of the air flow through the window, and the density of the air, determined the "mass flow rate" of the air coming into the room, the same three things determine the exit mass flow rate. In reality, all three of these things could be different at the exit (door, in this case), so, if we want to find the speed of the exiting air we must know both the area of the door and the density of the air at the door. However, there is no reason why the air flowing through the room would have changed density so we are safe in assuming "incompressible" flow, that is, density is constant. This gives us a simple equation:

$$(VA)_{out} = -(VA)_{in}.$$

So, if the door is 3 feet wide by 7 feet high, giving an area of 21 ft^2, while the window had an area of 8 ft^2, the speed of the air going out the door is:

$$V_{out}/V_{in} = -A_{in}/A_{out}$$

or

$$V_{out} = -V_{in}(A_{in}/A_{out})$$

Or in this case,

$$V_{out} = -20\ \text{ft/sec.}\ (8\ \text{ft}^2\ /\ 21\ \text{ft}^2) = -7.62\ \text{ft/sec.}$$

Now, why is there a minus sign with the exit velocity? This is because we, for no real reason, chose to give a positive sense to the velocity going in the window and since velocity is a vector; i.e., it has a direction, we have designated the flow of air into the room as positive. This means that the negative sign on the exit air velocity tells us it is going out of the room. While this may seem like an un-needed complication here, there are cases where it can help us figure out what is happening.

For example, suppose there are five windows and two doors in our room and we are told that air is coming into all five windows at a certain speed and is going out one door at a given speed, what is happening at the other door? Is the flow through that second door going into or out of the room?

We would have to write the complete equation for mass flow conservation to find both the amount and direction of the flow through the second door.

$$(\rho VA)_{w1} + (\rho VA)_{w2} + (\rho VA)_{w3} + (\rho VA)_{w4} + (\rho VA)_{w5} - (\rho VA)_{D1} + (\rho VA)_{D2} = 0$$

.

Note that we have assigned positive values to the flow through all the windows since we were told that the flow was coming into all of them. We have also assigned a negative value to the flow out the first door since the flow was said to be out of that door. Also note that we did not assign a sign (direction) to the flow through the second door because we have no idea which way it is going. Now, if we put all the needed information for the five windows and first door into the terms in the equation and if we know the area of the second door and assume that density is the same everywhere (incompressible flow), we can solve for the speed (velocity) of the flow using the mass conservation relationship above and find both the magnitude of the speed and its direction (sign).

Exercise 2.1

Try doing the above problem assuming that all five windows are 2 ft X 4 ft in size and that air is blowing in at 20 ft/sec.

Assume that the two doors are both 3 ft X 7 ft in size and that the flow out of the first door is measured at 50 ft/sec.

Find the speed and direction of the flow through the second door.

NOTE: Here we considered all flow INTO our “system” or “control volume” as POSITIVE, and all flow OUT of the system as NEGATIVE. If we do not know its direction, we assume it is positive in value and the solution of the equation will give us a negative answer if we assumed the wrong direction. Later, when we look at the Momentum Equation we will use a unit vector, n, to assign a positive direction within our chosen axis system for flow through an opening, and that unit vector will always point OUT of the system.

OK, that was simple enough, but how do we deal with mass conservation when we are looking at flow through a jet engine or a propeller?

Mass conservation through a propeller or a jet engine works just like mass conservation in a flow going through a room. In fact, for the jet engine it is even simpler than the average room because there is only one well defined entrance and exit, or is that really the case?

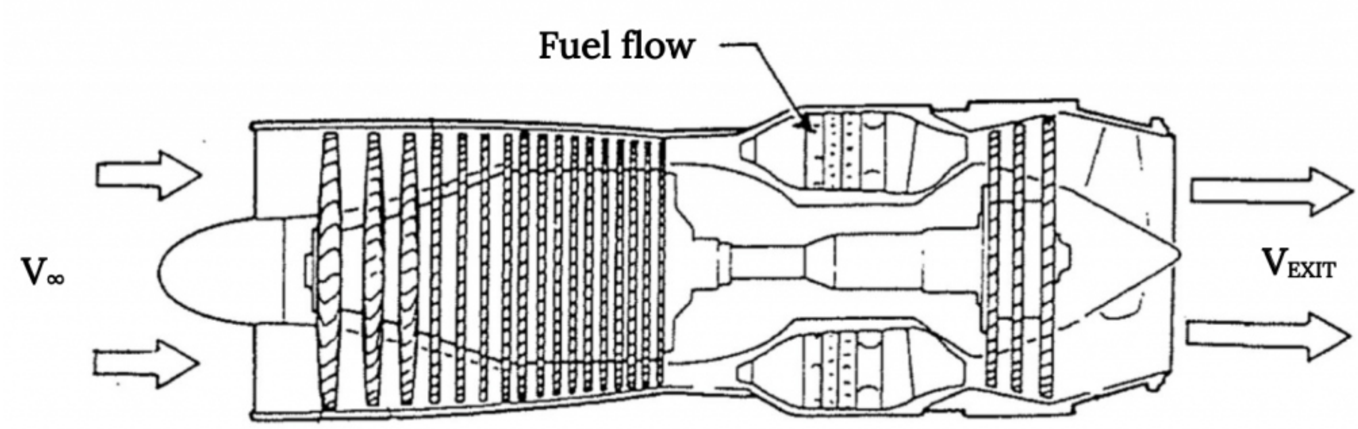

Figure R2.4: Mass Flows for a Turbo-Jet Engine

Technically, there is a second source of incoming mass in any jet engine and that is the mass flow of fuel coming into the engine. There is air coming into the engine inlet of a known area at (supposedly) a known speed and density, but the flow going out of the engine isn't really just air, it is the gas that comes from combustion of the incoming air and the incoming fuel. The mass rate of flow coming out of the exit must account for both the mass of the entering air and the entering fuel, so our mass conservation relationship must recognize this.

$$\mathbf{(dm/dt)_{inlet} + (dm/dt)_{fuel} + (dm/dt)_{exhaust} = 0}$$

We would normally write this as:

$$\mathbf{(\rho AV)_{inlet} + (dm/dt)_{fuel} = -\,(\rho AV)_{exhaust}}\ .$$

So we must know the mass flow rate of the fuel. Usually the mass flow rate of the fuel is very small compared to that of the inlet air so perhaps that term can be neglected. So what's the big deal? If we can neglect the fuel flow rate we are back to the one window, one door example and life is easy. Unfortunately there is another factor that we must not forget and that is density. Usually the flow through the exhaust of a jet engine is going pretty fast, near or greater than the speed of sound; i.e., we can no longer assume that density is constant as we did in the room ventilation example.

To solve this problem we have to know either the exit flow density or its speed in order to solve the equation for the "other" parameter (exit speed of density), and since the fuel mass flow contributes to this exit density we probably should not assume it to be negligible even if its velocity is almost negligible.

Making mass conservation for a jet even more complex is the fact that most of today's jets are "fan jets" where there are essentially two entrance flows, one that goes through the engine core, mixing with the fuel to form a high speed exhaust, and another, larger, flow that is accelerated through the fan. We might analyze this problem by accounting for two separate entrance flows and two separate exit flows, or by assuming (correctly in most cases) that the two exit flows mix before leaving the engine covering or "nacelle" to form a single, mixed exhaust.

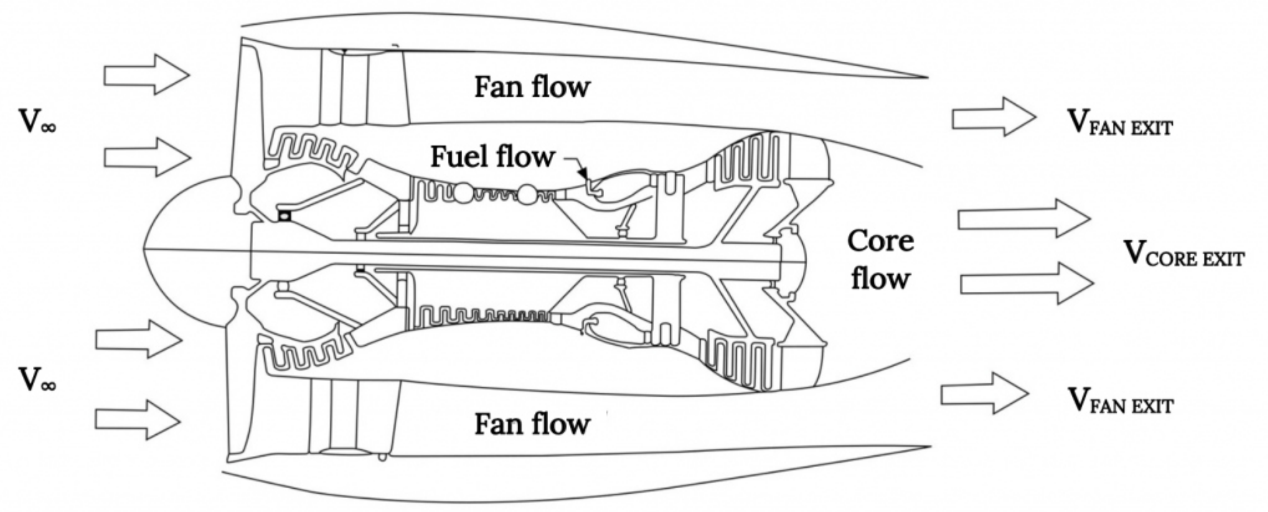

Figure R2.5: Flows through a Fan-Jet Engine

In any case, the jet engine flow problem is a little simpler for many people to understand than the propeller flow problem because the entrance and exit areas are normally pretty well defined. How do we define entrance and exit flows when we draw the flow through a propeller?

When a flow is going through a propeller, just what are the entrance and exit areas? There really is no physical entrance or exit. Of course, we know the flow goes through the propeller itself, so, is the propeller area used for both the flow "entrance" and its "exit"? This hardly makes sense. How can we talk about the changes in the flow between the entrance and exit when there is no physical distance between the entrance and exit?

Let's look at what we know intuitively about the flow through a propeller (or a fan). We know that the flow behind the propeller or fan is moving faster than the flow in front of it. We know that in some way, a way that can be analyzed in detail by looking at each propeller or fan blade as a little rotating wing that does work on the air, the propeller essentially adds energy to the flow. We also know, if we think about it a bit, that we cannot use Bernoulli's equation to compare the flow upstream and downstream of the prop or fan because energy is added at the prop or fan and Bernoulli's equation assumes that energy is constant through the flow. We also know that there are limits to what a fan or propeller can do to accelerate a flow due to tip speed limits on the blades themselves and these limits essentially mean that we can pretty safely assume incompressible flow through the system.

Putting all these facts together, we can draw a picture that looks something like the flow should appear through a propeller or fan. We know that somewhere upstream of the propeller the flow is undisturbed, it is at “free-stream” or atmospheric conditions. We know that somewhere downstream of the prop the static pressure in the mass of air that went through the propeller must return to its free-stream value.

We will imagine a “stream-tube”, or three-dimensional path of constant mass flow, that starts out in the undisturbed flow upstream of the prop, goes through the prop (becoming the same diameter as the prop at that location, and then continues downstream until the point we mentioned above where the static pressure has returned to the atmospheric value. What must that “stream-tube” look like?

A stream-tube is defined as a three-dimensional flow path in which the mass flow rate is the same at every point along its journey. Essentially, as shown in the following figure, the upstream cross sectional area of the stream-tube (its “capture” area) must have the same amount of mass flow rate through it as goes through the prop itself. Likewise, the “exit” area for our stream-tube must also allow passage of the same mass flow as went through the capture area and the prop “disk” area.

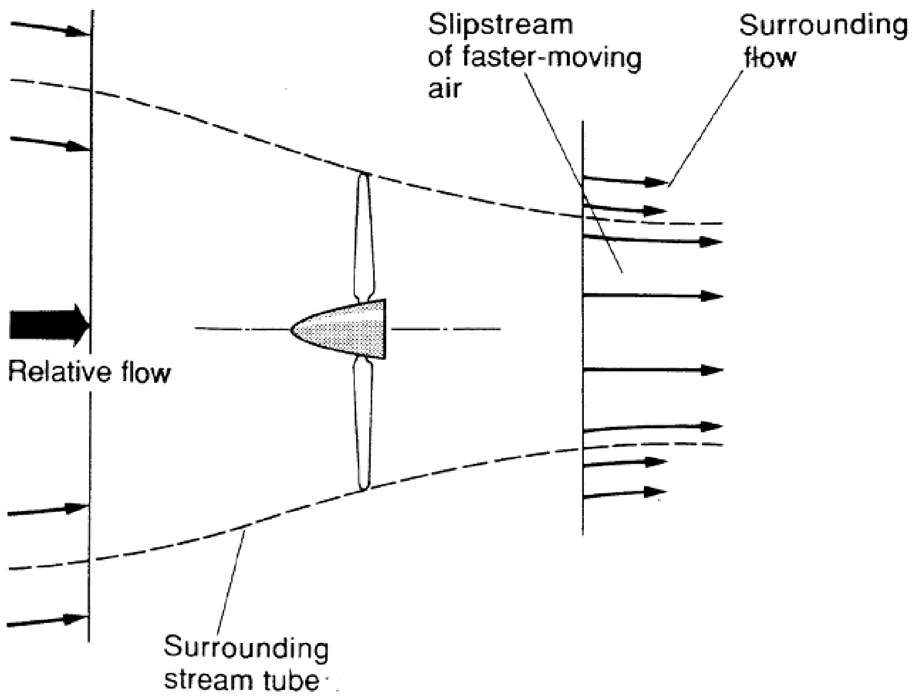

Figure R2.6: The Stream-tube Concept for a Propeller Flow

So why is the “stream-tube” in the figure above getting progressively smaller as the flow goes from the atmospheric pressure, free-stream capture area to the atmospheric pressure exit area somewhere downstream? First, we know the velocity in the exit area must be larger than in the capture (inlet) area; hence, if mass flow rate is the same and the flow is incompressible, the area must decrease in inverse proportion to the speed increases. But why do we assume that this area decrease (and speed increase) is smooth and continuous? Isn't there simply a big jump in speed across the propeller disk?

Well, we probably could analyze everything in terms of some kind of instantaneous jump in flow speed at the propeller disk based on an energy balance, assuming that the energy added by the prop produces a sudden increase in flow kinetic energy and speed. However, we know from real world measurements that this speed increase is not instantaneous and that part of the increase is seen in front of the propeller as the flow speeds up from its "free-stream" velocity to the velocity right at the front of the prop disk. We also know that it takes a couple of propeller diameters downstream before the flow in the "prop wash" reaches top speed. Based on this combination of reality and convenience, we choose to model the speed increase as a continuous one within a "stream-tube" shaped like a converging nozzle of circular cross section, as shown in the figure above.

This ideal picture, of course, ignores a lot of things such as the losses due to turbulence and rotational flow effects; nonetheless, it is one that works fairly well. So, what do we propose to do with this model and with the model of the flow through a jet engine? What we want to do is use these to determine how thrust is produced and find the properties that determine how thrust varies with speed and altitude.

R 2.4.2 Thrust

Our goal is to take a look at propulsion. How do we account for thrust or power in aircraft performance evaluations?

There are two ways to do this. One would look at energy additions to the flow and a conservation of energy. But, as noted in the propeller discussion above, this would be very tedious, requiring us to do aerodynamic analyses of each propeller blade, accounting for losses due to compressibility effects near the blade tips and for the interference between the flow over one blade and the following blade. There are books on how to do this, the oldest of which went under titles such as "Airscrew Theory", and this is the type of analysis that companies making propellers must use. The problem would be even more interesting in a jet engine with us having to account for energy gains and losses due to flow around compressor blades and turbine blades, combustion of fuel, and flow though internal nozzles.

It turns out that the simplest way to look at thrust is to look at momentum conservation.

R 2.4.3 Momentum conservation

Momentum conservation, like mass conservation and energy conservation, is one of the "big three" conservation "laws" that we all saw somewhere back in some Physics course. On the face of it, conservation of momentum is a simple concept. Just as in mass conservation of a flow we must account for all mass flows that enter or leave the flow-field under consideration, in looking at momentum conservation we must consider all things that could possibly account for momentum changes and, ultimately, in forces.

Essentially, the concept we are looking at is one that says that the change of momentum in a body or "system" with time must equal the forces on that body or system. The idea is that either forces on a body or system will cause its momentum to change or a momentum change within the system or body will result in a force.

$$\mathbf{(d/dt)(momentum) = (d/dt).(mv) = Force}$$

This is a simple idea that is often made to look very complicated when derived in most textbooks on fluid mechanics. If, for example, you kick a soccer ball, the force you impart to the ball will result in a change in momentum in the ball. If the ball was standing still before it was kicked, the force will change its momentum from zero to a value related to the force of the impact and the mass of the ball. If the ball was already moving, the kick may send moving in another direction, so this concept is directional; i.e., it is a vector concept, as would be expected when a force is involved.

In looking at aircraft propulsion we are interested in the reverse action; that is, creating a momentum change in order to get a force, changing the momentum of the flow through the engine or propeller to create thrust.

Just as in working with Bernoulli's equation we had a choice of modeling the flow as a moving fluid going past a wing or body, or as a body moving through still air, we have to make a similar choice here. We will, for example, choose to look at the flow through a jet engine or a propeller as if the engine (prop) is standing still and the flow is moving past it. This is really a choice between having to consider the momentum of the moving engine or the momentum of the moving air. Either view will give the same answer for the thrust, but the moving air model is usually a little easier to work with. Either way, we must be very careful to account for all possible momentum changes in both the engine and the flow.

We first need to look at what kinds of momentum changes might be present as well as what kinds of forces might be involved. To do this, let's look at one of the simplest of "jet" engines, but one of the hardest to analyze, a rubber balloon that is inflated and released.

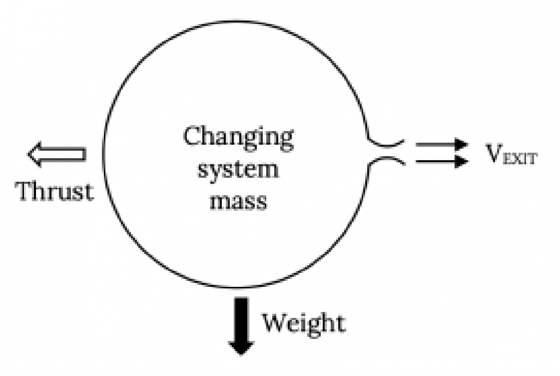

Figure R2.7: A Balloon as a Simple Jet

Let's look at the illustration above and list all of the ways that momentum might play a role as well as all the forces involved. There will be at least two sources of change of momentum for the balloon and at least three forces that might be involved.

Momentum change sources:

1. The change in momentum of the balloon (the "system") with time because of the change in mass of air inside the balloon with time and due to any changes in velocity of that mass. [As the balloon expels air through its inlet/outlet, the mass of the "system" itself is changing and, even if its speed was constant, the momentum of the system would change.]
2. The momentum of the flow exiting the "system" (balloon); i.e., the mass flow of air through the inlet/outlet (jet) multiplied by its velocity.

Both of these terms above are directional because of the velocities associated with them. The momentum of the balloon itself is related to the balloon's velocity and the momentum of the flow through the exit is obviously related to the direction of the flow through the exit.

Forces on the balloon:

1. The major force on the balloon will be the one we choose to call **thrust**. This is essentially what we are trying to find.
2. nother force on the balloon that we might not think of at first is that due to gravity; i.e., its **weight**.
3. Finally, there would be any **pressure forces** caused by pressures acting on areas. These might include pressure drag on the balloon itself or differences in pressure across system boundaries. Often we find that pressure forces tend to balance out or sum to zero but there are some cases where these must be considered.

4. We could also consider friction forces or even electromagnetic or other forces if we wished but we will limit ourselves to the first three forces mentioned above.

How do we describe each of these sources of momentum change or forces in a very general way? Let's look at each of these listed above.

1. The change in momentum of the "system" with time involves the changes in both mass and velocity of the system:

d/dt [(mass).(velocity)] ,

and, since the system mass can be written as its density times its volume, we might look at this as

d/dt [(density).(volume).(velocity)]

2. The change in momentum due to the flow out of (and in general) into the system with time is essentially the mass rate of flow (dm/dt) across any entrances or exits multiplied by the speed at which that mass is passing through the entrance or exit areas. We know that the mass rate of flow is the density multiplied by both the velocity and the flow cross sectional area, so this term is expressed as:

(dm/dt)(Velocity) = (density).(velocity).(area).(velocity) .

3. The weight is just the mass (density x volume) multiplied by the acceleration of gravity.

(Density).(volume)(g) .

4. The pressure forces are just pressures acting on an area:

(Pressure).(area)

Now, to work with all these we need to put them together in the form of some kind of equation. The equation must essentially say that the momentum changes must be balanced by the forces involved. This can be thought of as forces causing momentum change (the soccer player's foot kicks the ball) or momentum changes causing forces (the thrust from a released balloon). The equation that usually results from a much more formal derivation is a complicated looking, vector relationship called the momentum equation.

R 2.4.4 The Momentum Equation

$$\underbrace{\frac{d}{dt}\iiint_R \rho\bar{V}dR + \iint_s \rho\bar{V}\left(\bar{V}\cdot\hat{n}\right)dS}_{\text{momentum change in fluid}} = \underbrace{-Fe - \iint_S \rho\hat{n}ds + \iiint_R \rho\bar{g}dR}_{\text{Forces causing momentum change}}$$

Before you panic at the vector notation and the double and triple integrals, take a deep breath and see how these terms relate to the ones presented above.

A triple integral over "R" (the mathematical "region" or the "system") is nothing but the volume. If the density and velocity of everything contained in the region or system is the same; i.e., if it is a homogeneous system, then this term is nothing but the time derivative of the density times the volume times the velocity; i.e., of the system mass times its speed as it was stated in the section above.

So why do we make it so complicated looking? One reason might be just to impress our friends in liberal arts or to show our parents how hard our courses are. A better reason is to allow the momentum equation to account for non-homogenous system effects. Suppose, for example, that our "system" was not a balloon filled with nice homogenous air, but a baseball or golf ball with a solid filling made of several layers, each with different densities, and further, that someone had made the ball with its heavier core somewhat "off center". You can buy such "trick" golf balls at novelty shops and when you hit them with a golf club (impart a force to the system!), instead of traveling in a straight line they wobble around as if they were drunk. Because the momentum equation can account for this "non-homogeneity" it can account for the wobbly motion of the trick golf ball. In a similar way the last term on the right, the gravity or weight term, can account for gravitational effects on a non-homogeneous mass.

Two of the terms in the equation have double integrals. You might have guessed by now that the double integral over a distance "S" must relate to some kind of area, and looking at the terms would confirm that. The double integral term on the left relates to the momentum carried with a flow into or out of the system over an entrance or exit area. This term is written in this complex way to be able to account for non-uniform velocities over the entrance or exit and even for non-uniform densities over these areas. If we assume that all of the entrance or exit flow is the same fluid moving at the same speed then the density and velocity terms can come outside the integral and the integral itself becomes nothing but the entrance or exit area. So, again, why make it look this complicated? Well, in many cases the flow out of an opening is not uniform because friction forces cause it to move more slowly near the edges of the opening than at the center, and this comprehensive form of the momentum equation can account for that if we want it to do so. Similarly, the pressure term on the right hand side of the equation can account for pressure variation over a surface.

What about the vector notation, the **V•n** term, in the double integral term on the left? First, the momentum equation is a vector equation, meaning that each of the terms has a direction and the solution of the equation for a force such as thrust or drag will give both a magnitude and a direction for that force. Second, for one of the terms on each side of the equation, it is only the parameter "normal" to a defined surface or boundary that will cause a force and the "unit vector" **n** is used to designate that normal direction. We will always define the direction for this unit vector as pointing out of the system, even where the flow is coming into the system.

What then do these vector quantities mean? Each of the velocities can have up to three terms in them, one associated with each direction in a selected axis system. In the case of velocity in a conventional x, y, z axis system, we normally use the terms u, v, and w to designate the x, y, and z components of velocity, respectively. So we would write a velocity vector as:

$$\mathbf{V} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}\,,$$

Where **i**, **j**, and **k** are the unit vectors in the positive x, y, and z directions. In a similar manner, the gravity vector could have up to three components; however, we sometimes try to define our coordinate system so one axis is in the direction of gravitational acceleration to eliminate two of these components.

The **V•n** term is then the "dot product" of two vectors where both the **V** and the **n** vectors may have x, y, and z components, but only the like directed components multiply with each other, then sum to give a "scalar" quantity with a magnitude but no direction. So, if the velocity is in the same direction as the normal vector (as is often the case for flows into or out of a system) the result is simply the magnitude of the velocity. At the other extreme, if the velocity is at a 90 degree angle to the normal vector the dot product gives zero.

Again one might ask, why make things so complicated with all these integrals and vectors and dot products and the like? It is done this way because it is a very versatile equation that can account for fully three dimensional motion. For example, should that soccer player kick the ball at a 90 degree angle to its existing direction of motion, this relationship would, provided we knew the force of the kick and the mass and velocity of the ball, tell us the ball's new direction and speed even though the direction would be in neither the original direction of motion or in the direction of the kick. Similarly, if there is a bend in a pipe we can use the equation to find the magnitude and direction of the force that will occur when water flows through that bend in the pipe.

The trick to using the momentum equation is to follow the rule of thumb that often distinguishes an engineer from a pure scientist or mathematician; that is to use proper alignment of axis systems and to set system boundaries and to make good assumptions that will eliminate as much of the complexity as possible. Fortunately we can do a lot of this as we use the momentum equation to look at thrust.

R 2.4.5 Thrust (again)

Let's look at the flow through a jet engine in terms of the momentum equation.

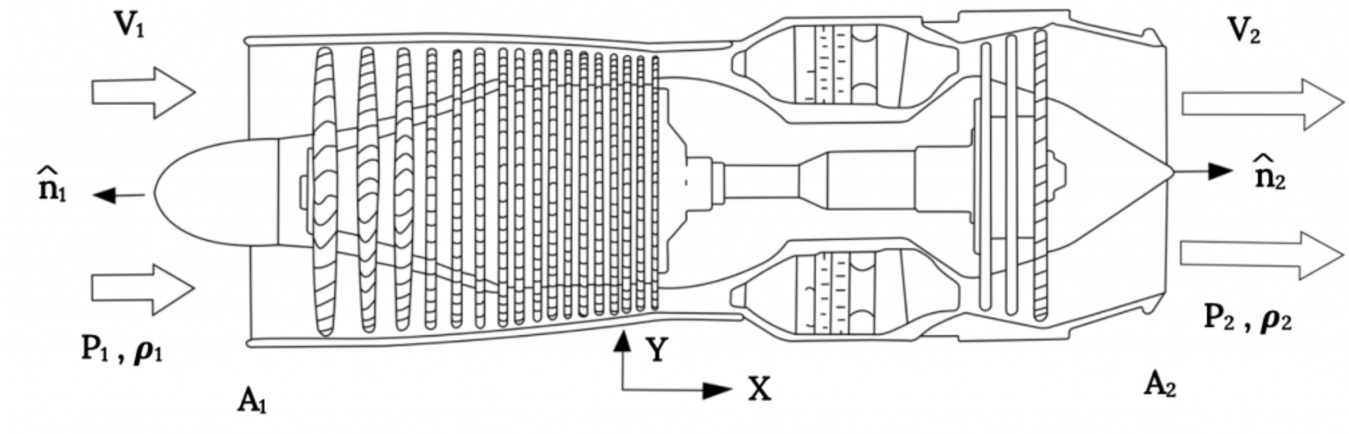

Figure R2.8: Momentum Equation Terms for Turbo-Jet

In the illustration above we have aligned the engine with the "x" axis and we have flow coming into the engine inlet in the x direction and another flow coming out of the engine, also in the x direction. We want to know the thrust as a function of this information. Let's look at what we can say about the various terms in the momentum equation.

The **first term on the left hand** side of the equation is a "time dependent" term to account for changes in momentum of the "system" itself with time. Here our system is the entire jet engine, and, if we assume that the engine (airplane) is in "steady" or constant speed flight, there is nothing in the term (density, velocity, or volume) that is changing with time. So, **this term is zero.**

The **second term on the left** accounts for the momentum carried into or out of the system as flow enters or leaves. Obviously, this term will not go away since we have air coming into the engine and combustion products going out the other end. First we need to ask if these two flows are "uniform" across their respective entrance or exit areas. If we can assume that they are uniform and can assume that all of the flow has the same density, then this term (actually two terms, one for the entrance and one for the exit) becomes:

$$\rho_1 \mathbf{V_1}(\mathbf{V_1} \bullet \mathbf{n_1})A_1 + \rho_2 \mathbf{V_2}(\mathbf{V_2} \bullet \mathbf{n_2})A_2 \,.$$

Now, what do we do with the vector business? The flows are both in the positive x direction. The first normal unit vector is in the negative x direction while the second is in the positive x direction. The result is:

$$-\rho_1 V_1^2 A_1 + \rho_2 V_2^2 A_2$$, (all in the x direction)

Ok, that takes care of the left hand side of the momentum equation. What happens to the terms on the right? The **first term on the right** is the "external" force which, in this case, is the thrust we want to find. The second term on the right is perhaps the hardest to understand physically so we will come back to this.

The **third term on the right** is the gravity term, really the weight. If we assume that this is acting at a 90 degree angle to the x axis or the direction of flight and thus is perpendicular to all the other forces and momentum changes in which we have an interest we might simply neglect this term. Actually it would be more proper to say that its component in the x direction is zero. In reality, this term would tell us that there must be a force to oppose the weight and this would be the aerodynamic lift which, in turn, would be related in the momentum equation to a vertical change in momentum of the flow as it moved around the wing and the corresponding pressure distribution around the wing. In essence we are choosing to ignore the vertical components of the forces and momentum changes.

Now let's go back to the **second term on the right**, the only term with pressures in it. This term looks at forces caused by pressures acting on areas. If we were looking at the lift force we would use this term to integrate the pressure distribution around a wing. On the engine we will assume that the flows over the outside of the engine casing or nacelle are symmetrical, that is that the same pressure distribution exists on the top as on the bottom of the nacelle, and that the net effect of these pressures (at least in the x direction) is zero. But what about pressures across the entrance and exit?

(Pressures across the entrance and exit?) How can this mean anything when there are no real surfaces here, just flows going in or out? This is where the concept of a "system" boundary gets interesting. When there is a real boundary such as the engine nacelle the bounds of the system are easy to understand. But these "open" ends of the "system" are also boundaries over which we must account for all the terms in the equation. In other words, just as we had to account for the flow through these somewhat imaginary boundaries, we must also account for pressure changes across them. But how can these pressures cause real forces when there are no "real" surfaces for them to act on? This becomes one of those "leaps of faith" that we often must take in applying equations to physical situations.

No, there are no surfaces at the entrance and exit where the pressure differences across the surface cause a force; however, we must account for them anyway if there is a pressure differential between the surrounding atmosphere and the flow into the entrance and out of the exit. This is probably the easiest to understand when we look at the exhaust flow.

Coming out of the exhaust is a flow of the combustion products of air and fuel that has been heated and pressurized in the engine combustor. After combustion we want to turn that added energy into as high a momentum (the second term on the left hand side of the momentum equation) as possible. This means that we want to "expand" the gas in a exit nozzle, lowering its pressure with a corresponding increase in speed (ala Bernoulli's equation) as much as possible to get a high momentum. The ideal situation is to expand it to the point where the exiting gas has the same pressure as the atmosphere into which it will exit. If it expands too much or too little there will be losses as the flow pressure comes to equilibrium with the atmosphere. It turns out (and the momentum equation essentially tells us this) that the losses from over or under-expansion are equivalent to the pressure force that would be on a surface with the same area as the exit with a pressure difference equal to that under or over-expansion delta-P. This is why some high performance jets have variable area exit nozzles on their engines.

The same problem can occur to a lesser degree at the engine inlet but a properly designed engine inlet and compressor section can eliminate most of the loss.

So, how do we deal with this pressure term? We either must know the differences between the atmospheric pressure and those of the entrance and exit flows or compute values for these terms, being careful to account properly for the unit vector signs, or we must assume that these losses are negligible. Let us take the easy way out and assume that these terms are of little consequence because we have a properly designed engine.

OK, where does this leave us? We have ended up with a relatively simple equation:

$$-\rho_1 V_1^2 A_1 + \rho_2 V_2^2 A_2 = -Fe$$

Rearranging this gives:

$$\text{Thrust} = Fe = \rho_1 V_1^2 A_1 - \rho_2 V_2^2 A_2\ .$$

Looking at this we see that the second term on the right will be much greater than the first term, so, the thrust will have a negative sign. Is this ok? Sure it is. It just says the thrust force is in the negative x direction, toward the left, just as we want it to be.

(That sure was a lot of work to get a fairly obvious answer; the thrust is equal to the momentum change from engine inlet to exit! Isn't this somewhat intuitive? Yes, it sort of is intuitive to many of us. On the other hand it does keep the mathematicians and theoreticians in our midst happy, and more importantly, it tells us that in arriving at this "intuitive" answer we have made some important assumptions about pressure behavior and axis system selection).

Ok, now that we have all that under our belts what important facts about propulsion can be drawn from this solution? To see this, let's play around with the equation above a little by accounting for conservation of mass.

Now, recognizing that V_1 is our "free stream" speed, V_∞, and that the entering air density is also that of the atmosphere, ρ_∞, we can write this as

$$\textbf{Thrust} = \rho_\infty V_\infty^2 A_1 - \rho_2 V_2^2 A_2$$

And looking only at the magnitude of the thrust (as said above, the relationship above gives a negative thrust, signifying simply that it is to the left in our original illustration of the engine moving from right to left)

$$\textbf{Thrust} = \rho_2 V_2^2 A_2 - \rho_\infty V_\infty^2 A_1$$

We now define the **"static thrust"** as $\mathbf{T_0}$, the thrust when the engine is standing still ($V_\infty = 0$). This is the amount of thrust that would be measured on an engine test stand and is a standard piece of information that would exist for any engine.

$$T_0 = \rho_2 V_2^2 A_2$$

This allows us to rewrite the general thrust relationship as:

$$\textbf{Thrust} = T_0 - \rho_\infty V_\infty^2 A_1 \,,$$

Or simply as:

$$T = T_0 - a\, V_\infty^2 \,,$$

Where:

$$a = \rho_\infty A_1$$

What does all this tell us? First, all the thrust equations tell us that thrust is a function of the atmospheric density. Unlike velocity, which we earlier found to vary with the square root of density, thrust decreases in direct proportion to the decrease of density in the atmosphere. Thus, we write:

$$T_{alt} = T_{sl}(\rho_{alt}/\rho_{SL})$$

<u>(This is an important relationship between thrust and altitude that we will use in all performance calculations).</u>

Second, we learn that, in general, the thrust of an engine varies with speed according to the relationship:

$$T = T_0 - a\, V_\infty^2$$

It should be noted, as always, that these equations involve important assumptions, such as the assumption that engine exit pressure and entrance pressure are both equal to the pressure in the free stream atmosphere. Pilots of jet aircraft will tell you that the thrust to static thrust relationship shown above doesn't, for example, account for an engine surge on initial acceleration down the runway as a "ram effect" into the engine inlet occurs. This "ram effect" is essentially one of these pressure effects that we chose to ignore.

R 2.4.6 Propeller Thrust

It should be noted that we would get essentially the exact same thrust equation looking at the flow through a propeller as we do with a jet engine. Keeping in mind an earlier discussion, we would draw our "system" as shown below using boundaries that represent a "stream tube" of constant mass flow. In this case we have no easy way of knowing the exact values for the entering flow area or the exit area but we would get exactly the same equation as we found for the jet and we would still find

$$\mathbf{T = T_0 - aV_\infty^2.}$$

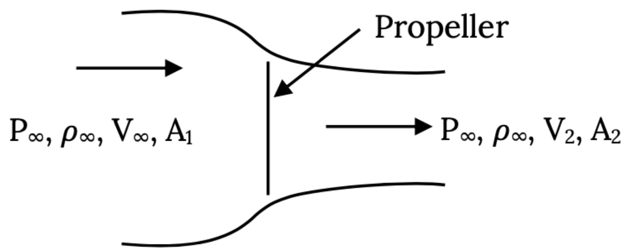

Figure R2.9: Momentum Equation Terms for Propeller Flow

In this chapter we have looked at the relatively simple models of aircraft propulsion that we will use in examining aircraft performance. In doing this we have used some basic physical concepts of conservation (mass and momentum), both of which can provide very powerful tools for evaluating forces and motions in fluid flows and other areas. We made a lot of simplifying assumptions along the way in order to understand some very basic concepts related to jet and propeller propulsion; in particular, to give a basis for modeling the way both thrust and power vary with speed and altitude. We will find these concepts very useful in later chapters.

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References Course supplement

Figure **R**2.1: Claire Colvin (2021). "TurboJet Engine Illustration." CC BY 4.0.

Figure **R**2.2: Claire Colvin (2021). "Illustration of a Turbo-Fan or Fan-Jet Engine." CC BY 4.0.

Figure **R**2.3: Claire Colvin (2021). "Turboprop Engine Illustration." CC BY 4.0.

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Figure **R**2.6: James F. Marchman (2004). "The Stream-tube Concept for a Propeller Flow." CC BY 4.0.

Figure **R**2.7: Kindred Grey (2021). "A Balloon as a Simple Jet." CC BY 4.0. Adapted from James F. Marchman (2004). CC BY 4.0. Available from https://archive.org/details/2.1_20210804

Figure **R**2.8: Claire Colvin (2021). "Momentum Equation Terms for Turbo-Jet." CC BY 4.0.

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Applications

Exercise1

• The following data apply to a turbojet flying at an altitude where the ambient conditions are 0.458 bar and 248 K.

• Speed of the aircraft: 805 km/h

• Compressor pressure ratio: 4:1

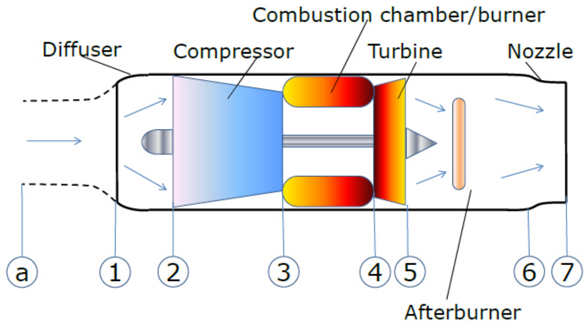
• Turbine inlet temperature: 1100 K

• Nozzle outlet area 0.0935 m2

• Heat of reaction of the fuel: 43 MJ/kg

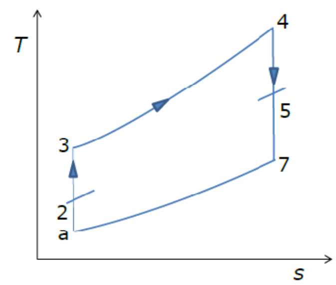
Find the thrust and TSFC assuming cp as 1.005 kJ/kgK and γ as 1.4

Ideal cycle for jet engines



Schematic of a turbojet engine and station numbering scheme

Ideal cycle for jet engines



Ideal turbojet cycle (without afterburning) on a T-s diagram

Solution1

- Speed of the aircraft =805x1000/3600=223.6 m/s
- Mach number = 223.6/√(γRT)= 223.6/ √(1.4x287x248)= 0.708
- Intake:

$$T_{02} = T_a\,[1+\frac{(\gamma-1)}{2}M^2]=248[1+\frac{1,4-1}{2}.0,708^2\,]=272,86°K$$

$$P_{02}=P_a\,[T_{02}/T_a]^{\gamma/(\gamma-1)}=0,548(272,86/248)^{1,4/(1,4-1)}=0,639\text{ bar}$$

- Compressor:

$$P03=\pi_c\,P_{02} = 4.0,639 =2,556\text{ bar}$$

$$T_{03} = T_{02}\,(\pi_c\,)^{(\gamma-1)/\,\gamma} =272,86.(4)^{(1,4-1)/1,4}=405,63°K$$

- Combustion chamber: From energy balance,

$H_{04} = h_{03} + f.Q_R$

Or , $f=[(T_{04}/T_{03}) -1]/Q_R/(C_p\ T_{03}) -(T_{04}/T_{03})$

$$=\frac{\left(\frac{1100}{405,63}\right)-1}{\left(\frac{43.1000000}{1005.405,63}\right)-1100/405,63} = 0,017$$

• Turbine: Since the turbine produces work to drive the compressor, $W_{turbine} = W_{compressor}$

$\dot{m}_t\, C_p\, (T_{04} - T_{05}) = \dot{m}_a\, C_p\, (T_{03} - T_{02})$

$T_{05} = T_{04} - (T_{03} - T_{02})/(1+f) = 1100-(405,63-272,86)/1+0,017) = 969,45°K$

Hence,

$P_{05} = P_{04}[T_{05}/T_{04}]^{\gamma/(\gamma-1)} = 2,556[969,45/1100]^{1,4/(1,4-1)} = 1,642$ bar

• Nozzle: we first check for choking of the nozzle.

• The nozzle pressure ratio is

$$P_{05}/P_a = 1.642/0.458 = 3.58$$

• The critical pressure ratio is

$$P_{05}/P^* = [\frac{\gamma+1}{2}]^{\gamma/(\gamma-1)} = [\frac{1,4+1}{2}]^{1,4/(1,4-1)} = 1,893$$

• Therefore the nozzle is choking.

• The nozzle exit conditions will be determined by the critical properties.

$T_7 = T^* = \left(\frac{2}{\gamma+1}\right) T_{05} = \frac{2}{1,4+1}\ .969,5 = 807,92\ °K$

$P_7 = P^* = P_{05}\ (1/[P_{04}/P^*]) = \frac{1,642}{1,893} = 0,867$

$\rho_7 = P_7/R\ T_7 = 0,867.\ 10^5\ /\ (287.807,\ 92) = 0,374\ kg/m^3$

Therefore,

$Ue = \sqrt{\gamma\, RT_7} = \sqrt{1,4\ .287\ .807,92} = 569,\ 75$ m/s

The mass flow rate is

$\dot{m} = \rho_7 A_7 u_e$=19.92 kg/s

The thrust developed is

$$\Gamma = \dot{m}\,[(1+f)\,u_e - u] + A_7\,(P^* - P_a)$$

$$= 19{,}92\,[(1+0{,}017).569{,}75 - 223{,}6]+0{,}0935.(0{,}867-0{,}458).10^5=10{,}912 \text{ kN}$$

Fuel flow rate,

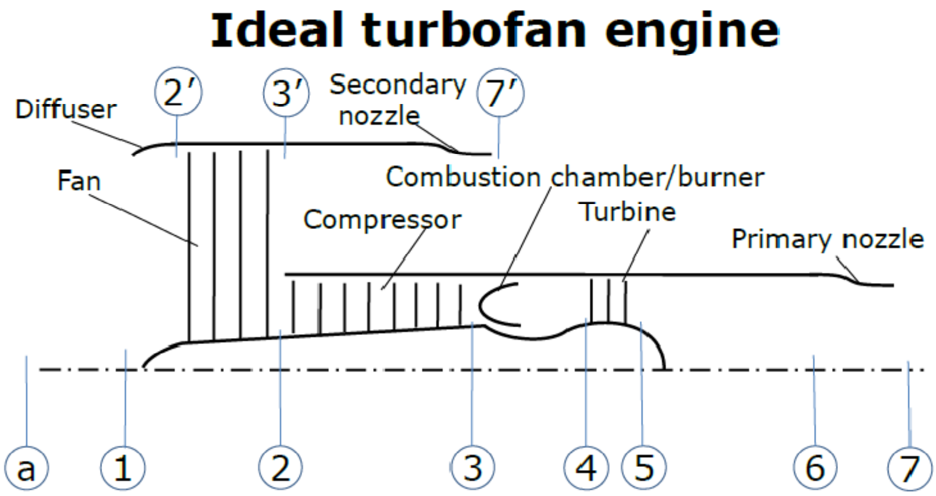
$$\dot{m}_c = f\,.\,\dot{m}_a = 0.017\ 19.92 = 0.3387 \text{ kg/s}$$

Therefore,

$$\text{TSFC} = \dot{m}_f/\Gamma = 3.1\ 10 \text{ kg/Ns} = 0.111 \text{ kg/N h}$$

Exercise 2

- The following data apply to a twin spool turbofan engine, with the fan driven by the LP turbine and the compressor by the HP turbine. Separate hot and cold nozzles are used.
- Overall pressure ratio: 19.0
- Fan pressure ratio: 1.65
- Bypass ratio: 3.0
- Turbine inlet temperature: 1300 K
- Air mass flow: 115 kg/s
- Find the sea level static thrust and TSFC if the ambient pressure and temperature are 1 bar and 288 K. Heat of reaction of the fuel: 43 MJ/kg



Schematic of an unmixed turbofan engine and station numbering scheme

- Since we are required to find the static thrust, the Mach number is zero.
- Intake:

$$T_{02} = Ta\ [1+\frac{\gamma-1}{2}.M^2] = 288\ °K$$

$$P_{02'} = Pa\ [T_{02'}/T_a]^{\gamma/(\gamma-1)} = 1\ bar$$

- Fan: Fan pressure ratio is known:

$$\pi_f = P_{03'}/P_{02'}$$

$$P_{03'} = \pi_f\ P_{02'} = 1{,}65\ bar$$

$$T_{03'} = T_{02'}\ (\pi_f)^{(\gamma-1)/\gamma} = 288\ .(1{,}65)^{(1{,}4-1)/1{,}4} = 332{,}35\ °K$$

<u>Solution2</u>

- Compressor:

Π_c = Overall pressure ratio / 1,65 = 19/1,65 = 11,515

$P_{03} = \pi_c \,.\, P02 = 11{,}515 \,.\, 1{,}65 = 19{,}0$ bar

$T_{03} = T_{02}\,(\pi_c\,)^{(\gamma-1)/\gamma} = 332{,}35\;.(11{,}515)^{(1,4-1)/1,4} = 668{,}53°K$

• Combustion chamber: From energy balance,

- $f=[T_{04}/T_{03}\ -1]/[Q_R\,/(C_p\,T_{03} - T_{04}/T_{03}\,)\,]$

$= [1300/668{,}53\ \ -1]/[43.10^6\ /(1005.668{,}53\ \ -1300/668{,}53\,)\,]= 0{,}01522$

• High pressure turbine:

$\dot{m}_t\,C_p\,(T_{04} - T_{05'}\,) = \dot{m}_{aH}\,C_p\,(T_{03} - T_{02})$

Here, is the temperature at the HPT exit.

$T_{05'} = T_{04} - (T_{03} - T_{02})/(1+f)$

$= 1300\text{-}(668{,}53\text{-}332{,}53)/(1+0{,}01522)= 969{,}04\ °K$

Hence,

$P_{05'} = P_{04}\,(T_{05'}/T_{04})^{\gamma/(\gamma-1)}$

$= 19.\ (969{,}04/1300)^{1,4/(1,4-1)} = 6{,}79$ bar

• Low pressure turbine:

$\dot{m}_t\,C_p\,(T_{05'} - T_{05}) = \dot{m}_{ac}\,C_p\,(T_{03'} - T_{02'})$

Here,T_{05} is the temperature at the HPT exit/LPT inlet.

$T_{05} = T_{05'} – B\ (T_{03'} – T_{02'})/\ (1+f)$, where , $B=\dot{m}_{ac} / \dot{m}_{aH}$

$=969,04-3.(332,35-288) / (1+0,01522)) =837,98\ °K$

And,

$P_{05} = P_{05'}\ (T_{05}/T_{05'})^{(\gamma/\gamma-1)}$

$= 6,79\ (837,98 / 969,04)^{1,4/(1,4-1)} = 4,08$ bar

• Primary nozzle: we first check for choking of the nozzle.

• The nozzle pressure ratio is

$$P_{05}/P_a=4.08/1=4.08 \text{ bar}$$

• The critical pressure ratio is

$$P_{05} / P^* = [\frac{\gamma+1}{2}]^{\gamma/(\gamma-1)} = [\frac{1,4+1}{2}]^{1,4/(1,4-1)} = 1,893$$

• Therefore the nozzle is choking.

• The nozzle exit conditions will be determined by the critical properties.

$$T_7 = T^* =(\frac{2}{\gamma+1}).T_{05} = \frac{2}{1,4+1} . 837,98 = 698,32\ °K$$

$$P_7 = P^* = P_{05}\ (1/(P_{05}/P^*) = 4,08 / 1,893 = 2,155 \text{ bar}$$

Therefore,

$$u_e = (\ \gamma\ R\ T_7\ )^{1/2} = \sqrt{1,4.287\ .698,32} = 529,7 \text{ m/s}$$

• Secondary nozzle:

• The nozzle pressure ratio is :

$$P_{03'}/P_a=1.65/1=1.65 \text{ bar}$$

• The critical pressure ratio is

$$P_{05} / P^* = [\frac{\gamma+1}{2}]^{\gamma/(\gamma-1)} = [\frac{1,4+1}{2}]^{1,4/(1,4-1)} = 1,893$$

• Therefore the nozzle is not choking.

$$U_{ef} = \{ 2.C_p\, T_{03'} \,[1-(Pa/P^{03'})^{(\gamma-1)/\gamma}]\}^{1/2}$$
$$= \{ 2.1003.332,35\, [1-(1/1,65)^{(1,4-1)/1,4}]\}^{1/2} = 298,52 \text{ m/s}$$

Thrust,

$$\Gamma = \dot{m}_{aH} [(1+f)u_e - u] + B\, \dot{m}_{aH} (u_{ef} - u)$$

Assuming $(P_e-P_a)A_e$ to be negligible

$$B= \dot{m}_{aC} / \dot{m}_{aH} =3,0 \;;\; \dot{m}_{aH} +\dot{m}_{aC} = 115 \text{ Kg/s}$$

$$=> \dot{m}_{aH} = 115 /4 = 28,75 \text{ Kg/s}$$

$$=> \Gamma = 28,75 [(1+0,01522)529,7 - 0] + 3.\ 28,75 (298,52 - 0)$$

$$= 40,74 \text{ kN}$$

Calculate the thrust by factoring the pressure thrust term as well. Hint: you can calculate the exit area from mass flow, density and exhaust velocity.

• TSFC,

Fuel flow rate,

$$\dot{m}_f=f.\ \dot{m}_a=0,01522\ .\ 28,75$$
$$= 0.4376 \text{ kg/s}$$

Therefore,

TSFC=ṁf /Γ

=1.075 10 kg/Ns = 0.0388 kg/N h

Exercise 3

• A helicopter using a turbo shaft engine is flying at 300 km/h at an altitude where the ambient temperature is 5°C. Determine the specific power output and thermal efficiency. The specifications of the engine are: compressor pressure ratio=9.0, turbine inlet temperature = 800°C.

• For a turboshaft engine, there is no nozzle thrust.

• u=300.1000/3600= 83,33 m/s

• T_a=278 °K

• Therefore, Mach number

$$M= 83,33/\sqrt{(1,4.287.278)} =0,25$$

• Intake:

$$T_{02} = T_a\,(1+ \frac{\gamma-1}{2}.M^2)= 278.(1+\frac{1,4-1}{2}\,.0,25^2\,) = 281,48\ °K$$

$$P_{02}= P_a\,(281,48 / 278)^{1,4/(1,4-1)} =0,835\ bar$$

Compressor:

$$P_{03} = \pi_c\, P_{02} = 9,0\ .\ 0,835 =7,52\ bar$$

$$T_{03} = T02\ (\pi_c\,)^{(\gamma-1)/\gamma}$$

$$=281,48\ .\ (9,0)^{(1,4-1)/1,4} =527,67\ °K$$

Specific work required to drive the compressor,

$$W_c = C_p\,(T_{03} - T_{04}) = 1{,}005(527{,}67 - 281{,}48) = 247{,}42 \text{ kJ/kg}$$

• Combustor:

$$f = [T_{04}/T_{03} - 1] / [(Q_R / C_p\, T_{03}) - (T_{04}/T_{03})]$$
$$f = [1073/527{,}67 - 1] / [(43.10^6 / 1005.527{,}67) - (1073/527{,}67)] = 0{,}013$$

Turbine:

$$P_{04}/P_{05} = P_{03}/P_a = (P_{03}/P_{02}).(P_{02}/P_a) = 9.\ 0{,}835/0{,}8 = 9{,}394$$
$$T_{04}/T_{05} = (P_{04}/P_{05})^{(\gamma-1)/\gamma}$$
$$= 9{,}394^{(1{,}4-1)/1{,}4} = 1{,}897$$
$$\Rightarrow T_{05} = 565{,}63°K$$

Work done by the turbine, (1) ( )

$$W_t = (1+f)\,C_p\,(T_{04} - T_{05})$$
$$= (1+0{,}013).1{,}005.(1073-565{,}63)$$
$$= 516{,}54 \text{ kJ/kg}$$

• Specific work output,

$$W_{net} = W_t - W_c = 516.54 - 247.42 = 269.12 \text{ kJ/kg}$$

• Thermal efficiency:

$$W_{net}/Q_{in}$$

• $Qin = cp(T_{04} - T_{03}) = 1{,}005(1073 - 527.67) = 548{,}05$ kJ/kg

• Therefore,

Thermal efficiency =269,12/548,05=0,49 or 49%

Exercise 4

• A turbojet engine inducts 51 kg of air per second and propels an aircraft with a uniform flight speed of 912 km/h. The enthalpy change for the nozzle is 200 kJ/kg. The fuel-air ratio is 0.0119 and the heating value of the fuel is 42 MJ/kg. Determine the thermal efficiency, TSFC, propulsive power.

• Ans: 0.34, 0.1034 kg/Nh, 8012 kW.

Exercise 5

• A twin spool mixed turbofan engine operates with an overall pressure ratio of 18. The fan operates with a pressure ratio is 1.5 and the bypass ratio is 5.0. The turbine inlet temperature is 1200 K. If the engine is operating at a Mach number of 0.75 at an altitude where the ambient temperature and pressure are 240 K and 0.5 bar.

• Determine the thrust and the SFC.

• Ans: 74 kN, 0.027 kg/N h

Exercise 6

• An aircraft using a turboprop engine is flying at 800 km/h at an altitude where the ambient conditions are 0.567 bar and -20^{o}C. Compressor pressure ratio is 8.0 and the turbine inlet temperature is 1100 K.

Assuming that the turboprop does not generate any nozzle thrust, determine the specific power output and the thermal efficiency.

• Ans: 311 kJ/kg, 0.44