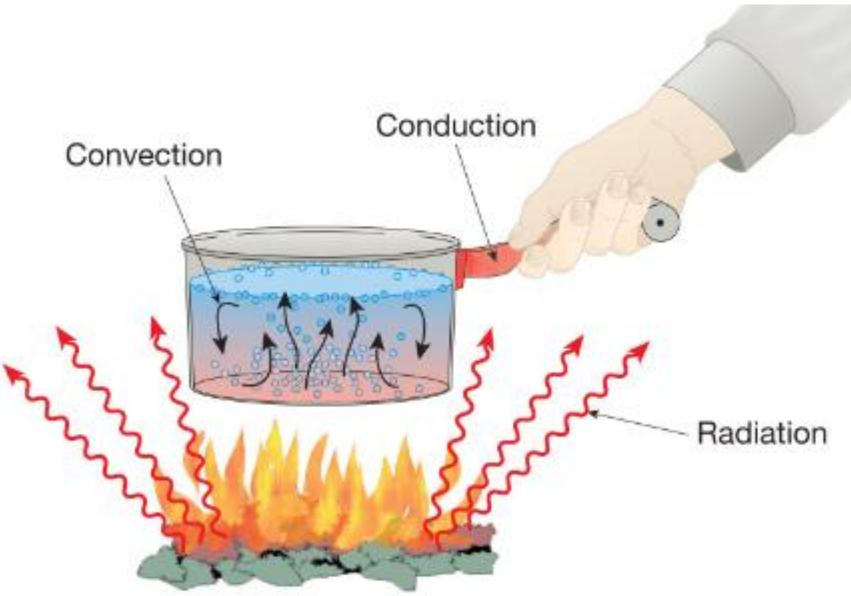
People's Democratic Republic of Algeria

Ministry of Higher Education and Research Scientifique

University of Rélizane

Faculty of Science and Technology

Department Mechanical Engineering



Heat transfer 1

Presented by:

Mr. Rachid ZAHI

Preface

This handout is made up of courses from the classic Mechanical Engineering program, taught to third year engineering students. Therefore, it is the fruit of several years of teaching. This part of the program is devoted to the three modes of heat transfer; conduction, convection and thermal radiation. Currently, it is aimed at LMD License students (for heat transfer module 1). We hope that it will be of great use in better understanding the principles of the three phenomena of heat transfer. The lessons are enriched by several examples and corrected exercises.

The handout is limited to four chapters, in the first chapter we will present heat transfer in a general way. The second chapter is reserved for basic heat transfer laws. The third chapter is devoted to one-dimensional conduction in steady state. Also, the problem of one-dimensional conduction with loss through the side surfaces. Thermal conductivity and orders of magnitude for common materials. Discussion of the parameters on which thermal conductivity depends. Energy equation, simplifying assumptions and different forms. Spatial and initial boundary conditions. The four linear conditions and their practical significance. Some solutions of the heat equation, in Cartesian, cylindrical and spherical coordinates, with linear conditions. Conductive systems with heat sources. Stationary electrical analogy. The longitudinal rectangular fin problem: Fin equation. Solving the problem. Calculation of fin efficiency. Generalization of the fin concept. Application to radial fin with uniform profile. The fifth and final chapter is devoted to convective heat transfer and the parameters involved in convective heat transfer. In the last chapter, the different types of convective heat transfer are highlighted: forced, natural and mixed convection. Cite common examples. Discern between laminar and turbulent convective transfer in both forced and natural modes.

We hope that this handout covering the main themes of the In-depth Heat Transfer module will be used as support for the course and that it will be useful for understanding the material taught.

Heat transfer1

Chapter 1.

Introduction to heat transfer and thermodynamics

General introduction:

From time immemorial, the problems of energy transmission, and in particular heat, have been of decisive importance for the design and operation of equipment such as steam generators, furnaces, heat exchangers, evaporators, condensers, etc., but also for chemical transformation operations.

Indeed, in some reaction systems, it is the rate of heat exchange rather than the rate of chemical reaction that determines the cost of the operation (in the case of highly endo- or exothermic reactions). What's more, with the relative increase in the cost of energy, the aim nowadays is to achieve maximum plant efficiency with minimum energy expenditure [1;2].

Heat transfer problems are numerous, and we can try to differentiate them by the goals pursued, the main ones being: increasing the energy transmitted or absorbed by a surface, obtaining the best efficiency from a heat source, reducing or increasing the flow of heat from one medium to another.

The potential that causes the transport and transfer of thermal energy is temperature. If two material points in a thermally insulated medium are at the same temperature, it can be said that there is no overall heat exchange between these two points, which are said to be in thermal equilibrium (this is because each of the material points emits a net thermal energy of the same modulus, but of opposite sign).

Heat transfer within a phase or, more generally, between two phases, takes place in three ways:

a) By conduction. **b) By convection.** **c) By radiation.**

In many thermal energy transformation problems, the three modes of heat transfer will coexist, but usually at least one of the three forms can be neglected, simplifying the mathematical treatment of the transfer apparatus. We can already say that, at ordinary temperatures, transport by radiation is negligible, but it can become significant and predominant as the temperature level rises.

We should also point out that some heat transfers are accompanied by a transfer of matter between two phases. The heat flux transferred in the presence of a phase change depends on the nature and physico-chemical properties of the phases involved.

This is the case for boiling and condensation, but also for humidification, drying, crystallization and so on.

In what follows, we present the general laws governing the three types of heat transport. We will then take a simple look at a few applications where the heat transfer mode studied is predominant [1;2]t.

1-1-Relationship between heat transfer and thermodynamics

Thermodynamics enables us to predict the total amount of energy a system must exchange with the outside world in order to move from one equilibrium state to another.

Thermodynamics or thermokinetics aims to describe quantitatively, in space and time, the evolution of the system's characteristic quantities, in particular the temperature between the initial and final states of equilibrium.

Thermodynamics: studying states of equilibrium

1st principle of thermodynamics: equivalence between heat and energy 1 cal = 4.18 joule, conservation of energy Ceded = Qabsorbed

The 2nd principle of thermodynamics: heat or thermal energy can only be transferred from a hot body to a cold one.

Heat transfer: study the process mechanism and transfer speed

- Calculate temperature distribution within bodies

- Calculate heat flow J/s (W) heat exchanged per unit time Transfer = exchange = transmission = propagation

Thermal = heat
Thermal transfer = heat transfer

Chapter 2.

Basic heat transfer laws

2-Definitions:

2-1 Temperature field: [1;2]

Energy transfers are determined from the evolution in space and time of temperature: T = (x,y,z,T)

The instantaneous value of temperature at any point in space is a scalar called the temperature field. We distinguish between two cases:

- Temperature field independent of time: the regime is said to be permanent or stationary.
- Evolution of the temperature field with time: the regime is said to be variable or unsteady.

2-2 Heat flow:

Heat flux is the quantity of heat transmitted to surface S per unit time.

$$\emptyset = \frac{dQ}{dt} \qquad \left[\frac{J}{s}\right] = [W] \tag{1.1}$$

2-3 Flux density:

The quantity of heat transmitted per unit time and per unit area of the isothermal surface is called the heat flux density:

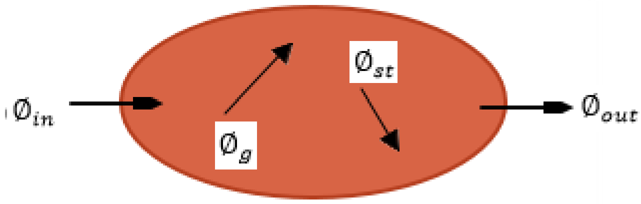
$$q = \frac{\emptyset}{A} = \frac{1}{A}\frac{dQ}{dt} \qquad \left[\frac{W}{m^2}\right] \tag{1.2}$$

Where A is the surface area (m2)

2-4 Energy balance

First, we need to define a system (S) by its boundaries in space, and then draw up an inventory of the various heat flows that influence the state of the system:

$$\emptyset_{in} + \emptyset_g = \emptyset_{st} + \emptyset_{out} \tag{1.3}$$



$\emptyset_{out}$: outgoing heat flow

$\emptyset_g$: heat flow generated

$\emptyset_{in}$: incoming heat flow

$\emptyset_{st}$: stored heat flow

2-4-1 Energy storage [1;2]

The storage of energy in a body corresponds to an increase in its internal energy over time (at constant pressure), hence:

$$\emptyset_{st} = \rho V C p \frac{\partial T}{\partial t} \tag{1.4}$$

With:
$\emptyset_{st}$: Stored heat flux [W]
ρ: Density (kg m-^{3})
V: Volume (m^3)
Cp: Heat density (J kg^{-1}°C^{-1}) T Temperature (°C)
t: Time (s)

2-4-2 Energy generation
This occurs when another form of energy (chemical, electrical, mechanical, nuclear) is converted into thermal energy. It can be written as:

$$\emptyset_g = \dot{q}V \quad [W] \tag{1.5}$$

With:
g: Flux of thermal energy generated (W)
$\dot{q}$: Density by volume of energy generated (W m^{-3})

2-5 The different modes of heat transfer [1;2]
There are three modes of heat transfer: Conduction, Convection, Radiation.

2-5-1 Conduction:
This is the transfer of heat within an opaque medium, without displacement of matter, under the influence of a temperature difference.
Conductive heat transfer within a body takes place by two distinct mechanisms: transmission by the vibrations of atoms or molecules, and transmission by free electrons.
Heat exchange between two points on a solid, or even an immobile, opaque fluid.

Fourier's law:
The French scientist J.B.J.Fourrier proposed the fundamental relationship of heat transmission by conduction in 1882. The heat flux density is proportional to the temperature gradient.

$$Q= - \lambda A \, dT/dx \tag{1.6}$$

Q : Conductive heat flux (W);
λ: Thermal conductivity of the medium (W/m °C);
x: space variable in the direction of flow (m);
A: Cross-sectional area of heat flow (m^2);
λ Varies with temperature for solids;
λ Varies with pressure for gases and liquids.

Chapter 3:

Heat conduction

3-Heat transfer by conduction [1; 2]

3.1 General conduction equation

In its one-dimensional form, this describes unidirectional heat transfer through a plane wall. Consider a system with thickness dx in the x direction and cross-sectional area S normal to the Ox direction. The energy balance for this system is written:

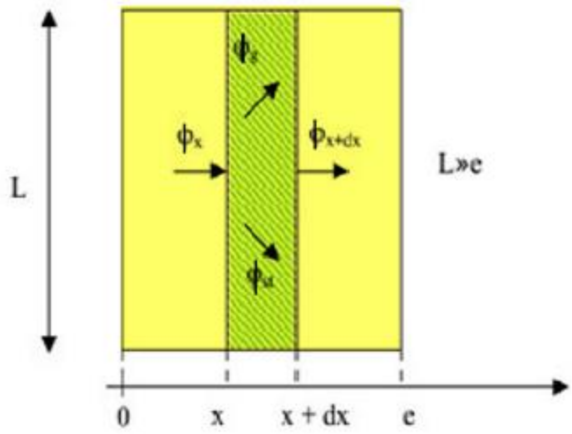


Figure :

$$\emptyset_x + \emptyset_g = \emptyset_{st} + \emptyset_{x+dx}$$

With:

$$\emptyset_x = -\lambda A \frac{dT}{dx}$$

$$\emptyset_g = \dot{q} A dx$$

$$\emptyset_{x+dx} = -\lambda A \frac{dT}{dx^{x+dx}}$$

$$\emptyset_{st} = \rho A dx\, Cp \frac{\partial T}{\partial t}$$

By transferring to the energy balance and dividing by dx, we obtain

$$\frac{\lambda A \frac{dT}{dx^{x+dx}} - \lambda A \frac{dT}{dx^{x}}}{dx} + \dot{q} A = \rho A Cp \frac{\partial T}{\partial t} \tag{3.1}$$

$$f'(x) = \frac{df}{dx} = \lim \frac{\Delta f}{\Delta x} = \lim \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

We have:

$$\frac{\partial}{\partial x}\left(\lambda_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda_z \frac{\partial T}{\partial z}\right) + \dot{q} = \rho Cp \frac{\partial T}{\partial t} \tag{3.2}$$

and in the three-dimensional case, we obtain the heat equation in the most general case

3.1.2 The simplifying hypotheses [1;2]

This equation can be simplified in a number of cases:

a) If the medium is isotropic: $\lambda x = \lambda y = \lambda z$

b) If λ is constant

Equation 3.2 becomes (Cartesian coordinate):

$$\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right)+\frac{\dot{q}}{\lambda}=\frac{\rho Cp}{\lambda}\frac{\partial T}{\partial t}$$

$$\Delta T+\frac{\dot{q}}{\lambda}=\frac{\rho Cp}{\lambda}\frac{\partial T}{\partial t}$$

$$\Delta T+\frac{\dot{q}}{\lambda}=\frac{1}{a}\frac{\partial T}{\partial t} \qquad (3.3)$$

Le rapport $a=\frac{\lambda}{\rho Cp}$ is called the thermal diffusivity (m^2/s)

3.1.3 Forms of the conduction equation:

- Medium with internal source in steady state (Poisson equation):$\Delta T+\frac{\dot{q}}{\lambda}=0$;
- Medium with internal source in steady state (Laplace equation):$\Delta T=0$;
- Medium with internal source in steady state (Fourier equation):$\Delta T=\frac{1}{a}\frac{\partial T}{\partial t}$.

3.1.4 Analytical expressions of the conduction equation: [1;2]

For the same simplifying assumptions

a) Cartesian coordinates (x,y,z) :

$$\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right)+\frac{\dot{q}}{\lambda}=\frac{\rho Cp}{\lambda}\frac{\partial T}{\partial t}$$

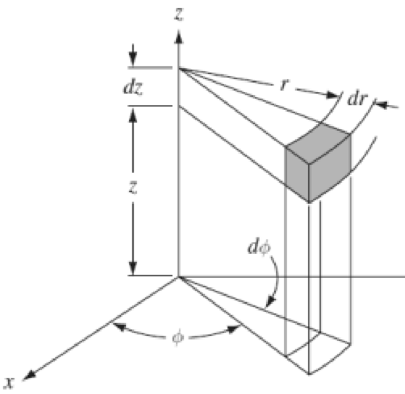
b) Cylindrical coordinates (r,θ,z) :

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\dot{q}}{\lambda}=\frac{\rho Cp}{\lambda}\frac{\partial T}{\partial t}$$

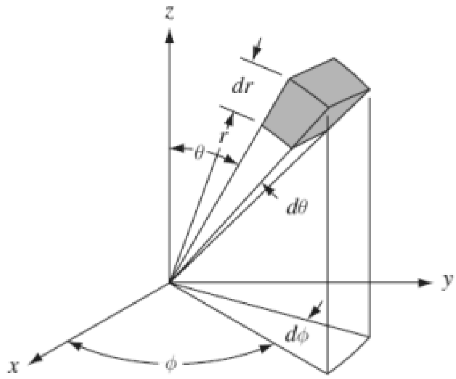
c) spherical coordinates:

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\phi \end{cases}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin\theta}\frac{\partial^2 y}{\partial\phi^2}+\frac{\dot{q}}{\lambda}=\frac{\rho Cp}{\lambda}\frac{\partial T}{\partial t}$$



Système de coordonnées cylindriques



Système de coordonnées sphériques

3.2 Constant unidirectional steady-state conduction: [1; 2]

3.2.1 Single wall:

Consider a wall of thickness e, thermal conductivity λ, and large transverse dimensions whose end faces are at temperatures $T_1$and T_2 ($T_1 > T_2$), and there is no energy generation or storage.

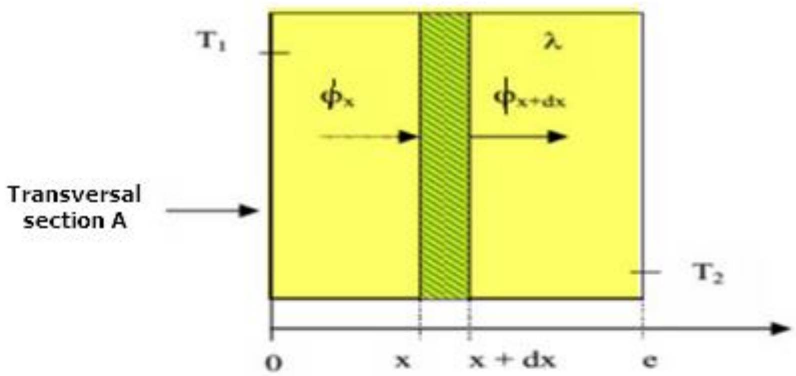


Figure:

Performing a heat balance on the system (S) consisting of the wall slice between abscissas x and x + dx gives: The general conduction equation:

$$\Delta T+\frac{\dot{q}}{\lambda}=\frac{1}{a}\frac{\partial T}{\partial t}$$

This is a continuous operation:

$$\frac{\partial T}{\partial t}=0,\quad \dot{q}=0$$

No heat source

$$\Delta T = 0 \leftrightarrow \frac{\partial^2 T}{\partial x^2} = 0 \longrightarrow \frac{dT}{dx} = A \quad and \quad T(x) = Ax + B$$

This is Laplace's equation

With boundary conditions: $T(x = 0) = T_1 \ and \ T(x = e) = T_2, \ T_1 = B, \quad T(x = e) = T_2 = A\,e + T_1$
Hence :

$$T(x) = T_1 - \left(\frac{x}{e}\right)(T_1 - T_2) \tag{3.4}$$

Equation (3.4) is the temperature distribution (the temperature profile). The heat flux density through the wall can be deduced by the relationship:

$$q = \frac{\emptyset}{A} = -\lambda \frac{dT}{dx} = \frac{\lambda}{e}(T_1 - T_2)$$

$$\phi = -\lambda A \frac{dT}{dx}$$

Heat flow

$$\phi = \frac{\lambda A}{e}(T_1 - T_2) \tag{3.5}$$

Flow density

$$q = \frac{\lambda}{e}(T_1 - T_2) \qquad \left[\frac{W}{m^2}\right] \tag{3.6}$$

Thermal resistance

3.2.2 Analogy between heat flow and electrical flow

Two systems are said to be analog when they obey the same equations and have identical boundary conditions. This means that the equation translating one of the systems can be transformed, to express the second system, by simply changing the symbols of the different variables. For example, the flow of heat through a thermal resistor is analogous to the flow of current in a DC electrical circuit, since both types of flow obey the same equations.

We retain the general definition of thermal resistance:

unité	Transfer thermique	↔	Courant électrique	unité
°C,K	Température T		Potentiel V ou U	V
J	Chaleur φ		Charge électrique q	Coulomb
W	Flux thermique $\emptyset = \frac{dQ}{dt}$		Intensité électrique $I = \frac{dq}{dt}$	A
W/m°C	Conductivité thermique λ		Conductivité électrique γ	$\Omega^{-1}m^{-1}$
W/m^2	Densité de flux. $\vec{q}$		Densité de courant $\vec{j}$	A/m^2
	Ø constant (régime permanent) $T_1 - T_2 = R_{th}\emptyset$		I constant (courant continu) $V_1 - V_2 = RI$	
°C/W	R_{th} résistance thermique $R_{th} = \frac{e}{\lambda S}$ conduction $R_{th} = \frac{1}{hS}$ convection		R résistance électrique $R_{th} = \frac{l}{\gamma S}$	Ω

The analogies established above show that the laws of association for thermal resistances are the same as those for electrical resistances.

$$R_{th} = \frac{T_1 - T_2}{\phi_{th}} \tag{3.7}$$

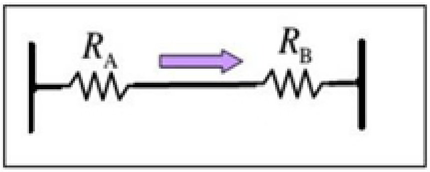
Electrical analogy is used extensively in the study of phenomena involving combinations of resistors. The laws of series and parallel circuits are often applied.

Equivalent resistance in series

The equivalent resistance of a set of series-connected resistors is equal to the sum of the resistances of the series-connected resistors:

$$R_{eq} = \sum_i R_i$$

$$R_{eq} = R_A + R_B$$

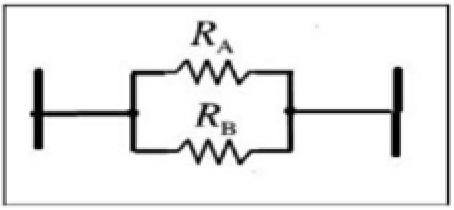


Equivalent resistance in parallel

The equivalent resistance of a set of parallel-connected resistors is equal to the inverse of the sum of the resistances of the resistors in parallel:

$$R_{eq} = \left[\sum_i \frac{1}{R_i}\right]^{-1}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_A} + \frac{1}{R_B}$$



3.2.3 Single wall in contact with two fluids: [1;2]

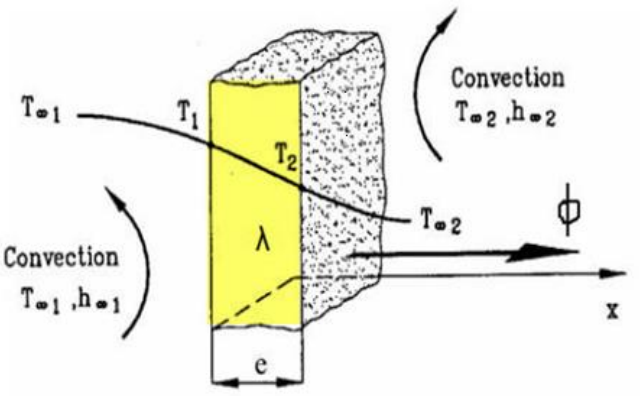
Assumptions :

⇒Steady state

⇒No radiation flow

⇒No heat generation

Assumptions ⇒ ∅ is constant



$$\emptyset_{\text{convection 1}} = \emptyset_{\text{conduction}} = \emptyset_{\text{convection 2}} \qquad (3.8)$$

$$\phi_{convection\ 1} = h_1 A(T_{\infty 1} - T_1) = \frac{(T_{\infty 1} - T_1)}{\frac{1}{h_1 A}} = \frac{(T_{\infty 1} - T_1)}{R_{th\ conv\ 1}}$$

$$\phi_{conduction} = \frac{\lambda A}{e}(T_1 - T_2) = \frac{(T_1 - T_2)}{\frac{e}{\lambda A}} = \frac{(T_1 - T_2)}{R_{th\ cond}}$$

$$\phi_{convection\ 2} = h_2 A(T_2 - T_{\infty 2}) = \frac{(T_2 - T_{\infty 2})}{\frac{1}{h_2 A}} = \frac{(T_2 - T_{\infty 2})}{R_{th\ conv\ 2}}$$

$$(3.8) \Longrightarrow \frac{(T_{\infty 1} - T_1)}{R_{th\ conv\ 1}} = \frac{(T_1 - T_2)}{R_{th\ cond}} = \frac{(T_2 - T_{\infty 2})}{R_{th\ conv\ 2}}$$

We have: $X = \frac{A}{B} = \frac{C}{D} = \frac{E}{F} = \frac{G}{H}$

We use this relationship:

$$\emptyset = \emptyset_{\text{convection 1}} = \emptyset_{\text{conduction}} = \emptyset_{\text{convection 2}}$$

$$\emptyset = \frac{(T_{\infty 1} - T_1)}{R_{th\ conv\ 1}} = \frac{(T_1 - T_2)}{R_{th\ cond}} = \frac{(T_2 - T_{\infty 2})}{R_{th\ conv\ 2}}$$

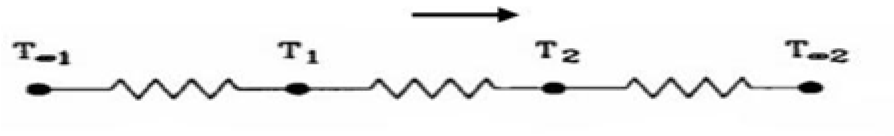
$$\emptyset = \frac{(T_{\infty 1} - T_1) + (T_1 - T_2) + (T_2 - T_{\infty 2})}{R_{th\ conv\ 1} + R_{th\ cond} + R_{th\ conv\ 2}}$$

$$\emptyset = \frac{(T_{\infty 1} - T_{\infty 2})}{R_{th\ conv\ 1} + R_{th\ cond} + R_{th\ conv\ 2}} = \frac{(T_{\infty 1} - T_{\infty 2})}{R_{th\ eq}} \quad (3.9)$$

$$R_{th\ eq} = R_{th\ conv\ 1} + R_{th\ cond} + R_{th\ conv\ 2}$$

$$R_{th\ eq} = \frac{1}{h_1 A} + \frac{e}{\lambda A} + \frac{1}{h_2 A} \quad (3.10)$$

The equivalent electrical diagram is as follows:



For the temperature profile at point x of thickness (e), the heat flux is constant for each point.

$$\emptyset = \frac{(T_{\infty 1} - T_{\infty 2})}{R_{th\ eq}} = \frac{(T_{\infty 1} - T(x))}{R_{th\ conv\ 1} + R_{th\ cond}(x)} = \frac{(T_{\infty 1} - T(x))}{\frac{1}{h_1 A} + \frac{x}{\lambda A}}$$

$$T(x) = T_{\infty 1} - \frac{R_{th\ conv\ 1} + R_{th\ cond}(x)}{R_{th\ eq}} (T_{\infty 1} - T_{\infty 2}) \quad (3.11)$$

Where: $R_{th\ cond}(x) = \frac{x}{\lambda A}$

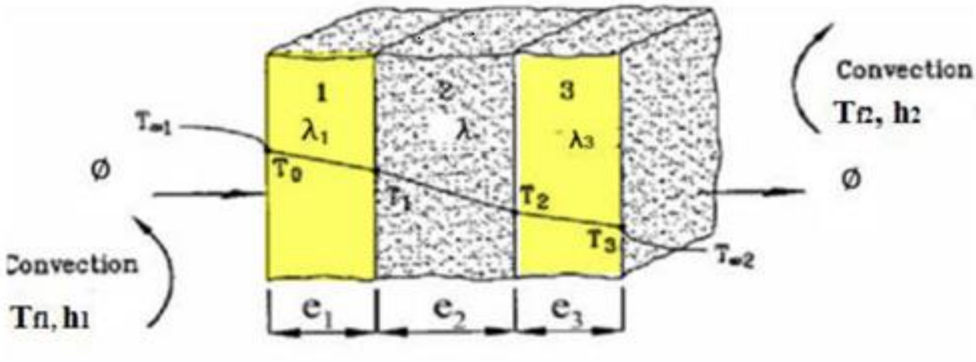
3.3 Composite wall in contact with two fluids:

This is the case with real walls made of several layers of different materials, where we only know the temperatures Tf1 and Tf2 of the fluids in contact with the two faces of the wall with lateral surface S. For the same assumptions, the flow is constant:

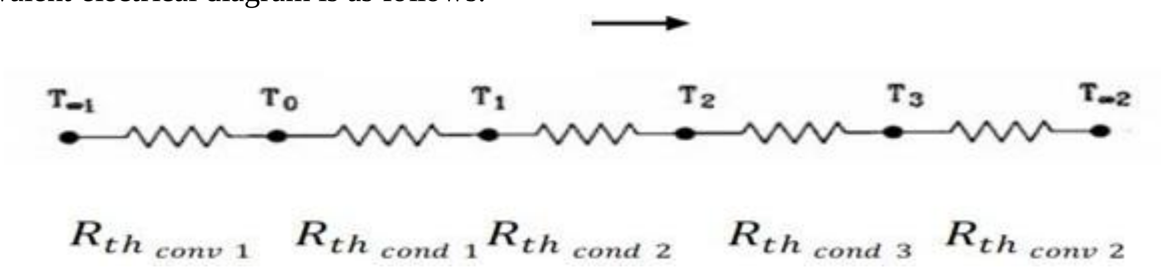
$$\emptyset_{conv\ 1} = \emptyset_{cnd\ 1} = \emptyset_{cnd\ 2} = \emptyset_{cnd\ 3} = \cdots \ldots \ldots \ldots \ldots = \emptyset_{cnd\ N} = \emptyset_{conv\ 2}$$

$$\emptyset = \frac{(T_{f1} - T_{f2})}{R_{th\ eq}}$$

$$R_{th\ eq} = \frac{1}{h_1 A} + \frac{e_1}{\lambda_1 A} + \frac{e_2}{\lambda_2 A} + \frac{e_3}{\lambda_3 A} + \cdots \ldots . + \frac{e_N}{\lambda_N A} + \frac{1}{h_2 A}$$



The equivalent electrical diagram is as follows:



For a wall with N layers:

$$R_{th\ eq} = \frac{1}{h_1 A} + \frac{1}{A}\sum_i^N \frac{e_i}{\lambda_i} + \frac{1}{h_2 A} \tag{3.12}$$

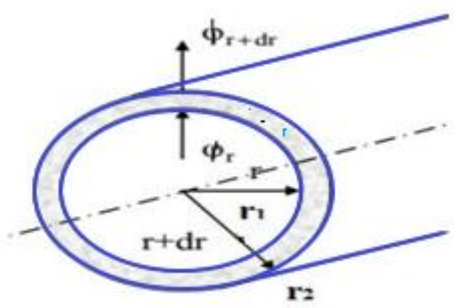
3.3.1 Long hollow cylinder (tube) with isothermal lateral surface

Consider a hollow cylinder of thermal conductivity λ, inner radius r1, outer radius r2, length L, with inner and outer surface temperatures T1, T2 respectively, and T1> T2. It is assumed that the longitudinal temperature gradient is negligible compared to the radial gradient.

T = T(r) (because independent of θ and z)

$$\frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial z} = 0$$

Let's carry out the heat balance of the system formed by the part of the cylinder between radii ***r*** and ***(r + dr)***:



The analytical expression of the conduction equation for cylindrical coordinates is equation (2.3):

Stationary case without heat generation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{q}}{\lambda} = \frac{\rho Cp}{\lambda}\frac{\partial T}{\partial t}$$

By integrating

With boundary conditions:

$$\Longrightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0 \Longrightarrow r\frac{\partial T}{\partial r} = C \Longrightarrow \frac{\partial T}{\partial r} = \frac{C}{r}$$

With boundary conditions:

$$r = r_1 \Longrightarrow T = T_1 = C\ln r_1 + B$$

$$r = r_2 \Longrightarrow T = T_2 = C\ln r_2 + B$$

$$C = \frac{T_1 - T_2}{\ln\left(\frac{r_1}{r_2}\right)}$$

$$T(r) = T_1 + \frac{T_1 - T_2}{\ln\left(\frac{r_1}{r_2}\right)}\ln\left(\frac{r}{r_2}\right) \qquad (2.18)$$

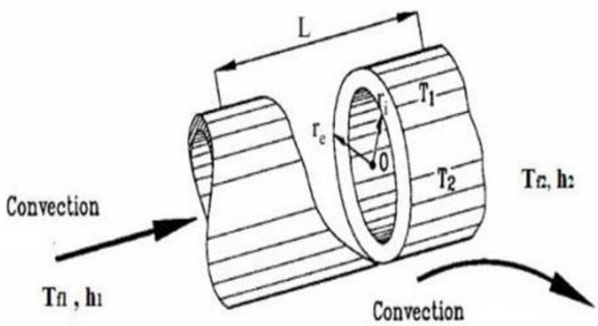
The temperature profile is logarithmic

Heat flux density:

$$q = \frac{\emptyset}{A} = -\lambda\frac{dT}{dr} = -\lambda\frac{T_1 - T_2}{\ln\left(\frac{r_1}{r_2}\right)}\frac{1}{r} \qquad (2.19)$$

3.3.2 Long hollow cylinder in contact with two fluids:

This is the practical case of a tube coated with one or more layers of different materials, where only the temperatures Tf1and Tf2of the fluids in contact with the inner and outer surfaces are known.



Figure

For cylinder case; $h_1$and h_2 are the convective heat transfer coefficients between the fluids and the inner and outer surfaces:

$$\emptyset = \emptyset_{conv\ 1} = \emptyset_{cond} = \emptyset_{conv\ 2}$$

$$\phi_{conv\ 1} = h_1 A\left(T_{f1} - T_1\right) = 2\pi r_1 L h_1\left(T_{f1} - T_1\right) = \frac{\left(T_{f1} - T_1\right)}{\frac{1}{2\pi r_1 L h_1}} = \frac{\left(T_{f1} - T_1\right)}{R_{th\ cv\ 1}}$$

$$R_{th\ cv\ 1} = \frac{1}{2\pi r_1 L h_1}$$

$$\phi_{cnd\ 1} = 2\pi L\lambda \frac{(T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)} = \frac{(T_1 - T_2)}{R_{th\ cod}}$$

$$R_{th\ cod} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi L\lambda}$$

Continuous operation, no heat source

$$\phi_{conv\ 2} = h_2 A_2 (T_2 - T_{f2}) = 2\pi r_2 L h_2 (T_2 - T_{f2}) = \frac{(T_2 - T_{f2})}{\frac{1}{2\pi r_2 L h_2}} = \frac{(T_2 - T_{f2})}{R_{th\ cv\ 2}}$$

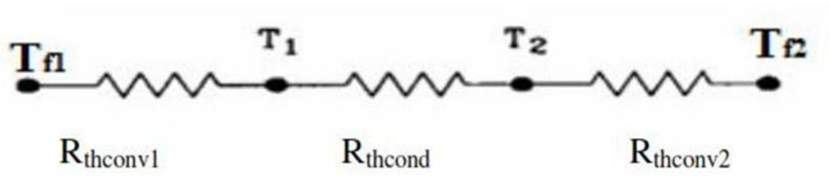
$$R_{th\ cv\ 2} = \frac{1}{2\pi r_2 L h_2}$$

$$\emptyset\ =\ \emptyset\text{conv } 1\ =\ \emptyset\text{cond} =\ \emptyset\text{conv } 2 \Longrightarrow \frac{(T_{f1} - T_1)}{\frac{1}{2\pi r_1 L h_1}} = \frac{(T_1 - T_2)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi L\lambda}} = \frac{(T_2 - T_{f2})}{\frac{1}{2\pi r_2 L h_2}}$$

$$\emptyset\ =\ \emptyset\text{conv } 1\ =\ \emptyset\text{cond} =\ \emptyset\text{conv } 2 \Longrightarrow \frac{(T_{f1} - T_{f2})}{\frac{1}{2\pi r_1 L h_1} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi L\lambda} + \frac{1}{2\pi r_2 L h_2}} = \frac{(T_{f1} - T_{f2})}{R_{th\ cv\ 1} + R_{th\ cod} + R_{th\ cv\ 2}}$$

$$\emptyset\ =\ \frac{(T_{f1} - T_{f2})}{R_{th\ equ}} \tag{3.20}$$

Equivalent circuit diagram

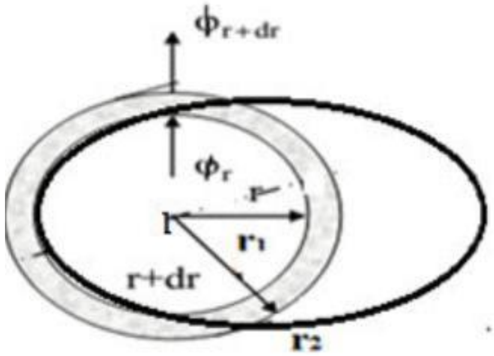


For concentric cylinders:

$$\emptyset\ =\ \frac{(T_{f1} - T_{f2})2\pi L}{\frac{1}{r_1 h_1} + \sum_{i=1}^{N} \frac{1}{\lambda_i} \ln \frac{r_i + 1}{r_i} + \frac{1}{r_2 h_2}} \tag{3.21}$$

3.3.3 Hollow sphere with isothermal surface

Consider a hollow sphere of thermal conductivity λ, inner radius r1, outer radius r_2, the temperatures of the inner and outer surfaces being T_1, T_2 respectively and that $T_1 > T_2$ It is assumed that T = T(r) (since independent of and ∅) ⇒



The analytical expression of the conduction equation for cylindrical coordinates is equation (3.4)

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin\theta}\left(\sin\theta\frac{\partial^2 T}{\partial\phi^2}\right)+\frac{\dot{q}}{\lambda}=\frac{\rho Cp}{\lambda}\frac{\partial T}{\partial t}$$

Stationary case without heat generation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=0\Longrightarrow\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=0\Longrightarrow\left(r^2\frac{\partial T}{\partial r}\right)=A$$

$$\frac{\partial T}{\partial r}=\frac{A}{r^2}\Longrightarrow T(r)=-\frac{A}{r}+B$$

With boundary conditions:

$$r=r_1\Longrightarrow T=T_1=-\frac{A}{r_1}+B$$

$$r=r_2\Longrightarrow T=T_2=-\frac{A}{r_2}+B$$

$$T_1-T_2=A\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$$

$$\Longrightarrow A=\left(\frac{r_1r_2}{r_1-r_2}\right)(T_1-T_2)\ \ et\ \ B=\left(\frac{r_1T_1-r_2T_2}{r_1-r_2}\right)$$

$$T(r)=T_1+\frac{\left(\frac{1}{r_2}-\frac{1}{r}\right)}{\left(\frac{1}{r_2}-\frac{1}{r_1}\right)}(T_1-T_2)$$

Heat flux density:

$$q=\frac{\phi}{A}-\lambda\frac{dT}{dr}=\lambda\frac{(T_1-T_2)}{\left(\frac{1}{r_2}-\frac{1}{r_1}\right)}\frac{1}{r^2}$$

The sphere's exchange surface:

$$A(r)=4\pi r^2$$

Thermal resistance for the spherical case:

$$\phi=q\lambda\frac{(T_1-T_2)}{\left(\frac{1}{r_2}-\frac{1}{r_1}\right)}\frac{1}{r^2}4\pi r^2=4\pi\lambda\frac{(T_1-T_2)}{\left(\frac{1}{r_2}-\frac{1}{r_1}\right)}$$

$$\phi = 4\pi\lambda \frac{(T_1 - T_2)}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)} = \frac{(T_1 - T_2)}{R_{th}}$$

3.4 Conduction in variable regime (transient or unsteady) [2,3]:

To carry out this study we generally consider two cases according to the thermal behavior
- Thermally thin body: a body is said to be thermally thin if it is internal resistance

$R_i = \frac{L}{\lambda A}$ is negligible. In this case, its temperature can be considered uniform at each instant t

-Thermally thick body: a body is said to be thermally thick if it is internal resistance
$R_i = \frac{L}{\lambda A}$ is not negligible. In this case, its temperature varies from one point to another at each instant t.

$$T = T(x, y, z, t)$$

The classification criterion is the "Biot" number [2,3].

Thermal classification of bodies ("Biot" criterion)

$$B_i = \frac{hL}{\lambda} = \frac{\frac{l}{\lambda A}}{\frac{1}{hA}} = \frac{R_i}{R_e} = \frac{conduction\ resistance}{convection\ resistance} \tag{3.21}$$

It is a dimensionless number.

l: characteristic length l=v/s
V: body volume
S: external exchange surface
- Wall thickness 2 γ heat exchange on both sides

$$l = \frac{V}{A} = \frac{\gamma 2\gamma}{2\gamma} = \gamma$$

- Wall thickness 2 γ single-sided heat exchange

$$l = \frac{V}{A} = \frac{\gamma 2\gamma}{\gamma} = 2\gamma$$

- Cylinder of radius R

$$l = \frac{V}{A} = \frac{\pi R^2 H}{\pi R H} = \frac{R}{2}$$

- sphere of radius R

$$l = \frac{V}{A} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3}$$

-Cube

$$l = \frac{V}{A} = \frac{a^3}{6a^2} = \frac{a}{6}$$

$B_i \leq 0.1$ thin body

$B_i > 0.1$ thin body

3.5 Thermally thin body [2,3]

The tramp of a hot solid in a cold liquid, we plane a solid probably heated to the initial temperature T_i in a fluid at temperature $T_f = T_\infty$.

Heat released by the body = heat absorbed by the fluid between t and t+ dt
The amount of heat transmitted to the fluid by convection over time dt= the decrease in internal energy in the solid.

$$\emptyset = \frac{dQ}{dt} = -\dot{m}Cp\frac{dT}{dt} = -\rho CpV\frac{dT}{dt} \quad et \quad \emptyset = hA\left(T - T_f\right) \tag{3.22}$$

Cp = constant it's a solid with boundary conditions

Variable-rate conduction

$$t = 0, \quad T = T_i$$
$$-\rho CpV\frac{dT}{dt} = hA\left(T - T_f\right)$$

We place

$$\dot{T} = T - T_f \Longrightarrow dT = d\dot{T}$$
$$-\rho CpV\frac{d\dot{T}}{dt} = hA\dot{T} \Longrightarrow -\rho CpV\frac{d\dot{T}}{\dot{T}} = hAdt$$
$$ln\dot{T} = \frac{hA}{\rho CpV}t + C$$
$$t = 0, \quad T = T_i \Longrightarrow \dot{T} = T - T_f = \dot{T}_i$$
$$C = ln\dot{T}_i$$

$$ln\dot{T} = \frac{-hA}{\rho CpV}t + ln\dot{T}_i \Longrightarrow ln\frac{\dot{T}}{\dot{T}_i} = -\frac{hA}{\rho CpV}t$$
$$\frac{\dot{T}}{\dot{T}_i} = e^{-\frac{hA}{\rho CpV}t} \Longrightarrow \frac{T - T_f}{T_i - T_f} = e^{-\frac{hA}{\rho CpV}t}$$
$$\phi = hA\left(T - T_f\right) = e^{-\frac{hA}{\rho CpV}t} \tag{3.23}$$

Fourier number :

$$F_0 = \frac{\lambda t}{\rho Cpl^2}$$
$$a = \frac{\lambda t}{\rho Cp} \Longrightarrow F_0 = \frac{at}{l^2}$$

a: thermal diffusivity of the material

$$\frac{T - T_f}{T_i - T_f} = e^{-\frac{hA}{\rho CpV}t} = e^{-\frac{htl}{\rho Cpl^2}} = e^{-\frac{hl}{\lambda}\frac{\lambda}{\rho Cpl^2}t}$$

$$\frac{T - T_f}{T_i - T_f} = e^{(-B_i F_i)}$$

4.3 Thermally thick body B i<0.1 [2,3]

Assuming that the heat transfer problem allows heat flow to be neglected in the y and z directions and furthermore that λ is constant.

The heat equation becomes:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{a}\frac{\partial T}{\partial t} \tag{3.24}$$

Let's look for a solution product of the

$$T(x,t) = X(x)G(t)$$

The previous equation becomes

$$\frac{1}{x}\frac{d^2X}{dx^2} = \frac{1}{a}\frac{1}{G}\frac{dG}{dt} = -\omega^2$$

So

$$\frac{d^2X}{dx^2} + \omega^2 X = 0 \quad et \quad \frac{dG}{dt} + a\omega^2 G = 0$$

With ω as the constant separating the variables.

The solution for X is

$$X = C_1 \cos(\omega x) + C_2 \sin(\omega t)$$

and for G

$$G = exp(-a\omega^2 t)$$

$$G = exp(-\omega^2 at)$$

Therefore, the geal solution is:

$$T(x,t) = [C_1 \cos(\omega x) + C_2 \sin(\omega t)]C_3 exp(-a\omega^2 t) \tag{3.25}$$

Chapter 4.

Convective heat transfer

4 Convective heat transfer
4.1 A reminder of dimensional analysis
4.1.1 Fundamental dimensions

Physical quantities can be expressed in terms of a limited number of fundamental dimensions. Examples: Velocity: $L \cdot T^{-1}$; dynamic viscosity: $M \cdot L^{-1} \cdot T^{-1}$; force: $M \cdot L \cdot T^{-2}$.
In these examples, the number of fundamental dimensions is 3: Mass (M), Length (L), and Time (T).
These three fundamental dimensions are not always sufficient. For heat transfer problems, it is necessary to add a 4th dimension: Temperature. And, when the exchange of energy between mechanical quantities and thermal quantities is not measurable, the quantity of heat (Q) will be added as a 5th dimension.
Note: Q, which is homogeneous to work and expressed in terms of the fundamental dimensions M, L, and T as $Q = M \cdot L \cdot T^{-2}$, is not a true fundamental dimension.
The method of dimensional analysis, based on the principle of dimensional homogeneity of the terms in an equation, is known as the Vasha-Buckingham theorem or the theorem of grouping [3,4].

4.1.2 Principle of the method

If 1 can mathematically represent a physical law by expressing a physical variable G1 as a function of a number of other independent physical variables G2, ...
., Gn, i.e. if G1 = f (G2, G3,..., Gn) or f (G1, G2,. , Gn) = 0, the problem can be simplified as follows simplified as follows:

- For each variable Gi, we write the dimension equation as a function of the fundamental dimensions. We then have n equations, which require p fundamental dimensions to characterize all the physical quantities.

- We take p of these n equations and consider them as basic equations. Although the choice of equations is arbitrary, each fundamental dimension must appear at least once in all p equations.

- The remaining (n-p) equations then take the form of (n-p) dimensionless ratios called groupings, which are "reduced quantities". The result is reduced to equation:

$$g(\pi_1, \pi_2, \dots \pi_{n-p}) = 0$$

A grouping is the ratio of a dimensional equation of a physical quantity not belonging to the set of basic equations to the product of the basic equations, each of which is raised to a certain power [3,4] :

$$\pi_i = \frac{[G_i]}{[G_1]^{a_i}[G_2]^{b_i} \dots [G_p]^{e_i}}$$

For each fundamental dimension M, L, T, , Q in the denominator, the exponents are summed and identified with the exponent of the same dimension in the equation for the dimension of the physical quantity in the numerator. The result is a linear system of p equations, whose resolution enables us to determine the p exponents of the basic equations in the denominator.

The ratio can then be written as a function of the physical quantities attached to the initial dimensional equations.

4.1.3 Application example

Let's consider a fluid in forced circulation in a cylindrical pipe, for which we want to determine the convection coefficient h for the fluid-wall heat transfer corresponding to forced convection [3, 4]:

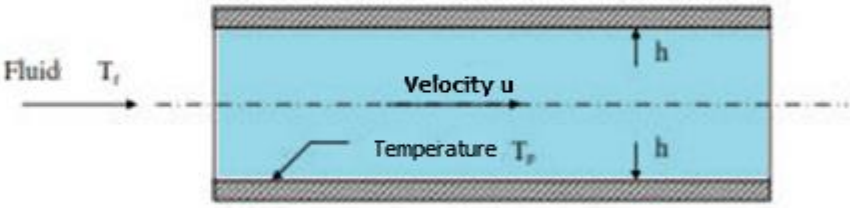


Figure 4.1: Diagram of the configuration studied

Determination of physical quantities:

All the parameters on which the heat flux density (linked to h Q) depends must be determined:
- Fluid characteristics:
- λ coefficient of thermal conductivity
- cp mass heat
- ρ density
- μ dynamic viscosity
- Flow characteristics
- u mean fluid velocity
- Exchange surface geometry
- D pipe diameter
- Wall-fluid temperature difference

Hence:$f(\lambda, Cp, \rho, \mu, u, D, \Delta T, \phi) = 0$
Dimensional equation for each quantity:

Next, we need to write the equation in the fundamental dimensions M, L, T, θ, Q of each of the quantities, which is written here:
λ : Q. T-1.L-1. -1
cp : Q.M-1. -1
ρ : M.L-3
μ : M.T-1.L-1 u : L.T-1
D : L
Φ : Q.T-1.L-2

Determining π-groupings:

We now need to choose 5 basic equations (All fundamental dimensions have been used) so that all 5 fundamental dimensions appear at least once in the set of equations.

Take, for example: λ, ρ, u, D, ΔT, leaving Φ, cp and μ.

We then write the 3 dimensionless ratios corresponding to these variables in the form:

$$\pi_1 = \frac{\varphi}{T^{a_1}\ \lambda^{b_1}\,\rho^{c_1}\ D^{d_1}\ u^{e_1}};\ \ \pi_2 = \frac{Cp}{T^{a_2}\ \lambda^{b_2}\,\rho^{c_2}\ D^{d_2}\ u^{e_2}};\ \pi_3 = \frac{Cp}{T^{a_3}\ \lambda^{b_3}\,\rho^{c_3}\ D^{d_3}\ u^{e_3}}$$

For each ratio, we replace the physical quantities by their dimensional equations, which gives for example for π_1:

$$[\pi_1] = \frac{QT^{-1}L^{-2}}{\theta^{a1}(QT^{-1}L^{-1}\theta^{-1})^{b1}(ML^{-3})^{c1}L^{d1}(LT^{-1})^{e1}}$$

For each fundamental dimension, we identify the power exponents between numerator and denominator relative to the same dimension, which leads to the system [3,4]:

(Q): $\quad 1 = b_1$
(T): $\quad -1 = -b_1 - c_1$
(L): $\quad -2 = -b_1 - 3c_1 + d_1 + e_1$
(θ): $\quad 0 = a1 - b1$
(M): $\quad 0 = c_1$

The ratio $\pi 1$ is therefore

$$\pi_1 = \frac{\phi D}{\Delta T \lambda}$$

This, with $\Phi = h\ \Delta\theta$, can still be written as:

$$\pi_1 = \frac{hD}{\lambda}$$

In the same way :

$$\pi_2 = \frac{\rho\, u\, D\ Cp}{\lambda} \qquad and \quad \pi_3 = \frac{\mu}{\rho\, u\, D}$$

Le théorème de Vaschy-Buckingam nous permet d'affirmer que la relation :

$$f(\lambda, Cp, \rho, \mu, u, D, \Delta T, \phi) = 0$$

Between 8 variables can be expressed using the three dimensionless numbers 1, 2 and 3 as:

$$f(\pi_1, \pi_2, \pi_3) = 0\ where\ \pi_1 = f(\pi_1, \pi_2)$$

Physical significance of these groupings:

$$\pi_1 = \frac{hD}{\lambda}$$

The Nusselt number, it can also be written as

$$Nu = \frac{\frac{D}{\lambda}}{\frac{1}{h}}$$

This is the ratio of conduction thermal resistance to convection thermal resistance. The higher the ratio, the more convection than conduction. It characterizes the type of heat transfer. It is the inverse of the Reynolds number, which characterizes the flow regime in the pipe.

$$\pi_3 = \frac{\mu}{\rho D u} = \frac{1}{Re}, \quad \pi_2 = \frac{\rho D u\ Cp}{\lambda}$$

It is the Peclet number; it is also 1'small to write:

$$Pe = \frac{\rho D u}{\mu}\frac{Cp\ \mu}{\lambda}$$

In addition, reveal a new dimensional number:

$$Pr = \frac{Cp\ \mu}{\lambda}$$

Called the Prandtl number. This number can be calculated for a given fluid independently of experimental conditions (it depends only on temperature) and characterizes the influence of the fluid's nature on convective heat transfer.

We therefore prefer to look for a relationship in the form [3, 4]:

$$\text{Nu} = f\ (\text{Re}, \text{Pr}) \tag{4.1}$$

4.1.4 Advantages of using reduced sizes

They essentially concern the representation, comparison and research of experimental results:

- The representation of the experimental results is simplified, we can have a curve linking 2 variables or an abacus linking 3 reduced variables instead of a relationship linking (3 + p) parameters.
- Comparison of experimental results is also very quick and easy, regardless of the researcher, even if the unit system used is different since the reduced quantities are dimensionless.
- The search for experimental results is made easier and orderly: if it is enough to draw a curve between two reduced variables, it is because it is enough to carry out a single series of experiments.

Notice :

However, it must be understood that the method of dimensional analysis which provides the reduced quantities does not give the form of the relationship which links them; the search for this relationship is the subject of the analysis of the experimental results.

Groupement	
$Re = \frac{\rho u D}{\mu}$	Nombre de Reynolds
$Pr = \frac{c_p\ \mu}{\lambda}$	Nombre de Prandtl
$Nu = \frac{h D}{\lambda}$	Nombre de Nusselt
$Pe = \frac{\rho u D c_p}{\lambda}$	Nombre de Peclet
$Ma = \frac{h}{\rho u c_p}$	Nombre de Margoulis
$Gr = \frac{\beta g \Delta T \rho^2 L^3}{\mu^2}$	Nombre de Grashof
$Ra = \frac{c_p \beta g \Delta T \rho^2 L^3}{\lambda \mu}$	Nombre de Rayleigh

Heat transfers that take place simultaneously with mass transfers are known as convective heat transfers. This mode of heat exchange exists in fluid media, where it is generally predominant.

Natural and forced convection

A distinction is made between natural and forced convection, depending on the nature of the mechanism that

causes the fluid to move:
- Free or natural convection: the fluid is set in motion solely by differences in density resulting from temperature differences at the boundaries and a field of external forces (gravity).
- Forced convection: fluid movement is induced by a cause independent of temperature differences (pump, fan, etc.).
The study of heat transfer by convection enables us to determine the heat exchanges taking place between a fluid and a wall.

Flow regime
Given the link between mass transfer and heat transfer, it is necessary to take into account the flow regime. As an example, let's consider the flow of a fluid in a pipe:

- In laminar flow, the fluid flows in practically indep

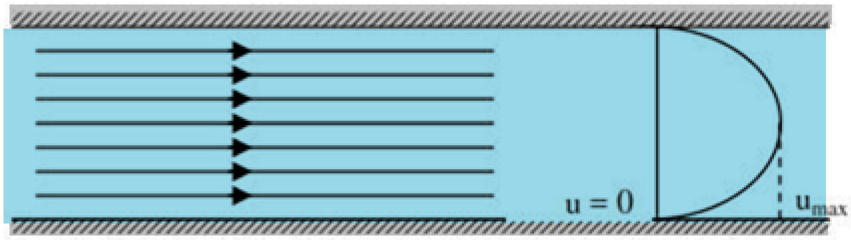


Figure 4.2: Laminar flow diagram

Heat exchange between two adjacent fluid threads therefore takes place :
- By conduction only if we consider a direction normal to the fluid threads.
- By convection and conduction (negligible) if the direction is not normal to the fluid threads.

- Turbulent flow is not unidirectional:

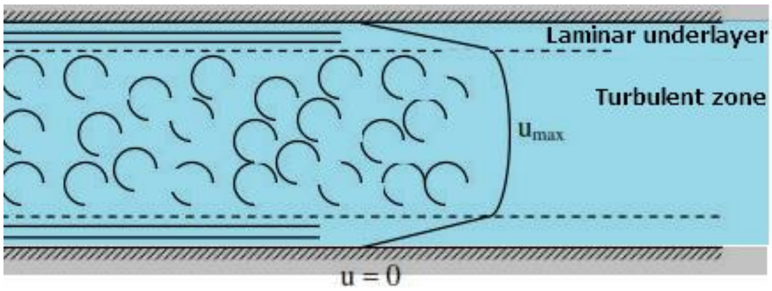


Figure 4.3: Diagram of a turbulent flow

Heat exchange in the turbulent zone takes place by convection and conduction in all directions. Molecular conduction is generally negligible compared with convection and "turbulent diffusion" (mixing of the fluid due to turbulent agitation) outside the laminar sublayer.

4.1.1 Heat flow expression

Reynolds analogy

Just as gas viscosity is explained at the molecular level by the transmission of molecular momentum during intermolecular shocks, heat transfer is explained by the transmission of kinetic energy during these same shocks.
This intimate connection between the phenomena of viscosity and heat transfer leads to the Reynolds analogy:

in a fluid flow with heat transfer, the velocity profile and the temperature profile are linked by a relationship of similarity schematized in figure 4.4. This similarity will be demonstrated later in the case of flow over a heated flat plate[3,4].

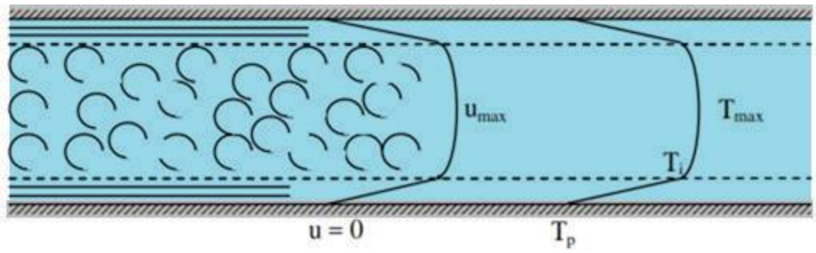


Figure 4.4: Representation of the Reynolds analogy for turbulent flow in a tube

Dynamic and thermal boundary layers

Whatever the flow regime, there remains a dynamic boundary layer in which the flow is laminar, and whose thickness is reduced as the Reynolds number increases. The thickness of this boundary layer
This boundary layer varies according to a number of parameters: fluid type, temperature, wall roughness, etc.
The Reynolds analogy shows that the thermal gradient is particularly steep in the vicinity of the wall, in a thermal boundary layer that develops analogously to the dynamic boundary layer. Whatever the fluid flow regime, thermal resistance is assumed to be entirely located in this thermal boundary layer, which acts as an insulator.
This corresponds to the Prandtl model shown in figure 4.5 as an example of turbulent fluid flow in a pipe.

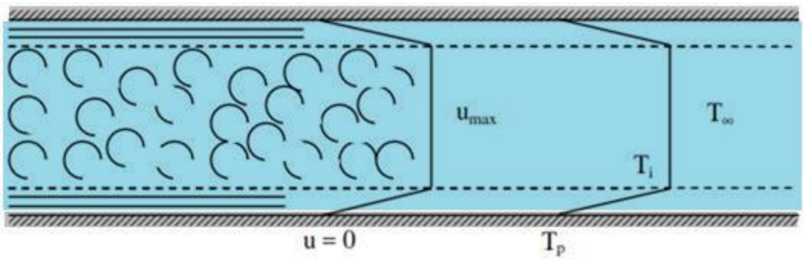


Figure 4.4: Prandtl model for turbulent flow in a pipe

Flow expression

Whatever the type of convection (free or forced) and whatever the fluid flow regime (laminar or turbulent), heat flow is given by Newton's law:

$$\Phi = h\ S\ \Delta\theta \quad (5.4)$$

The major problem to be solved before calculating the heat flow is to determine the convective heat transfer coefficient h, which depends on a large number of parameters: characteristics of the fluid, flow, temperature, shape of the exchange surface, etc.

Table 4.1 shows the order of magnitude of the convective heat transfer coefficient for different configurations [4,5].

Table 4.1: Order of magnitude of the convective heat transfer coefficient

Configuration	h (Wm^{-2} °C^{-1})
<u>Natural convection</u> In a gas In a liquid	 2-10 100-1000
<u>Forced convection</u> With a gas With a liquid	 10-200 100-5000
<u>Boiling water</u> In a container Flow through a tube	 2500-35000 5000-100000
<u>Condensation of water at 1 atm</u> On a vertical surface Outside horizontal tubes	 1000-11000 10000-25000

4.1.1 Calculating heat flow in forced convection [4,5] Exact calculation

In certain simple cases, a theoretical calculation can lead to an analytical expression for the heat flux exchanged by convection between a fluid and a wall. As an example, we'll deal here with the classic case of steady-state laminar flow of a fluid with constant physical properties at temperature T∞ over a flat wall of length L maintained at temperature Tp (see figure 4.6).

We can see that the fluid velocity evolves from a zero value at the wall to a value close to u∞ in a zone of thickness (x) called the dynamic boundary layer. Similarly, the fluid temperature evolves from a value of Tp at the wall to a value close to T∞ in a Zone of thickness (x) called the thermal boundary layer.

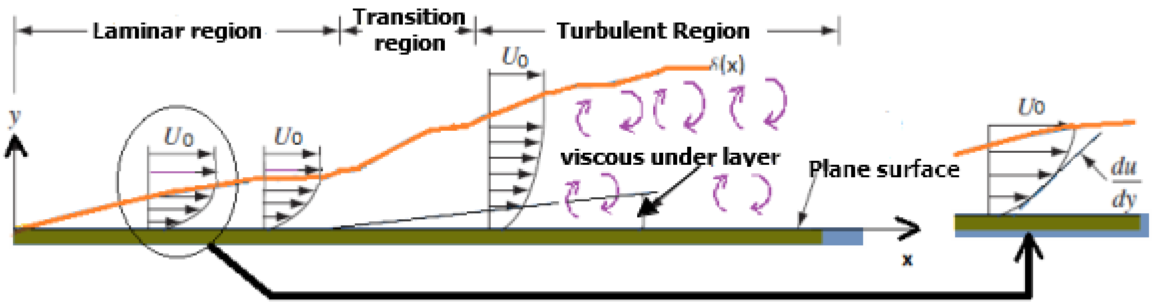


Figure 4.6: Schematic diagram of dynamic boundary layer development on a flat plate

The conservation of mass equation is written in integral form:

$$\int_{\Lambda} \frac{\partial \rho}{\partial t} dv + \int_{\Sigma} \rho \vec{V}.\vec{n} dS = 0$$

Where n is the external normal to ∑.

In steady state. Let's apply this relationship to the volume [abcd] shown in figure 4.6 :

$$\int_{\Sigma} \rho\vec{V}.\vec{n}dS = \underbrace{-\rho\left(\int_0^{\delta} u\,dy\right)_x}_{mass\ flow\ outgoing\ by\ ab} + \underbrace{\rho\left(\int_0^{\delta} u\,dy\right)_{x+dx}}_{mass\ flow\ outgoing\ by\ cd} + \underbrace{\int_{\mathrm{bc}} \rho\vec{V}_{\infty}.\vec{n}dA}_{mass\ flow\ outgoing\ by\ bc} = 0$$

We reduce to :

$$\int_{\mathrm{bc}} \rho\vec{V}_{\infty}.\vec{n}dS = -\rho\frac{d}{dx}\left(\int_0^{\delta} u\,dy\right)dx$$

The equation for conservation of momentum in the steady state (Euler's Theorem) is written:

$$\int_{\Sigma} \rho\vec{V}(\vec{V}.\vec{n})dA = \int_{\Lambda} \rho\vec{f}dv + \int_{\Sigma} \rho\vec{f}dv = \int_{\Sigma} \vec{T}dA \quad ; \quad f = 0$$

Where T are the external forces (per unit area) exerted by contact on the faces of the surface delimiting the volume. Let’s apply this relationship to the volume [abcd]:

$$\int_{\Sigma} \rho\vec{V}(\vec{V}.\vec{n})dA = -\rho\left(\int_0^{\delta} \vec{V}u\,dy\right)_x + \rho\left(\int_0^{\delta} \vec{V}u\,dy\right)_{x+dx} + \int_{\mathrm{bc}} \rho\vec{V}_{\infty}(\vec{V}_{\infty}.\vec{n})dA$$

Projection on (Ox):

$$\int_{\Sigma} \rho u(\vec{V}.\vec{n})dA = -\rho\left(\int_0^{\delta} u^2\,dy\right)_x + \rho\left(\int_0^{\delta} u^2\,dy\right)_{x+dx} + \int_{\mathrm{bc}} \rho u_{\infty}(\vec{V}_{\infty}.\vec{n})dA$$

$$\int_{\Sigma} \rho u(\vec{V}.\vec{n})dA = -\rho\left(\int_0^{\delta} u^2\,dy\right)_x + \rho\left(\int_0^{\delta} u^2\,dy\right)_{x+dx} + \rho u_{\infty}\int_{\mathrm{bc}} (\vec{V}_{\infty}.\vec{n})dA$$

Let's analyze the forces at work along Ox:

- On [ad] there is parietal friction
- On [bc], since the velocity profile is uniform, there is no friction
- There are no pressure forces, since pressure is uniform in the flow, we can therefore write:

$$\int_{\Sigma} \vec{T}dA = -\tau_p dx = \rho\frac{d}{dx}\left(\int_0^{\delta} u^2 dy\right)dx - \rho u_{\infty}\frac{d}{dx}\left(\int_0^{\delta} u dy\right)dx; \qquad \int_{\Sigma} \rho\vec{V}(\vec{V}.\vec{n})dA = \int_{\Sigma} \vec{T}dA$$

We deduce :

$$\frac{\tau_p}{\rho u_{\infty}^2} = \frac{d}{dx}\left[\delta\int_0^1 \frac{u}{u_{\infty}}\left(1 - \frac{u}{u_{\infty}}\right)d\left(\frac{y}{\delta}\right)\right] \qquad (a)$$

We look a priori for the speed in the simple forme of a parabolic profile:

$$\frac{u}{u_{\infty}} = a + b\left(\frac{y}{\delta}\right) + c\left(\frac{y}{\delta}\right)^2$$

The speed is zero at the wall: $u(y = 0) = 0$

The continuity of speed and friction at the boundary of the following conditions:

$$u(y = \delta) = u_\infty$$

$$u\frac{du}{dy}(y = \delta) = 0$$

We deduce that:

$$\frac{u}{u_\infty} = \frac{y}{\delta}\left(2 - \frac{y}{\delta}\right) \qquad (b)$$

And $\tau_p = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = 2\mu\frac{u_\infty}{\delta}$

Relations (a) and (b) lead to: $\frac{2\mu}{V_\infty \delta} = \frac{2}{15}\frac{d\delta}{dx}$

$$\delta^2 = 30\frac{\mu\, x}{\rho u_\infty};\ \left(\frac{\delta}{x}\right)^2 = 30\frac{\mu}{\rho u_\infty\, x} = \frac{30}{Re_x}$$

Then by integration

$$\tau_p = 2\mu\frac{u_\infty}{\delta} = 2\mu u_\infty\sqrt{\frac{\rho u_\infty}{30\mu\ x}}$$

This gives the expressions for the coefficient of friction:

$$C_{fx} = \frac{\tau_p}{\frac{1}{2}\rho {u_\infty}^2} = \frac{2\mu u_\infty\sqrt{\frac{\rho u_\infty}{30\mu\ x}}}{\frac{1}{2}\rho {u_\infty}^2}$$

The result is: $C_{fx} = \frac{0.73}{{Re_x}^{0.5}}$

A more precise analysis (local equations and no assumptions on the shape of the velocity profile) would lead to a constant of 0.664 instead of 0.73.

At constant pressure, the enthalpy variation of a system is equal to the heat supplied to it. Applying this principle to a volume ( ) of surface ( ) and neglecting viscous dissipation (the internal heat source corresponding to the degradation of mechanical energy into heat), we obtain:

Where H is the enthalpy of the fluid and q is the heat flux density vector.

$$\int_\Lambda \frac{\partial}{\partial t}(\rho H)dV + \int_\Sigma \left(\rho H\vec{V} + \vec{q}\right).\vec{n}\, dA$$

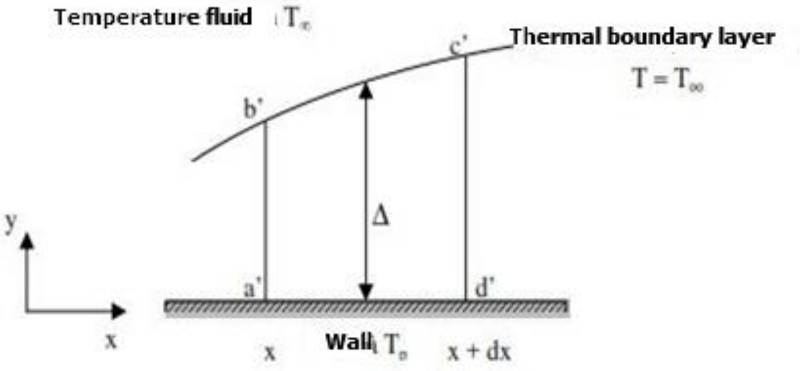


Figure 4.7: Diagram of the thermal boundary layer on a flat plate

Let's apply this steady-state relationship to the volume (a'b'c'd') shown in figure 5.6for a fluid such that $H = Cp(T - T_0)$, (The conductive heat flux density is zero on the surface (b'c'), since outside the thermal imitation layer the temperature is uniform and equals T_x.
on the other hand, the longitudinal heat flow (alongOx) is neglected in front of the transverse heat flow (alongOy).the temperature varies much more rapidly in theOy direction than in theOx direction (boundary layer hypothesis):

$$\int_{b'c'} \rho\vec{V}\vec{n}dA = \rho\int_{b'c'} u\,dA = -\rho\frac{d}{dx}\left(\int_0^{\Delta} u\,dy\right)$$

Where:

$$\frac{d}{dx}\left(\rho Cp\int_0^{\Delta} u\,T\,dy\right)dx - \rho CpT_{\infty}\frac{d}{dx}\left(\int_0^{\Delta} u\,dy\right)dx - q_p dx = 0$$

$$\frac{q_p}{\rho Cp\ \ u_{\infty}(T_p - T_{\infty})} = +\frac{d}{dx}\left[\Delta\int_0^1 \frac{u}{u_{\infty}}\left(1 - \frac{T - T_p}{T_{\infty} - T_p}\right)d\left(\frac{y}{\Delta}\right)\right] \qquad (c)$$

$$\frac{T - T_p}{T_{\infty} - T_p} = a + b\left(\frac{y}{\Delta}\right) + c\left(\frac{y}{\Delta}\right)^2$$

A priori, we look for the temperature in the form:

$$\frac{T - T_p}{T_{\infty} - T_p}(y = 0) = 0$$

$$\frac{T - T_p}{T_{\infty} - T_p}(y = \Delta) = 1$$

$$\lambda\frac{dT}{dy}(y = \Delta) = 0$$

We deduce that:

$$\frac{T - T_p}{T_{\infty} - T_p} = \frac{y}{\Delta}\left(2 - \frac{y}{\Delta}\right) \qquad (d)$$

$$q_p = -\lambda\left(\frac{\partial T}{\partial y}\right)_{y=0} = 2\lambda\frac{T_p - T_{\infty}}{\Delta}$$

Relations (c) and (d) allow us to write:

$$\frac{q_p}{Cp\, u_\infty (T_p - T_\infty)} = +\frac{d}{dx}\left[\Delta \int_0^1 \frac{y}{\delta}\left(2 - \frac{y}{\delta}\right)\left(1 - \frac{y}{\Delta}\left(-2\frac{y}{\Delta_p}\right)\right) d\left(\frac{y}{\Delta}\right)\right]$$

Where $\Delta < \delta$ and $r = \frac{\Delta}{\delta} \leq 1$,

The new relation is:

$$\Delta \frac{d\Delta}{dx} = \frac{12\lambda}{\rho\, Cp\, u_\infty\, r\left(1 - \frac{r}{5}\right)}$$

After the integration:

$$\left(\frac{\Delta}{x}\right)^2 = \frac{24}{Re_x\, Pr} \frac{1}{r\left(1 - \frac{r}{5}\right)}$$

$$\left(\frac{\delta}{x}\right)^2 = \frac{30}{Re_x}$$

$$r^2 = \frac{4}{5\,\mathrm{Pr}\left(1 - \frac{r}{5}\right) r} \qquad (e)$$

In the case Pr = 1, the solution to equation (e) is r = 1, the dynamic and thermal boundary layers have the same thickness, and there is complete analogy between heat and momentum transfer. This is the case for gases for which Pr ≈ 1.

The case r <1 corresponds to the case Pr > 1, which is the case of water, for example (Pr ≈ 7). An approximate solution of in more complex cases, where an analytical solution cannot be established, we use correlations deduced from experiments.

Bibliographic references:

[1] Donald Pitts, “Theory and problems of heat transfer”, second edition, Schaum’s, Mc Graw-Hill,1998.

[2] Jean-Luc Battaglia, Andrzej Kusiak, Jean-Rodolphe Puiggali, « aux transferts thermiques: Cours et exercices corrigés », Dunod, 2014.

[3] Michael J. Moran, “Introduction to thermal Systems Engineering: Thermodynamics, Fluid Mechanics, and Heat Transfer”, John Willey & Sons Inc. 2003.

[4] Yves JANNOT "TRANSFERTS THERMIQUES", Ecole des Mines Nancy 2éme année2012

[5] G. BRUHAT, Thermodynamique, Edition Masson

J.P. BARDON, Temperatures de surface, Notions fondamentales (R2730) Edition Techniques de l'ingenieur

[6] C. CHAVSSIN, C. HILLY et J. BARRALS Chaleur et thermodynamique, EditionLavoisier

[7] O. PERROT, Cours de rayonnement, 3eme Semestre, departernent Genie

[8] Thermique et energie, LV.T. de Saint-Orner Dunkerque.

[9] Devendra Gupta, “Diffusion processes in advanced Technological Materials”.

[10] Y. Adda, J.M. Dupouy, J. Philibert et Y. Quéré, "élements de métallurgie physique, tome 4, Diffusion, transformations", Chap. 28 (2e édition, INSTN), 1990.

[11] D.W. Richardson, “Modern Ceramic Engineering”, (Marcel Dekker).

[12] Diffusion dans les métaux’ par Pierre GUIRALDENQ Professeur des universités à l’École Centrale de Lyon (ECL) et au Conservatoire National des Arts et Métiers (Centre Associé de Lyon) Directeur de Recherches à l’ECL